The baroclinic instability in accretion discs

Mechanism and secondary instabilities

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Giant vortex in Naruto straight (Japan)

Hall discs SEVENTH FRAMEWORK PROGRAMME
cnrs dépasser les frontières
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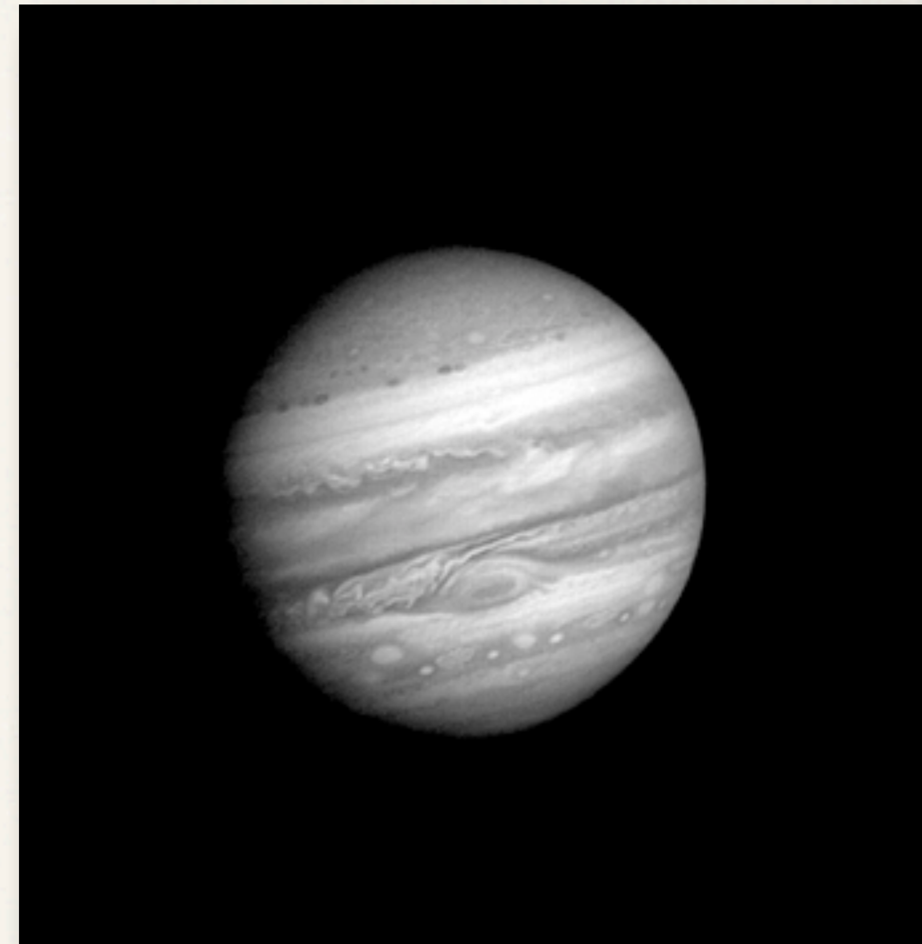
Outline

- A brief introduction
  - Vortices in accretion discs?
  - Planetary formation and transport
- The baroclinic instability in 2D
  - Shearing box model
  - Instability main properties
  - Phenomenological description
- The instability in accretion discs
  - Compressibility
  - (3D stability)
- Conclusions

Vortices in nature

- Well known in planetary atmospheres

Cyclones on Earth

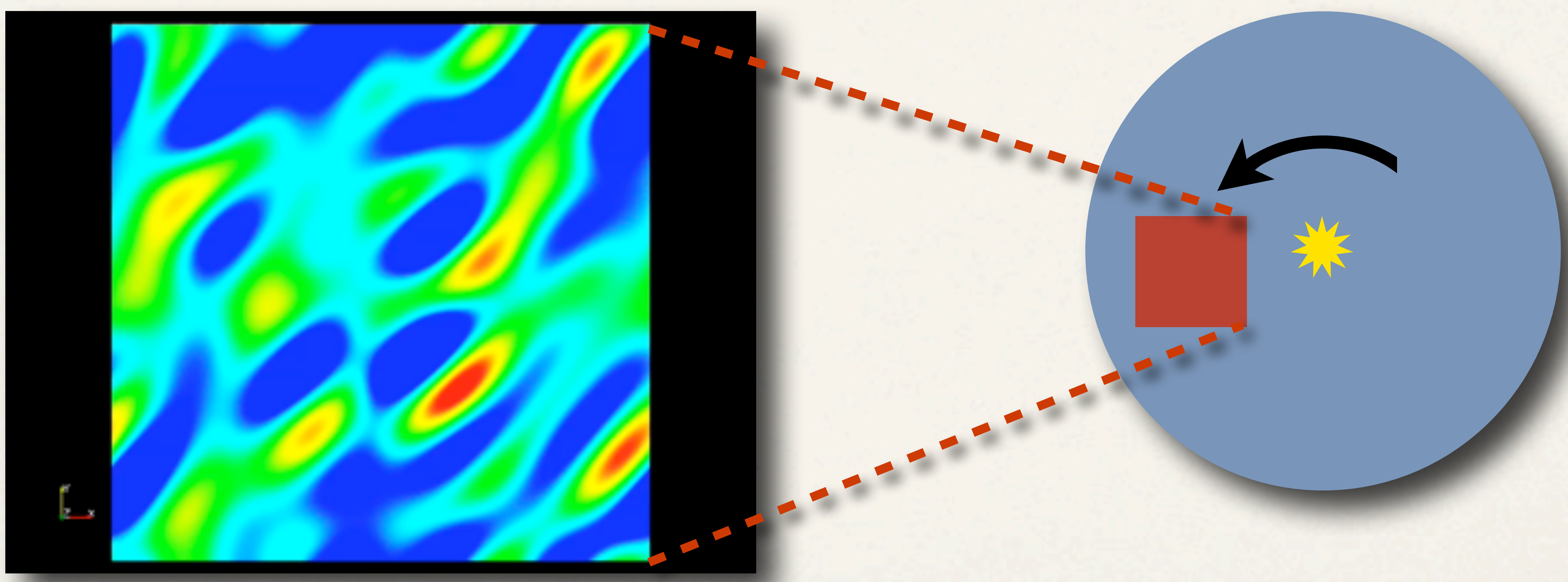


Great red spot

- Generally associated with quasi 2D configuration and rotation/shear

Vortices in accretion discs?

