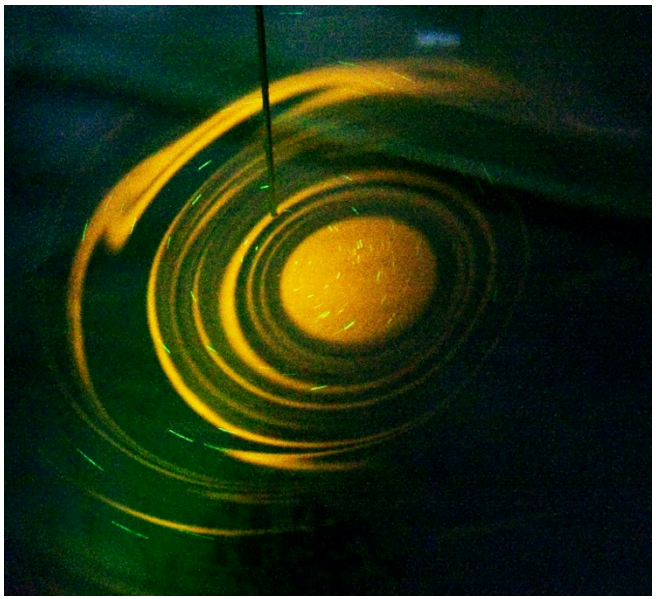
Vortices in rotating stratified flows: aspect ratio and sustainability



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Vortices in rotating stratified flows

Rotation vs stratification

Rotation homogeneizes the flow along the axis of rotation

$$f = 2\,\Omega \sin\phi$$

Coriolis parameter

Stratification limits the motion in the direction of the density gradient

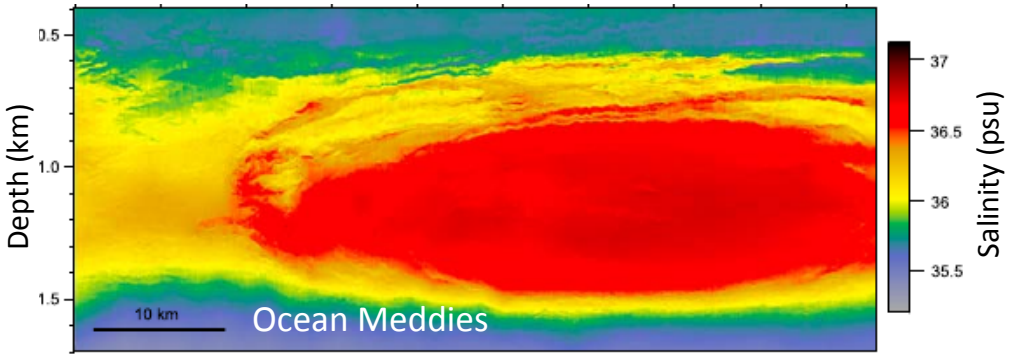
$$\bar{N} = \sqrt{-\frac{g}{\rho_0}\frac{d\bar{\rho}}{dz}}$$

Brunt-Väisälä frequency

Recurrent long-lived anticyclonic pancake vortices:
in atmospheres, oceans, proto-planetary disks…

