Planet-Disk Interaction revisited

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1. Introduction

We present results on our investigations of planet–disk interaction in protoplanetary disks. For the hydrodynamic simulations we use a second order semi–discrete total variation diminishing (TVD) scheme for systems of hyperbolic conservation laws on curvilinear grids.
Our previously used method conserves the momentum in two dimensional systems with rotational symmetry. Additionally, we modified our simulation techniques for inertial angular momentum conservation even in two dimensional rotating polar coordinate systems.
The basic numerical practices are outlined briefly. We show relevant test cases for the conservation of angular momentum in typical setups. In addition we present the results of a common planet–disk interaction setup.

2. 2D advection solver FOSITE

FOSITE (Illenseer and Duschl (2009), fosite.sf.net) implements second order semi-discrete central schemes for systems of hyperbolic conservation laws on curvilinear grids. Let (ξ, η, φ) be the coordinates of such a grid with the geometrical scaling factors h_ξ, h_η, h_φ. It is convenient to define new spatial differential operators

$$\mathcal{D}_\xi = \frac{1}{\sqrt{g}}\frac{\partial}{\partial \xi} h_\eta h_\varphi \qquad \mathcal{D}_\eta = \frac{1}{\sqrt{g}}\frac{\partial}{\partial \eta} h_\xi h_\varphi$$

using the $\sqrt{g} = h_\xi h_\eta h_\varphi$. Systems of hyperbolic conservation laws on curvilinear grids can than be described by

$$\partial_t u + \mathcal{D}_\xi F(u) + \mathcal{D}_\eta G(u) = S(u).$$

The numerical scheme used by FOSITE generalizes the two-dimensional central-upwind schemes developed by Kurganov and Tadmor (2000). Geometrical source terms of vectorial conservation laws are formulated in a general prescription for various orthogonal curvilinear coordinates.
In contrast to many other astrophysical softwares used for planet-disk interaction simulations (e.g. FARGO by Masset (2000), RODEO by Paardekooper and Mellema (2006) and others in de Val-Borro et al. (2006), as well as RAPID by Mudryk and Murray (2009)), FOSITE solves the Euler equations in one step without depending on techniques such as operator splitting.
In conclusion we abstract the most import features of FOSITE:

- finite volume scheme for hyperbolic conservation laws
- semi discrete: second order in space, up to fifth order in time
- total variation diminishing with a variety of flux limiters
- arbitrary orthogonal curvilinear grids
- upwind: accounts for propagation of information
- Fortran 95, object-oriented design, GPL
- parallelised using MPI, vectorized for NEC SX8/9
- outputs: ASCII, gnuplot, VTK, netcdf, hdf5, binary
- integrated python based plotting framework

3. Inertial angular momentum transport

If exact conservation of angular momentum in the inertial frame is of great interest, it is possible to reformulate the Euler equations for transport of inertial angular momentum l. This includes the Coriolis and centrifugal forces in the rotating frame of reference with angular velocity Ω:

$$l = h_\eta (v_\eta + h_\eta \Omega)$$

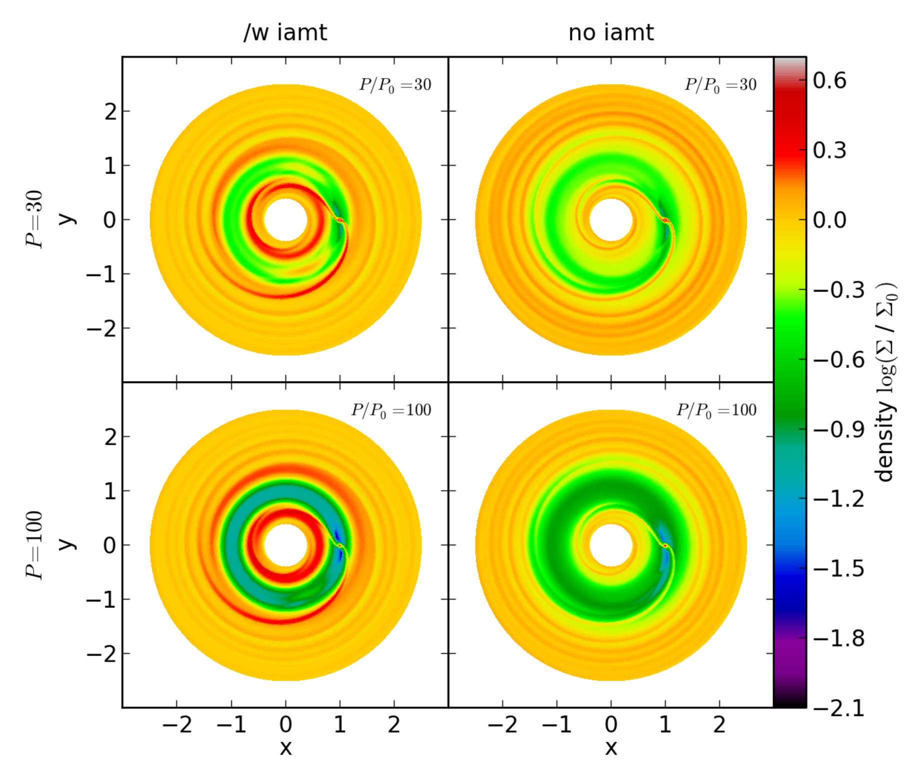
Here h_η is the geometrical scaling factor along the second coordinate, e.g. for polar coordinates $h_\eta = r$. This leads to a new system of hyperbolic conservation laws, which imply exact conservation of mass and inertial angular momentum. Using the local isothermal speed of sound approximation yields $p = \rho c_s^2$ for the pressure, we derive:

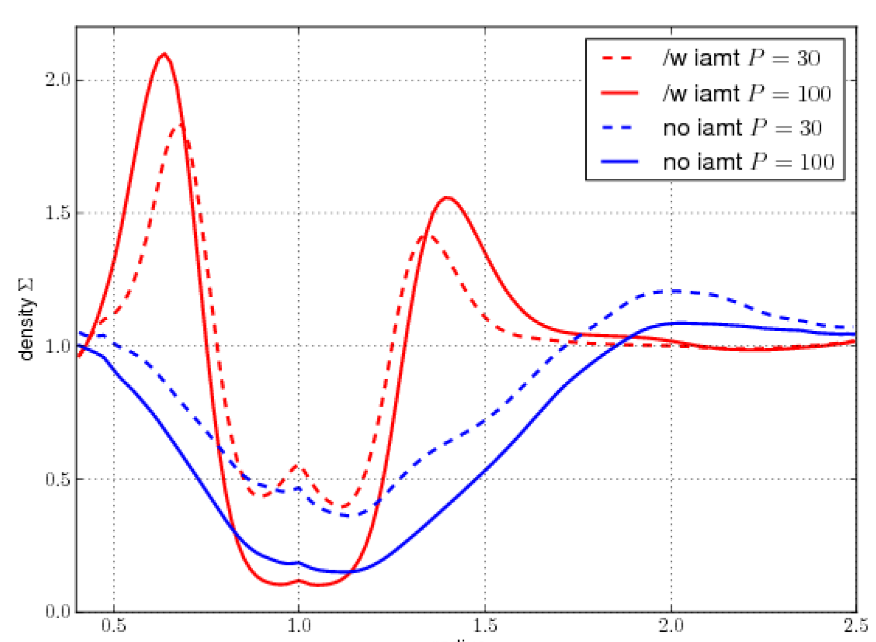
$$u = \begin{bmatrix} \rho \\ \rho v_\xi \\ \rho l \end{bmatrix}, \quad F = \begin{bmatrix} \rho v_\xi \\ \rho v_\xi^2 + p \\ \rho v_\xi l \end{bmatrix}, \quad G = \begin{bmatrix} \rho v_\eta \\ \rho v_\xi v_\eta \\ \rho v_\eta l + h_\eta p \end{bmatrix},$$

$$S = \begin{bmatrix} 0 \\ \rho h_\eta \left(\frac{v_\eta}{h_\eta} + \Omega\right)^2 + p \frac{1}{h_\eta}\frac{\partial h_\eta}{\partial \xi} \\ 0 \end{bmatrix}.$$