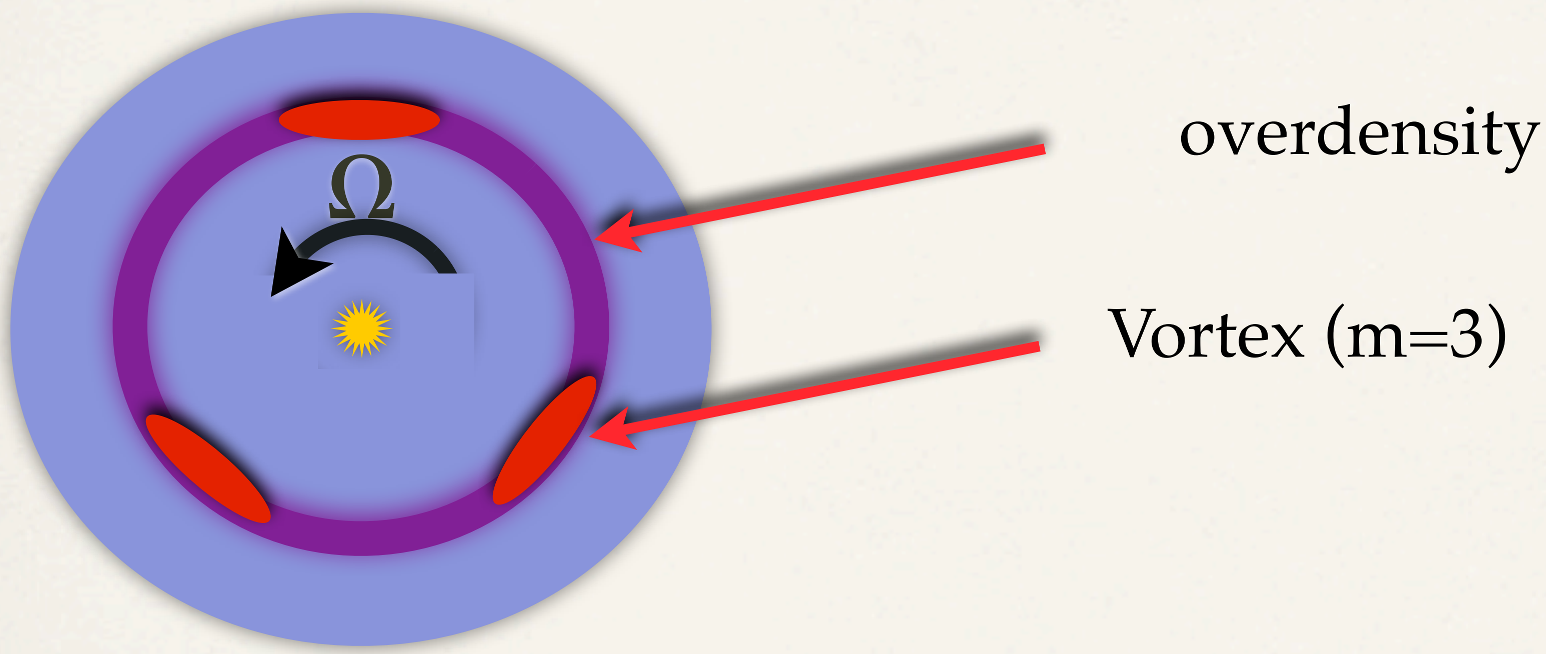
- Initially suggested by von Weizsäcker (1944) to explain planetary formation.
- Reintroduced by Barge & Sommeria (1995) : dust accumulation.
- In discs, only anticyclonic (counter rotating) vortices can survive.



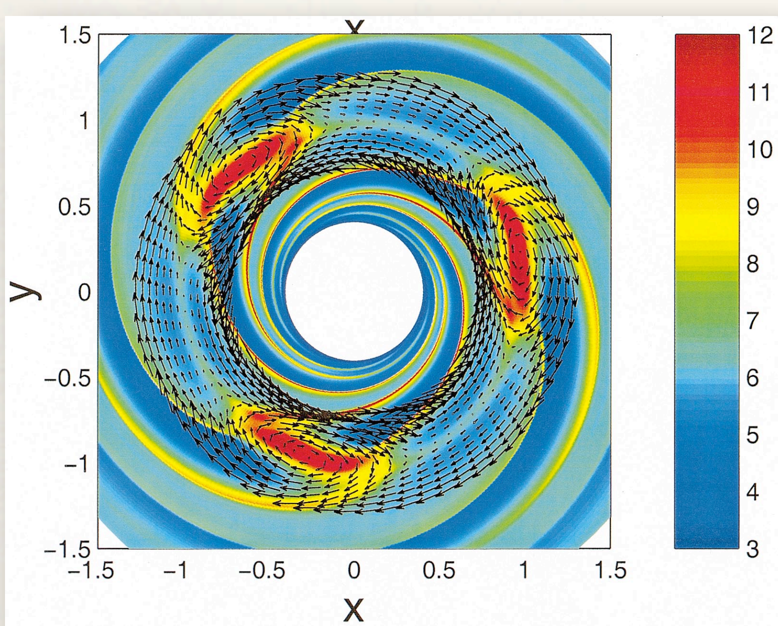
➡ How do we generate vortices?

Rossby wave instabilities

- Observed when a radial structure is imposed (e.g high density annulus).