NASA/JPL

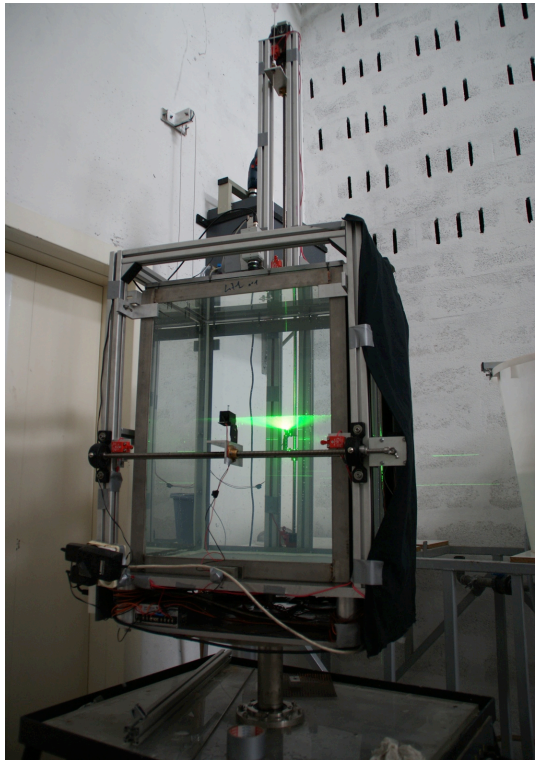
f ≈ 1.4 10^{-4} rad/s, N ≈ 0.016 rad/s

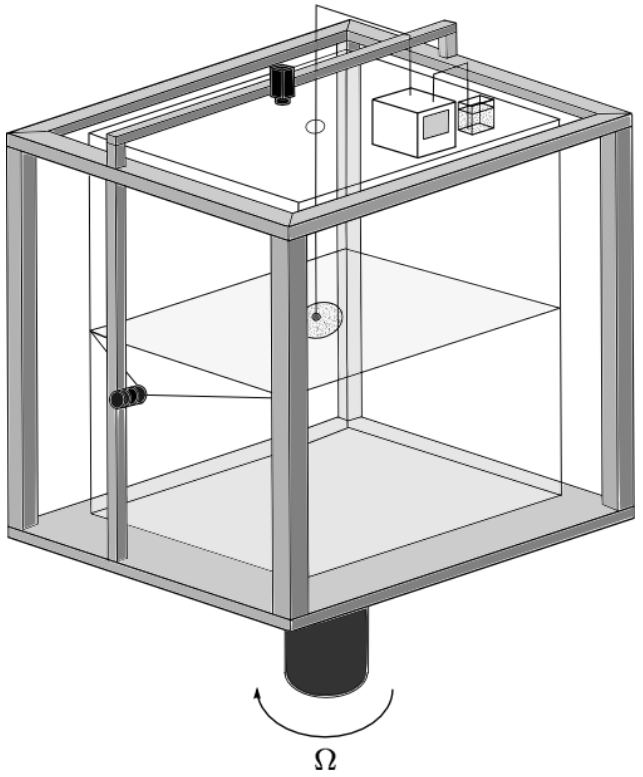


Papenberg *et al.,* 2011

f ≈ 8 10^{-5} rad/s, N ≈ 0.003 rad/s

An experimental study





Parameters

- Rotating table: f = 1 to 7 rad/s
- Linearly stratified salty water: N = 0.6 to 2.4 rad/s

Rossby number

$$Ro = \frac{V}{L\,f} = \frac{\Omega_v}{2\,\Omega}$$

Initial condition

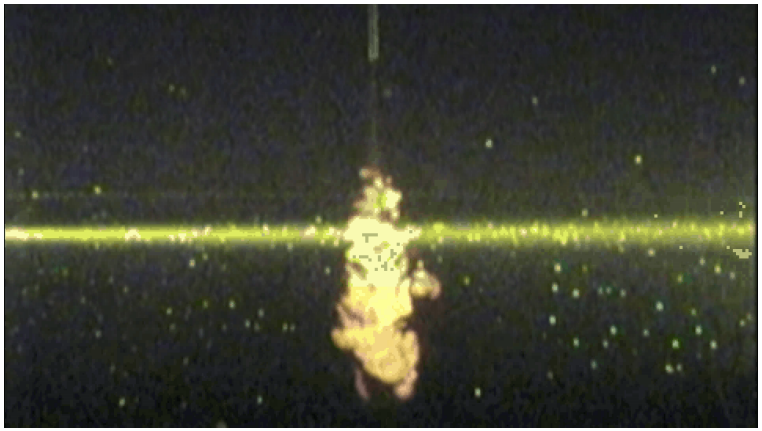
$$\Omega_v(0) = -\Omega \longrightarrow Ro(0) = -0.5$$

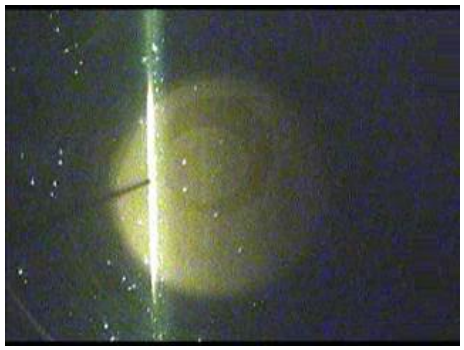
An experimental study: injection methods

Brief injection:

freely-decaying vortex

Decay of Ro

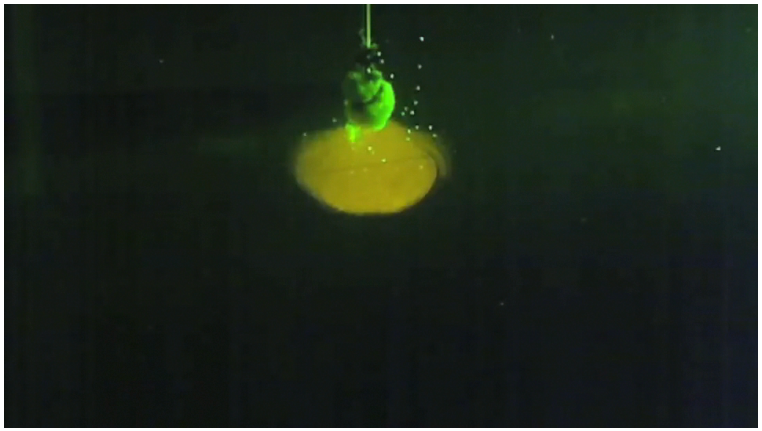


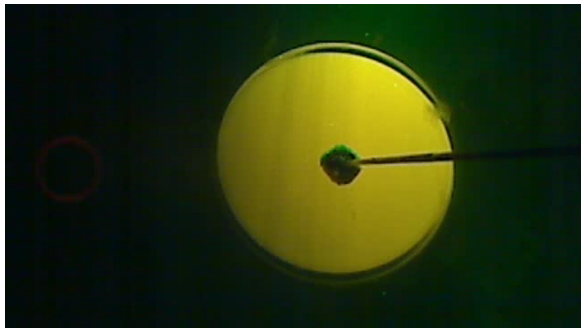


Continuous injection:

sustained vortex

Ro remains high



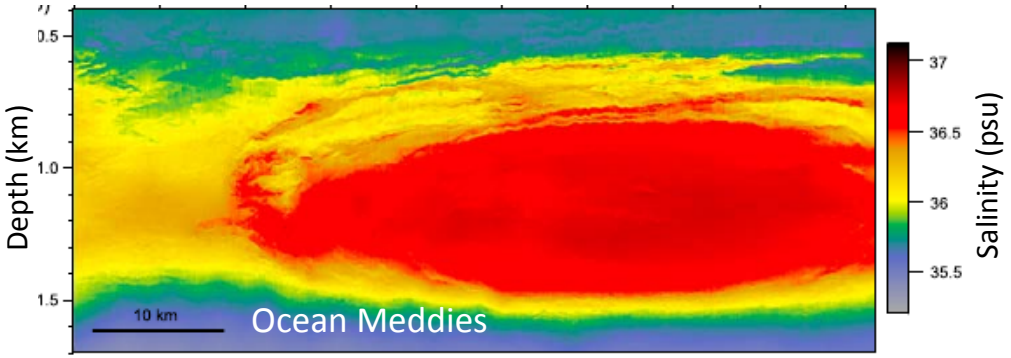


Questions/studies

Vortices in a rotating stratified flow that are big enough to sense the Coriolis force

- **Shape and aspect ratio**
- **Why anticyclones?**
- **Lifetime**
- **Layering**

NASA/JPL



Papenberg *et al.,* 2011

Law for the aspect ratio of vortices in rotating stratified fluids

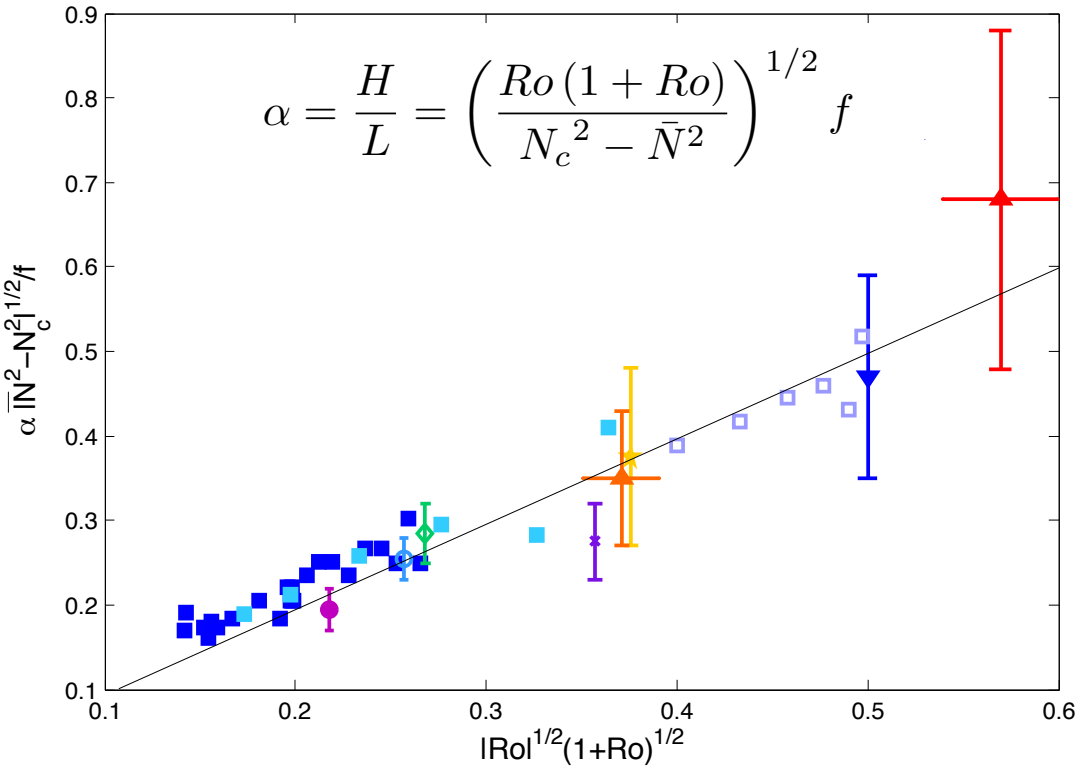
- **Shape and aspect ratio**
- **Why anticyclones?**
- **Lifetime**
- **Layering**