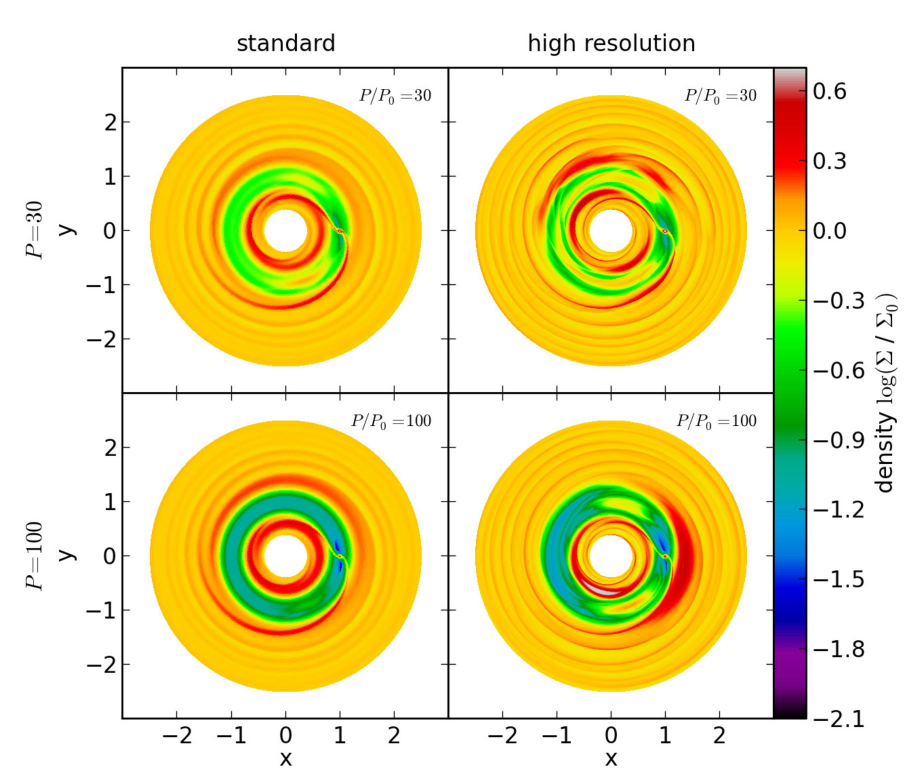
4. Planet-disk interactions

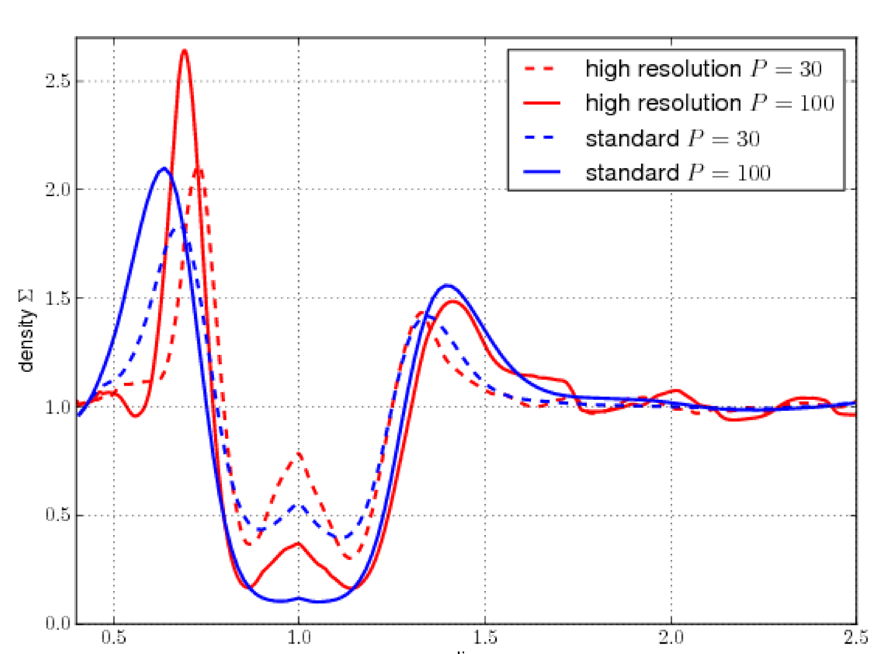
We validate FOSITE with the planet-disk interaction standard simulations as proposed by de Val-Borro et al. (2006). First we compare a standard jupiter mass simulation using inertial angular momentum transport to our past method. The figures shown below display the surface densities and the radial density profiles after 30 and 100 planet orbits. While the depth of the gap is similar in both simulations, there is a big difference at the gap edges in the radial density plot. Furthermore the Lagrange points L4 and L5 can only be seen in the simulation with inertial angular momentum transport, since the other is a lot more diffusive.





Secondly we compare results using the standard resolution 128×384 to high resolution 512×1536 simulations. Generally these simulations agree quite well. However the high resolution simulation shows different behavior regarding the Lagrange points, which vanish a lot faster in the standard simulations. In the high resolution simulation the outer edge of the gap is dominated by a big vortex which rotates with a lower angular velocity than the proto planet.