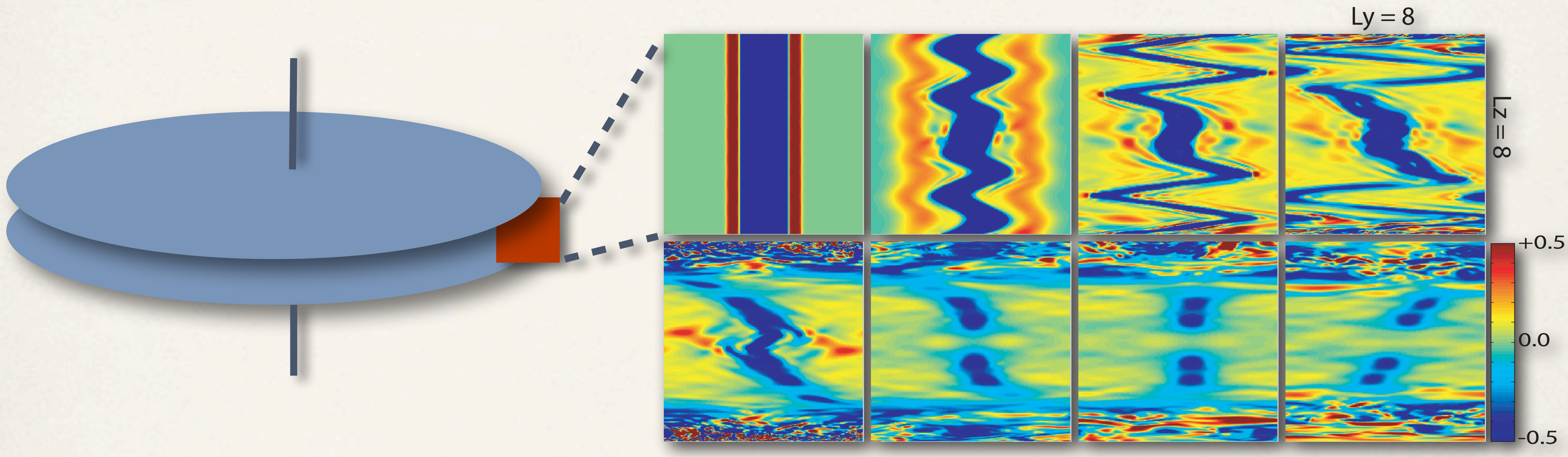
- Instability saturation produces vortices on the radial structure



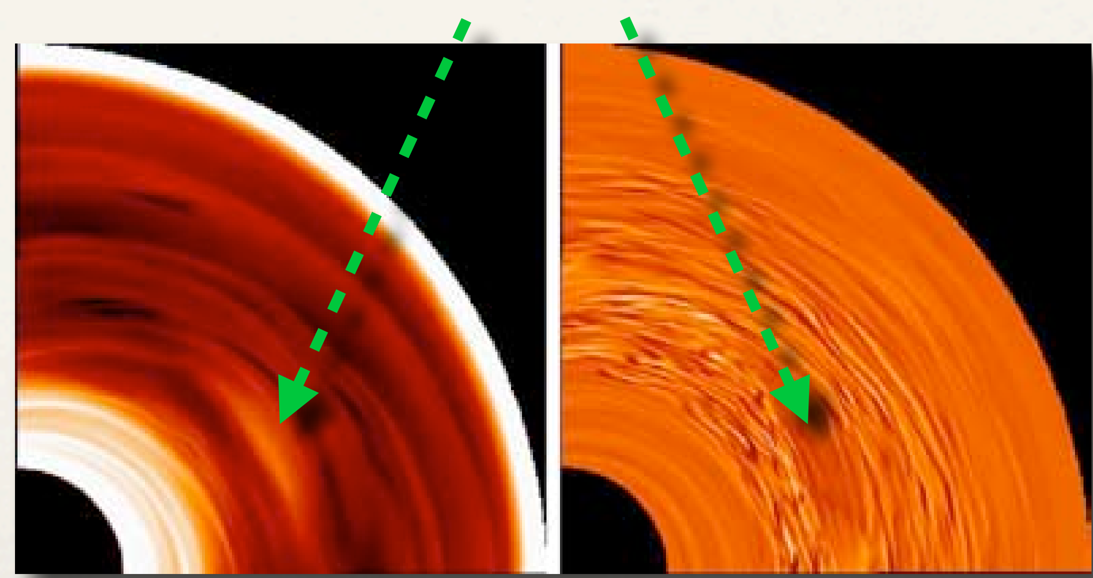
Rossby wave instabilities in a numerical simulation
Li *et al.* (2001)

3D processes

- Off midplane generation (Barranco & Marcus 2005). *Mechanism unknown.*



- Spontaneous production in MRI turbulence (Fromang & Nelson 2005)