cyclo-geostrophic and hydrostatic balances

$$\begin{cases} \dfrac{v^2}{r} + f\,v = \dfrac{1}{\rho_0}\dfrac{\partial p'}{\partial r} \\ 0 = \dfrac{\partial p'}{\partial z} + g\,\rho' \end{cases}$$

thermal-wind equation

$$\longrightarrow \quad \frac{\partial v}{\partial z}\left(\frac{2v}{r} + f\right) = -\frac{g}{\rho_0}\frac{\partial \rho'}{\partial r}$$



Hassanzadeh *et al.*, JFM, 2012
Aubert *et al.*, JFM, 2012

Laboratory vortices
- freely-decaying
- sustained
- Bush&Woods (JFM,1999)

Meddies
- Ceres
- Hyperion
- Encelade
- Sharon
- Bobby

Jovian vortices
- Oval BA
- Great Red Spot

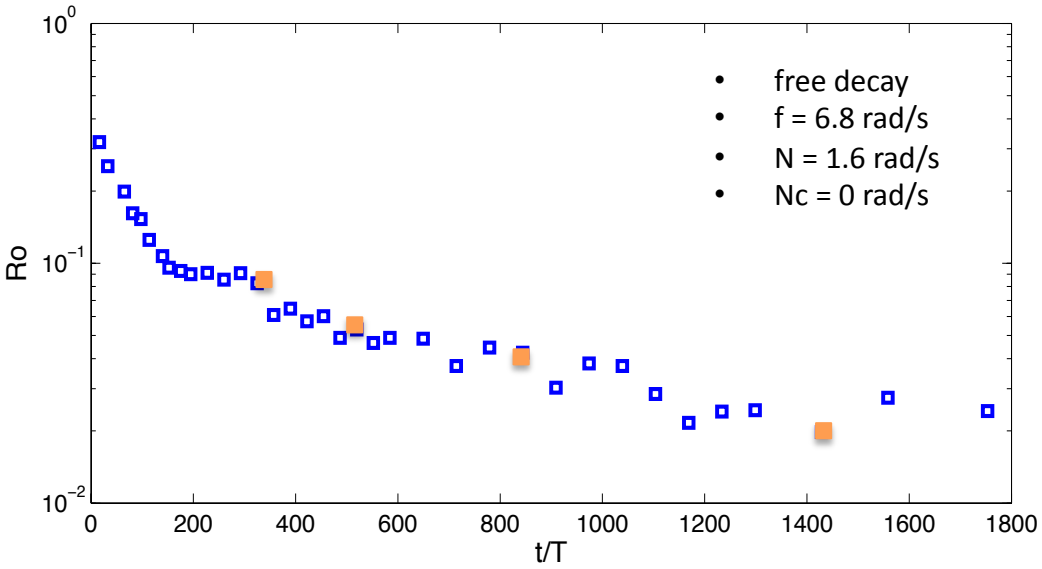
Self-similar shape at low Rossby number

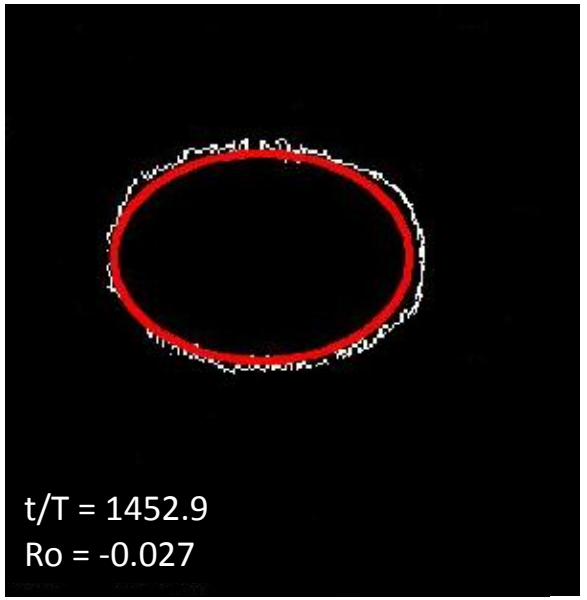
- **Shape and aspect ratio**
- **Why anticyclones?**
- **Lifetime**
- **Layering**