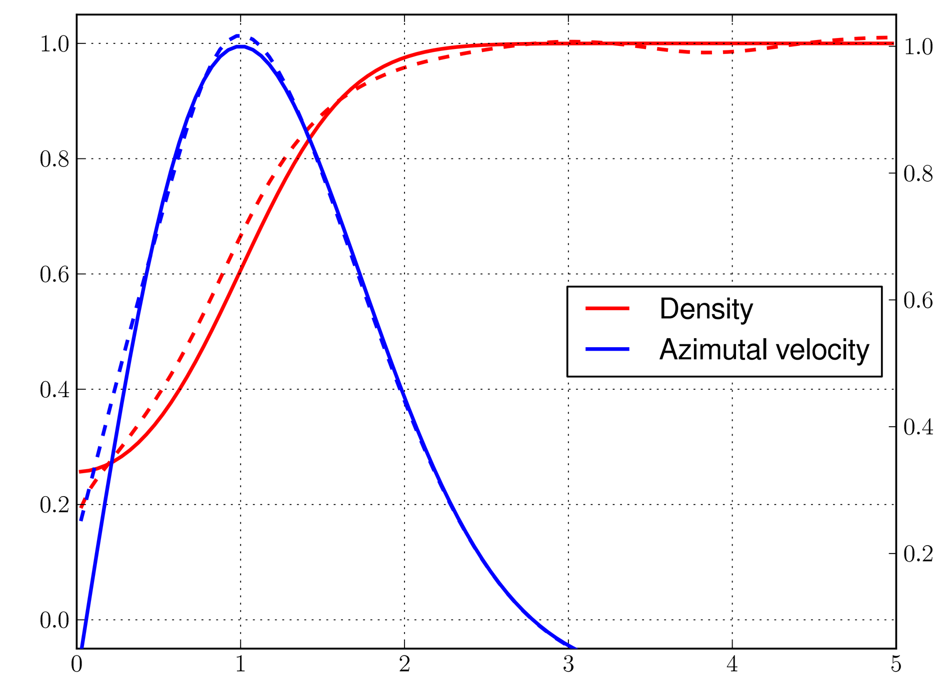


5. Local isothermal isentropic vortex test

The isentropic vortex (e.g. Yee et al. 1999) is a sophisticated pure hydrodynamical numerical test for angular momentum transport. Until now it has been necessary to solve the energy equation to do the test, which sometimes is not desirable. As a novel modification of the isentropic vortex test, we propose to approximate the demanded pressure by a local isothermal equation of state. Hence as long as the state remains stationary, the solution can be reproduced. The corresponding speed of sound is:

$$c_s = \sqrt{\frac{p}{\rho}}.$$

The figure shows the density and azimuthal velocity of the initial state (solid lines) and after 100 dynamical timescales at the radius 1. The simulation was performed in a rotating frame of reference with an angular velocity of 1.

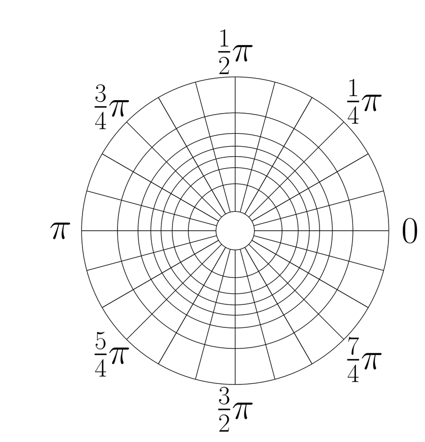


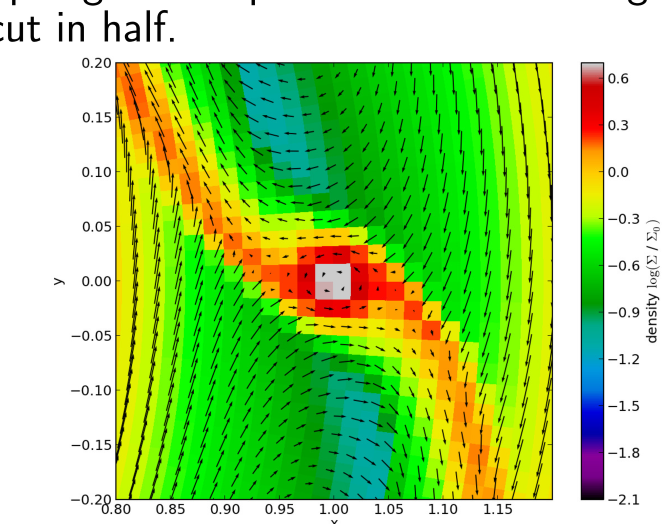
6. Static mesh refinement

The possibility to use any curvilinear orthogonal mesh is a special feature of FOSITE. Although inertial angular momentum transport is restricted to polar coordinates, arbitrary radial scalings are allowed. Besides well known linear or logarithmic scalings this permits for higher resolution at any annulus of the mesh without computational overhead. For example a mesh using radial sinushyperbolicus scaling according to

$$\boldsymbol{r} = (\kappa_1 \cdot \sinh(\xi - \kappa_3) + \kappa_2) \begin{pmatrix} \cos(\eta) \\ \sin(\eta) \\ 0 \end{pmatrix}$$

with appropriately chosen constants $\kappa_1, \kappa_2, \kappa_3$ can achieve the same resolution of the gap region compared to linear scaling with the amount of cells cut in half.





7. Conclusions

- Exact inertial angular momentum conservation can be achieved, while solving the Euler equations at once
- FOSITEs results agree well with standard simulations
- A local isothermal isentropic vortex can be used as pure hydrodynamical numerical test
- Higher resolutions at the same price are possible due to static mesh refinement

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