Density *Vorticity*

Baroclinic instabilities

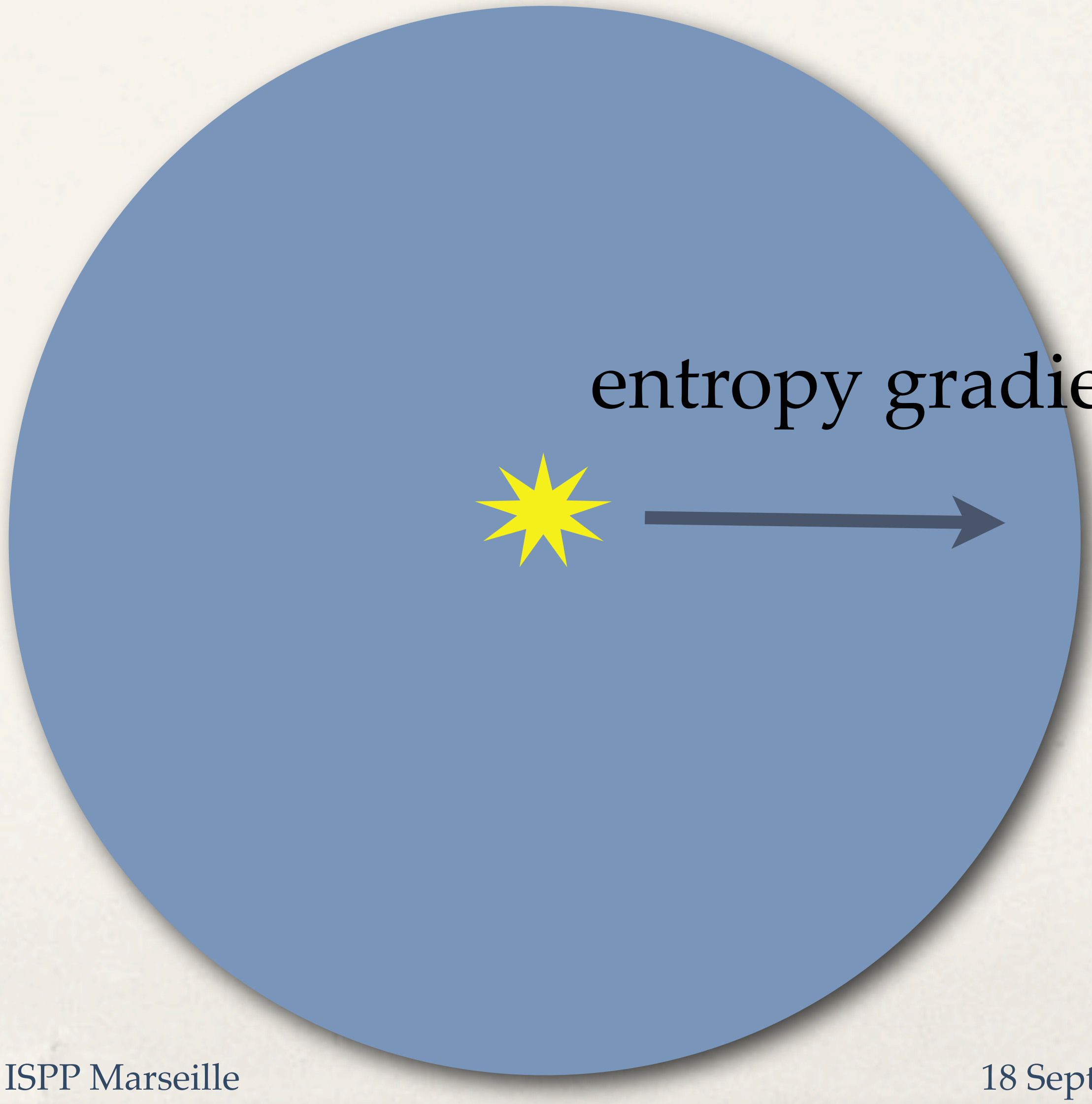
- Baroclinic instabilities are driven by the missalignement of isobar and isodensity contours: $\nabla P \times \nabla \rho$

- Well known in geophysics (even seen in labs!)