From a simple model of a homogeneous vortex in solid-body rotation:

$$p_{00} = -\frac{\rho_0}{2} f^2 \, Ro \, (1 + Ro) \, r_{cont}^2 + \frac{\rho_0}{2} ({N_{out}}^2 - {N_{in}}^2) \, z_{cont}^2$$

with $$p_{00} = \frac{\rho_0}{4} \left(\frac{9}{2} \frac{V^2 f^4 Ro^2 (1 + Ro)^2 {N_{out}}^2}{\pi^2} \right)$$





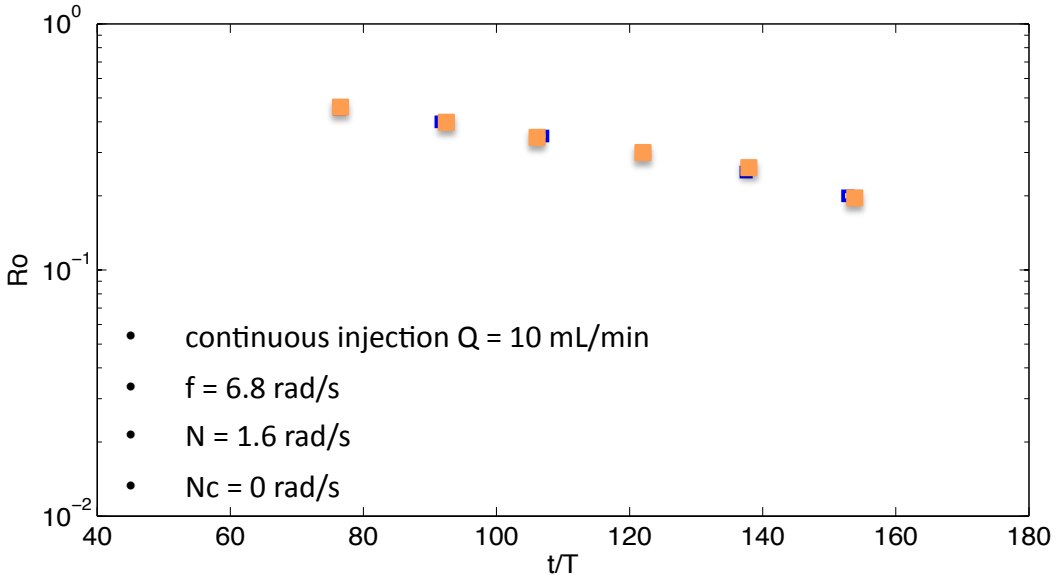
Self-similar shape at high Rossby number

- **Shape and aspect ratio**
- **Why anticyclones?**
- **Lifetime**
- **Layering**

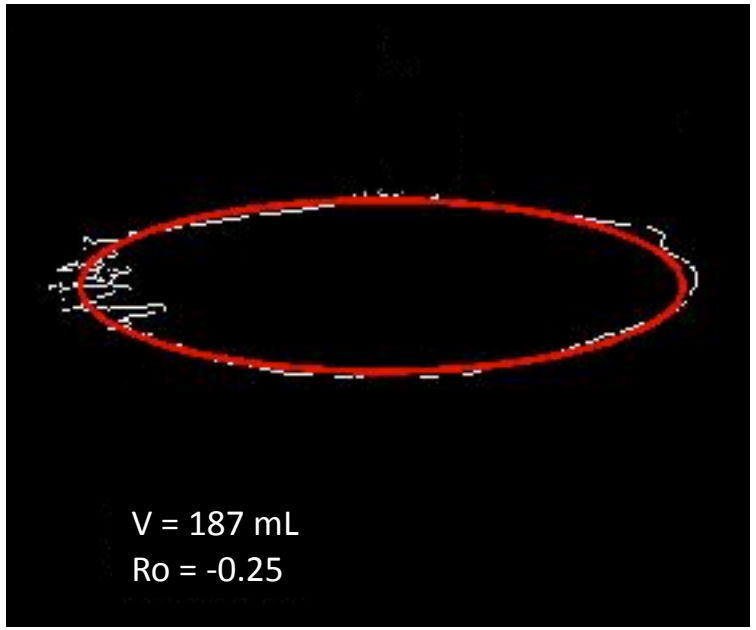
From a simple model of a homogeneous vortex in solid-body rotation:

$$p_{00} = -\frac{\rho_0}{2} f^2\, Ro\,(1+Ro)\, r_{cont}^2 + \frac{\rho_0}{2}({N_{out}}^2 - {N_{in}}^2)\, z_{cont}^2$$

with $$p_{00} = \frac{\rho_0}{4}\left(\frac{9}{2}\frac{V^2 f^4 Ro^2 (1+Ro)^2 {N_{out}}^2}{\pi^2}\right)$$



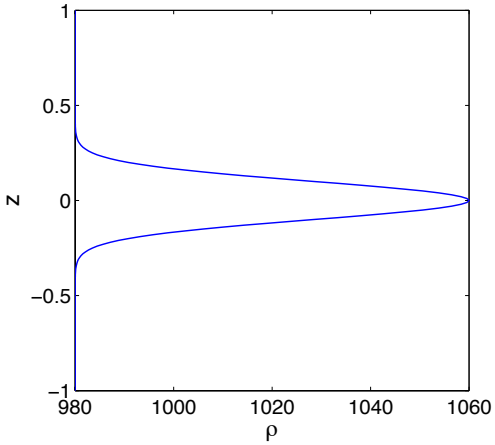
Cyclo-geostrophic equilibrium



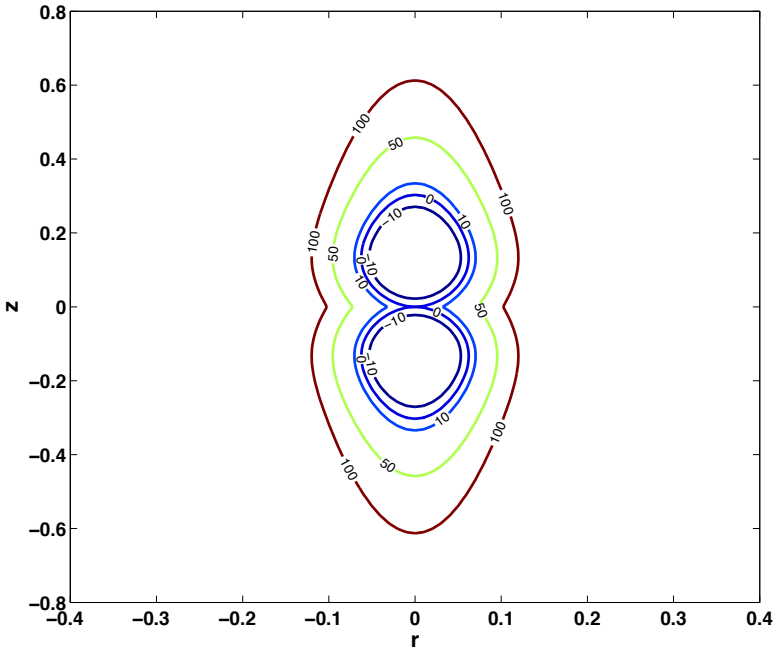
Application to vortices in PPD

- **Shape and aspect ratio**
- **Why anticyclones?**
- **Lifetime**
- **Layering**

From a linear to a Gaussian stratification:



- Self-similar shape
- Vortices split above and below the mid-plane



+ effect of shear:

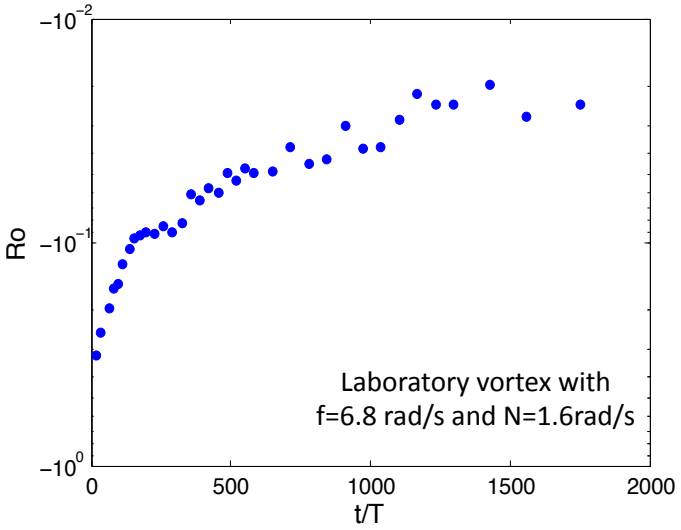
⟶ favours anticyclones

⟶ elliptic shape

Energy conservation law of a freely-decaying vortex

- Shape and aspect ratio
- Why anticyclones?
- **Lifetime**
- Layering

Laboratory vortices lifetime



Meddies lifetime

more than 2 years…

Time evolution

$$\begin{cases} \dfrac{\partial v}{\partial t} + (\mathbf{u}.\nabla)\, v + f\, u = \nu \nabla^2 v \\ \dfrac{\partial \rho'}{\partial t} + (\mathbf{u}.\nabla)\, \rho' = \dfrac{\rho_0\, \bar{N}^2}{g} w \end{cases}$$