Credit: Yakov Afanasyev & Peter Rhines

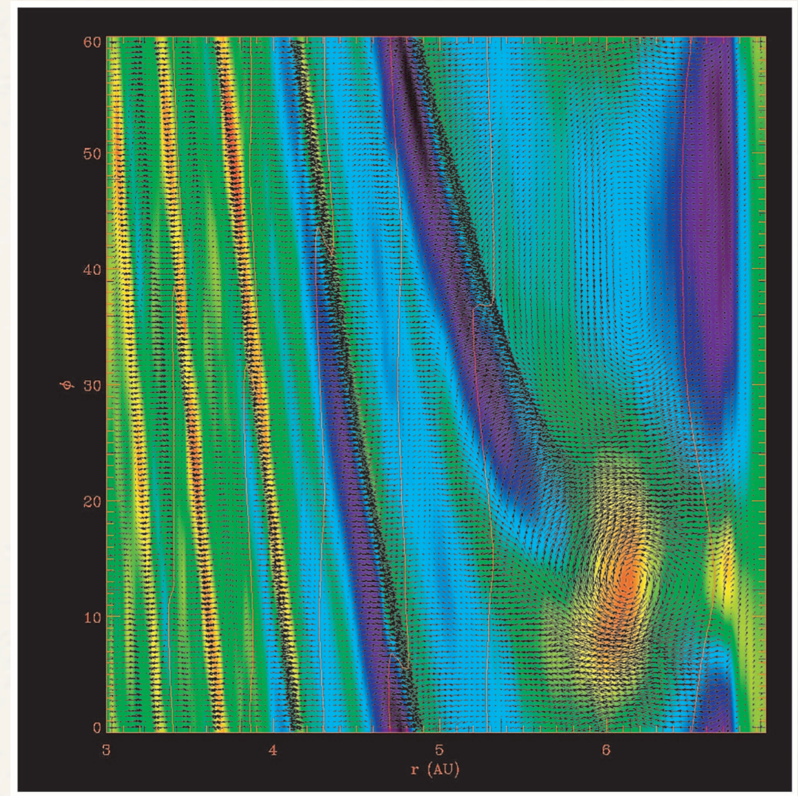
Baroclinic instabilities in discs

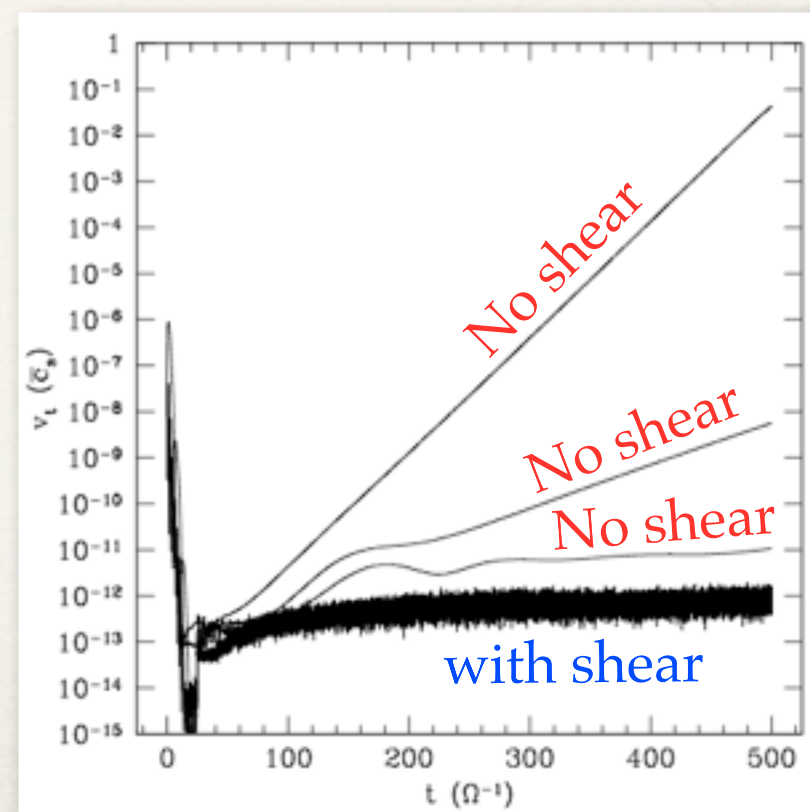
- Baroclinic instabilities in discs are essentially driven by the radial entropy structure of the disc.
- 2D configuration
- Different from the geophysical case (3D)



Baroclinic instabilities: an overview (1)

- Baroclinic instabilities are driven by the radial entropy structure of the disc.
- Initially suggested in global simulations by Klahr & Bodenheimer (2003).
  Many numerical problems (Boundary conditions, numerical convergence)
- Local linear and numerical studies (Johnson & Gammie 2005, 2006) did not find anything.





Baroclinic instabilities: an overview (2)

The Astrophysical Journal, 636:63–74, 2006 January 1

NONLINEAR STABILITY OF THIN, RADIALLY STRATIFIED DISKS

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ABSTRACT

We perform local numerical experiments to investigate the nonlinear stability of thin, radially stratified disks. We demonstrate the presence of radial convective instability when the disk is nearly in uniform rotation and show that the net angular momentum transport is slightly inward, consistent with previous investigations of vertical convection. We then show that a convectively unstable equilibrium is stabilized by differential rotation. Convective instability is ... gradients on the order of the disk radius have Ri ≳ −0.01 and are therefore stable to local nonaxisymmetric disturbances. One implication of our results is that the "baroclinic" instability recently claimed by Klahr & Bodenheimer is either global or nonexistent. We estimate that our simulations would detect any genuine growth rate ≳0.0025Ω.

2 ingredients missing:

-finite amplitude perturbations

-thermal diffusion / relaxation

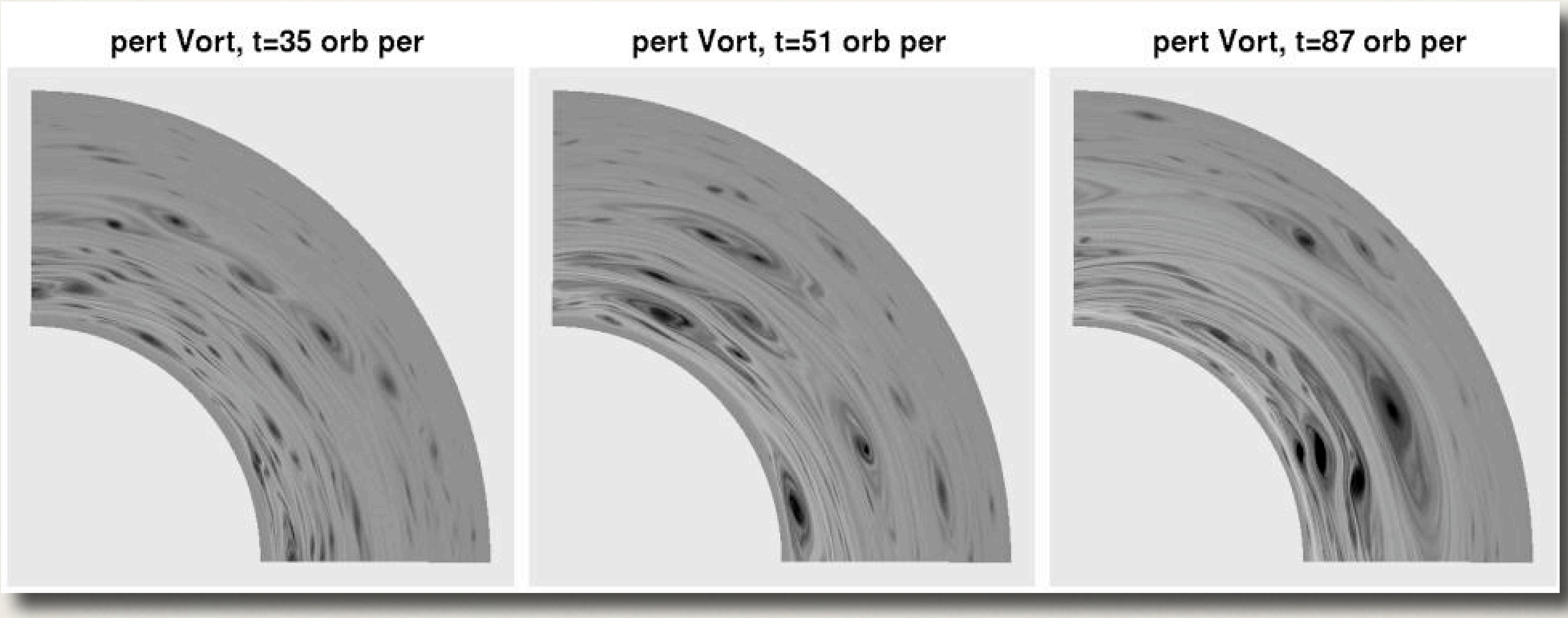
must be removed from the disk material and transported outward. In many disks, this outward angular momentum transport is likely mediated internally by magnetohydrodynamic (MHD) turbulence driven by the magnetorotational instability (MRI; see Balbus & Hawley 1998). A key feature of this transport mechanism is that it arises from a local shear instability and is therefore very robust. In addition, MHD turbulence transports angular momentum *outward*; some other forms of turbulence, such as

weakly ionized disks.

In a companion paper (Johnson & Gammie 2005, hereafter Paper I), we have performed a linear stability analysis for local nonaxisymmetric disturbances in the shearing-wave formalism. While the linear theory uncovers no exponentially growing instability (except for convective instability in the absence of shear), interpretation of the results is somewhat difficult due to the nonnormal nature of the linear differential operators:[2] one has a cou-

Baroclinic instabilities: an overview (3)

- Petersen et al. (2007) revived the idea, with anelastic spectral simulations showing vortex amplification.
- They also included a new ingredient: thermal diffusion.



Petersen et al. 2007

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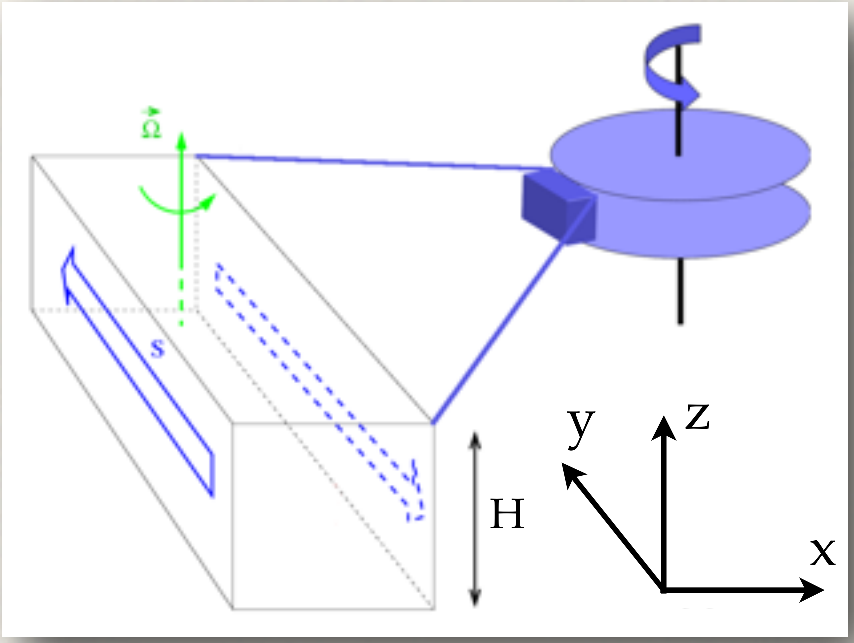
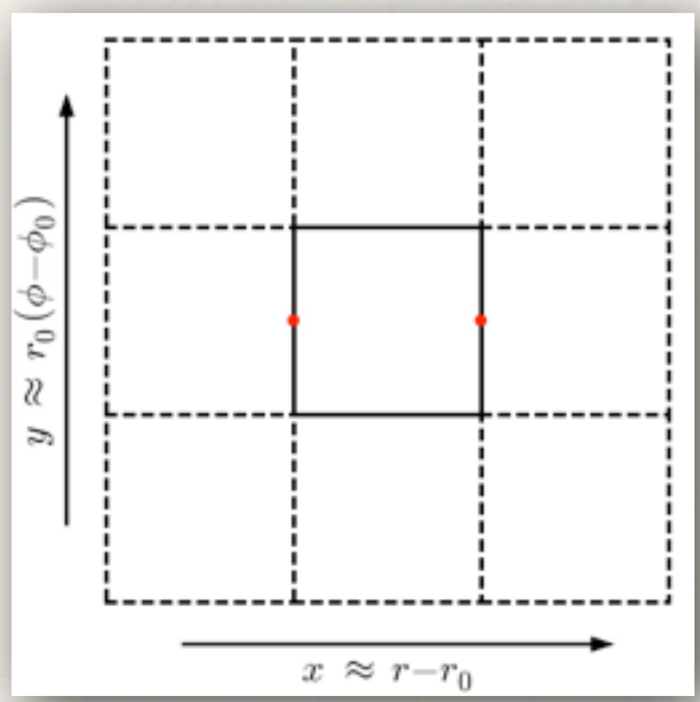
The shearing box model





- Local approximation:
  - Neglect curvature effects
  - Almost incompressible (incompressible approximation valid in first approximation)
- Have to include the radial stratification to take into account baroclinic effects (Boussinesq).

(some) equations

- Incompressible equations in 2D $(x,y)=(r,\phi)$
- Stratification in the Boussinesq approximation
- Buoyancy frequency:

$$N^2 = -\frac{1}{\gamma\Sigma}\frac{\partial P}{\partial R}\frac{\partial}{\partial R}\ln\left(\frac{P}{\Sigma^\gamma}\right)$$

- In 2D, stratification is a source of vertical vorticity through the baroclinic term

$$\begin{aligned} \partial_t\omega + \boldsymbol{u}\cdot\boldsymbol{\nabla}\omega &= \Lambda N^2\partial_y\theta + \nu\Delta\omega \\ \partial_t\theta + \boldsymbol{u}\cdot\boldsymbol{\nabla}(\theta + x/\Lambda) &= \mu\Delta\theta \end{aligned}$$