Kinetic energy

$$KE = \frac{1}{2} \rho_0\, v^2$$

Available potential energy

$$APE = \rho'\, g\, z$$

Energy budget of a vortex

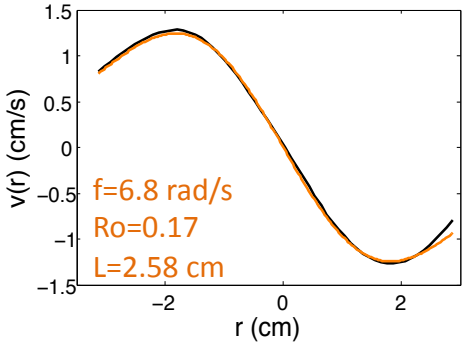
$$\frac{\partial}{\partial t} \int_V (KE + APE)\, \mathrm{d}V + \oint_S (KE + APE)\, \mathbf{u}.\mathbf{dS} + \int_V (3\, KE + APE)\, \frac{u}{r}\, \mathrm{d}V$$

$$= \eta \int_V v\, \nabla^2 v\, \mathrm{d}V + \rho_0\, \bar{N}^2 \int_V w\, z\, \mathrm{d}V - \rho_0\, f \int_V u\, v\, \mathrm{d}V + g \int_V w\, \rho' \mathrm{d}V$$

Modeling and lifetime

- Shape and aspect ratio
- Why anticyclones?
- **Lifetime**
- Layering

PIV

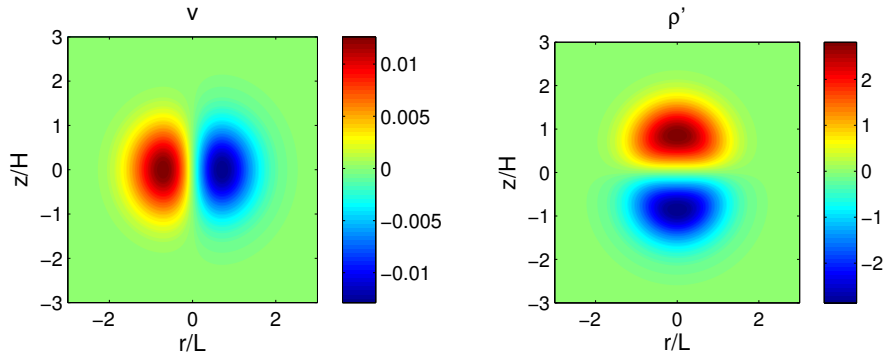


Gaussian vortex

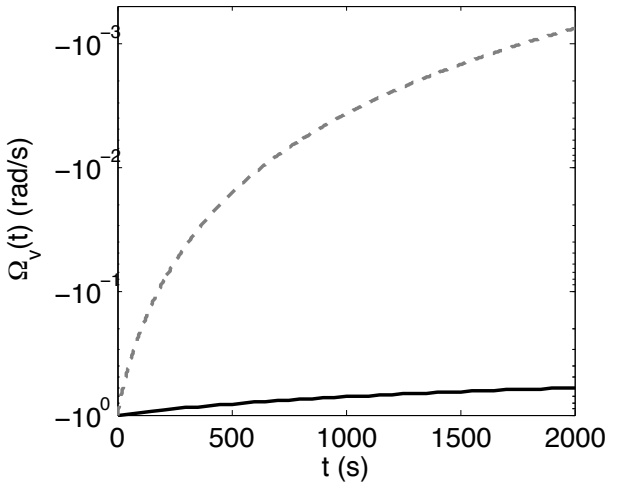
solution of the thermal-wind equation:

$$\begin{cases} v(r,z) = f\,Ro\,r\,e^{-(r/L)^2-(z/H)^2} \\ \rho'(r,z) = -\frac{\rho_0}{g}\left(\frac{L}{H}\right)^2 f^2 Ro\,z\,e^{-(r/L)^2-(z/H)^2} \end{cases}$$

$$(1 + Ro\,e^{-(r/L)^2-(z/H)^2})$$



Influence of rotation vs analytical solution



rotation of the background (f=2 rad/s)
no rotation of the background (f=0 rad/s)