→ Non axisymmetric temperature perturbations can *locally* produce vorticity

Linear stability

- Dimensionless number comparing stratification and rotation (shear):

$$Ri = \frac{N^2}{\Omega^2}$$

- The stability to axisymmetric disturbances is given by the Solberg-Hoïland criterion

$$Ri > -1 \Rightarrow \text{Stability}$$

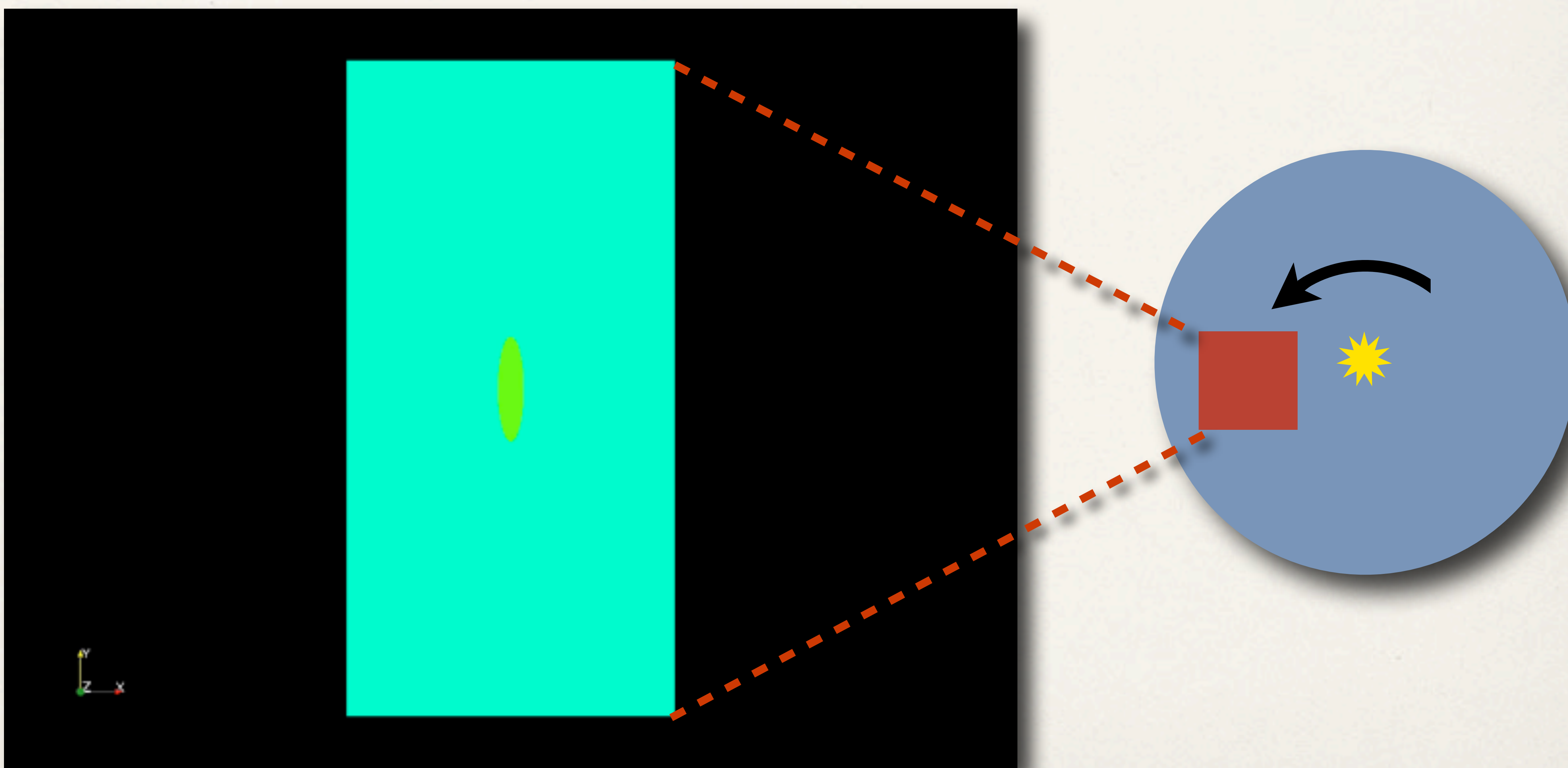
- In discs,

$$|Ri| \lesssim 10^{-2}$$

→ Radially stratified discs are linearly stable to axisymmetric disturbances

The baroclinic instability: vortex amplification

- Initial condition: box-centred Kida vortex
- Radial (x) stratification with $N^2<0$
- Integrated for 20 orbits
- Vertical vorticity plot