- Shape and aspect ratio
- Why anticyclones?
- **Lifetime**
- Layering

Lifetime

- Density anomaly
- Restoring Coriolis force

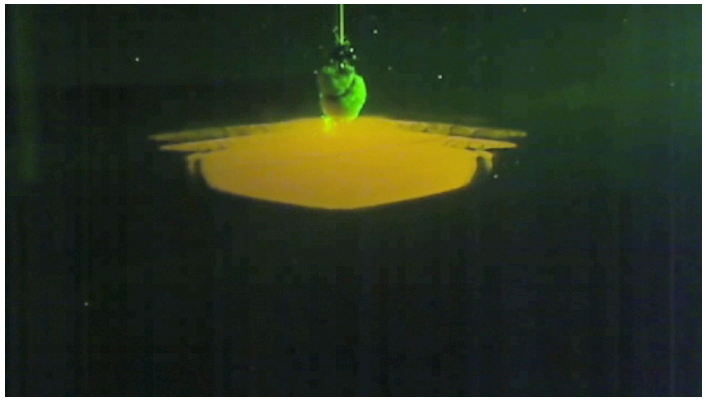
Vortices in PPD -and GRS- (sustained vortices)

+ effect of shear ⟶ sustaining torque

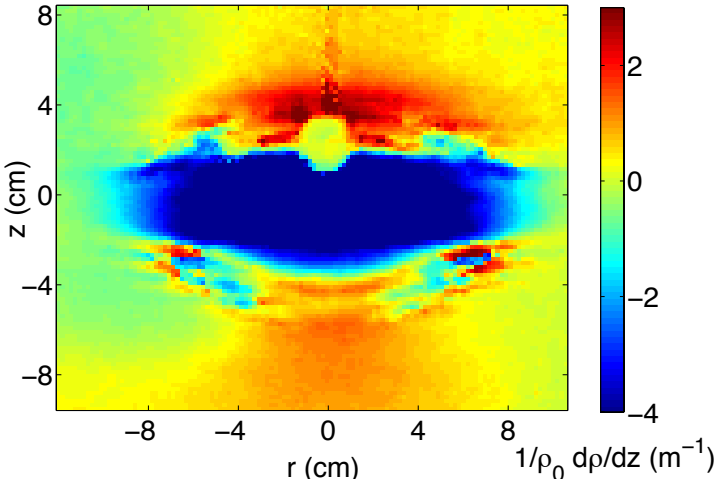
Layering phenomenon of sustained vortices

- Shape and aspect ratio
- Why anticyclones?
- Lifetime
- **Layering**

Experimental investigation

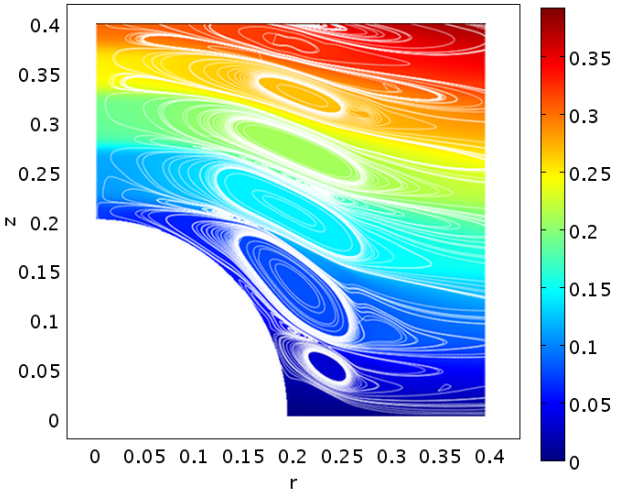


f = 2.7 rad/s, N = 2.2 rad/s



f = 3.6 rad/s, N = 1.8 rad/s

Numerical investigation



- Shape and aspect ratio
- Why anticyclones?
- Lifetime
- **Layering**

McIntyre's double-diffusive instability

Double-diffusive instabilities of salty water

competition between two diffusive phenomena, one much faster than the other

$$Pr = \frac{\nu}{\alpha} = \frac{\text{viscous diffusion rate}}{\text{thermal diffusion rate}} \approx 7$$

$$\sigma = \frac{\nu}{D} = \frac{\text{viscous diffusion rate}}{\text{molecular diffusion rate}} \approx 700$$

McIntyre's intability, 1970

$$\frac{\sigma}{Pr} = \frac{\alpha}{D} = \frac{\text{thermal diffusion rate}}{\text{molecular diffusion rate}} \approx 100$$

Application to vortices in PPD?

Conclusions and work in progress

Vortices in rotating stratified fluids

- **Shape and aspect ratio**
- **Why anticyclones?**

$$\alpha = \frac{H}{L} = \left(\frac{Ro\,(1+Ro)}{{N_c}^2 - \bar{N}^2} \right)^{1/2} f$$

  - Self-similar shape
  - Experimental study of (superstratified) cyclones

- **Lifetime** → Reservoirs of energy (density anomaly) and restoring Coriolis force

- **Layering** → McIntyre's double-diffusive instability
  - Systematic experimental study
  - Application to Meddies