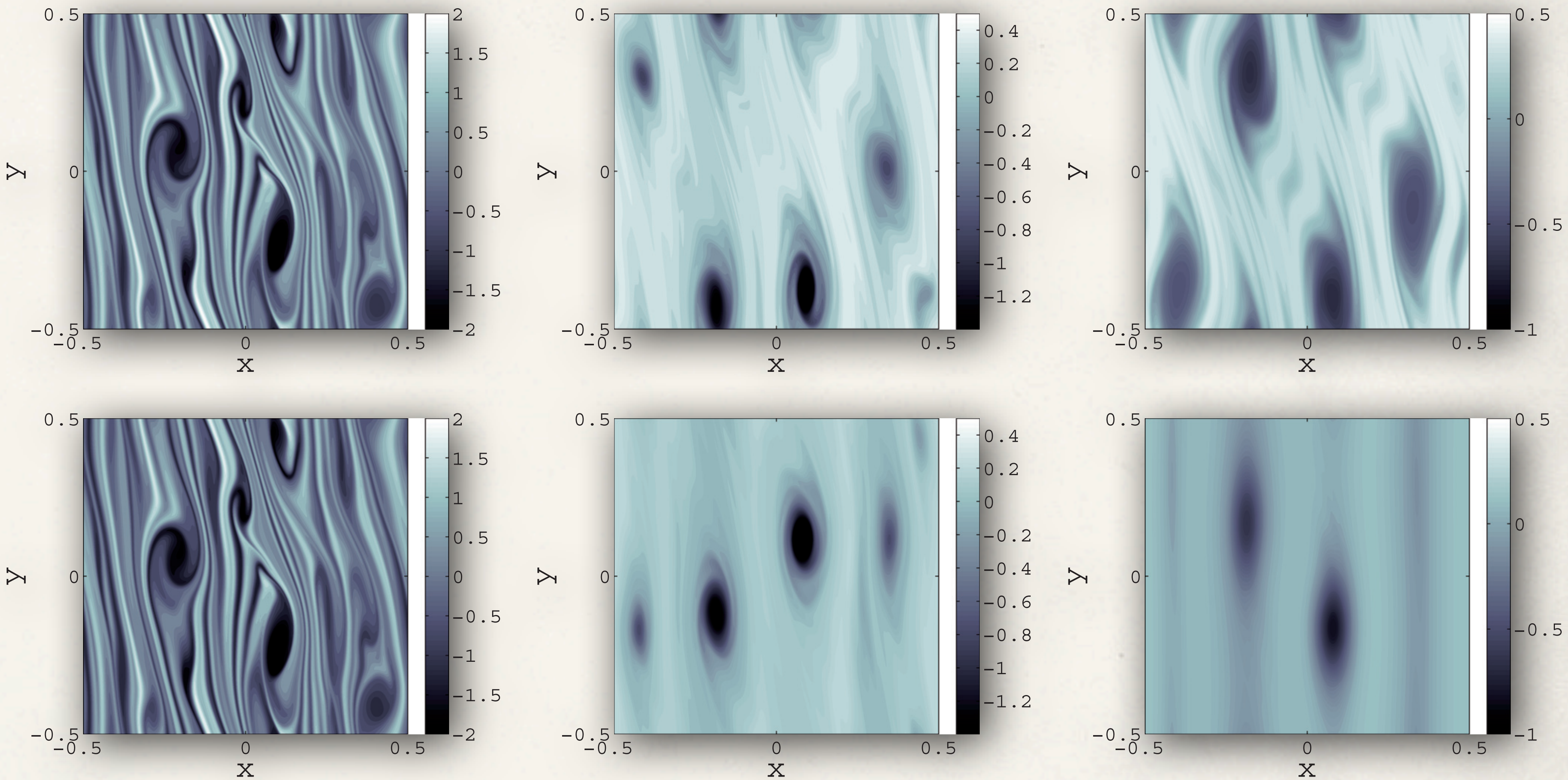
The effect of stratification

t=0.1 orbits | t=10 orbits | t=50 orbits

$$\frac{N^2}{\Omega^2} = -0.022$$

$$\frac{N^2}{\Omega^2} = 0$$

(no stratification)



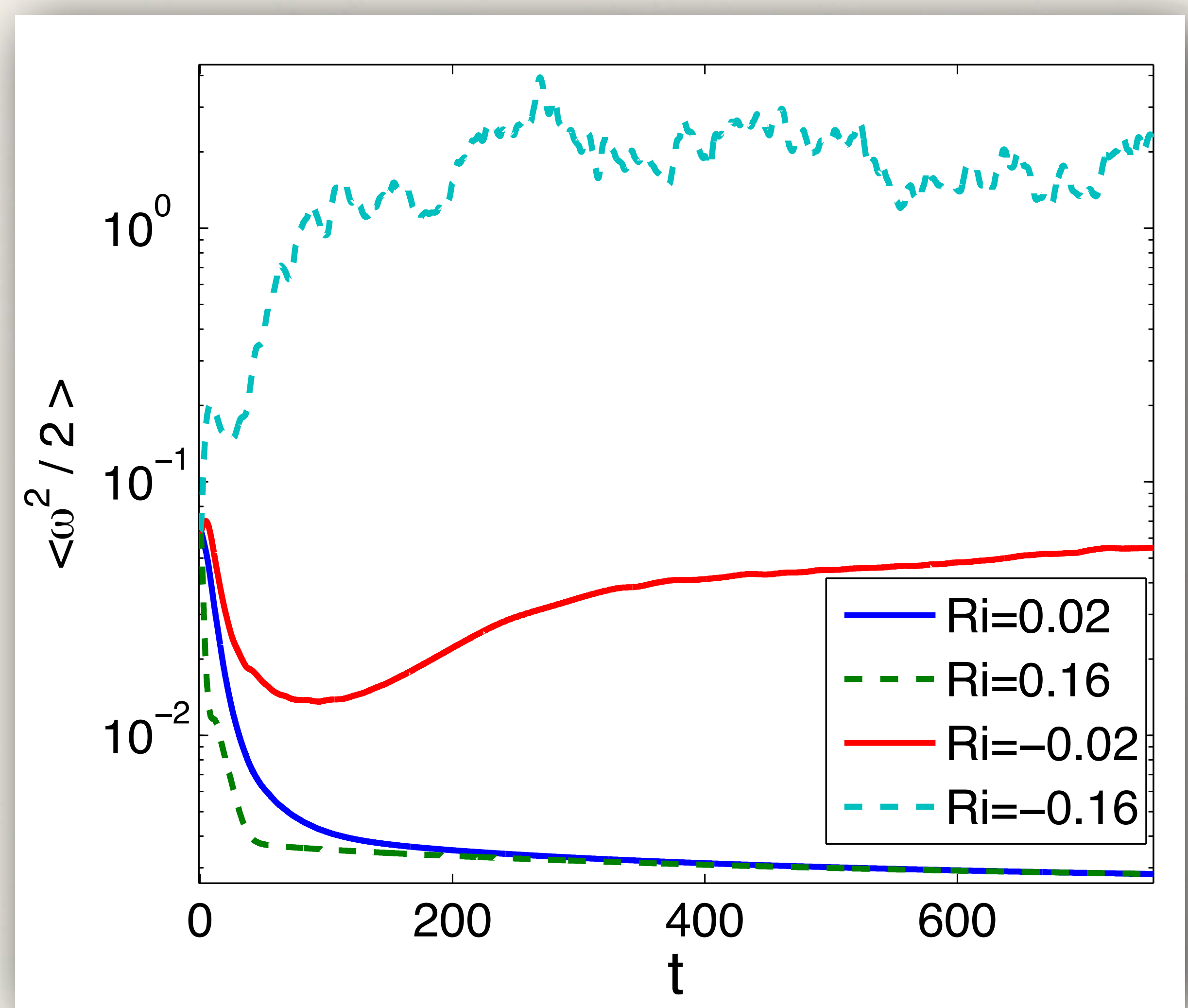
Lesur & Papaloizou 2010

Vortex amplification is due to the stratification.

Requires $N^2<0$ (or equivalently $T \sim r^d$ with $d < -0.5$)

The effect of the stratification (cont'd)

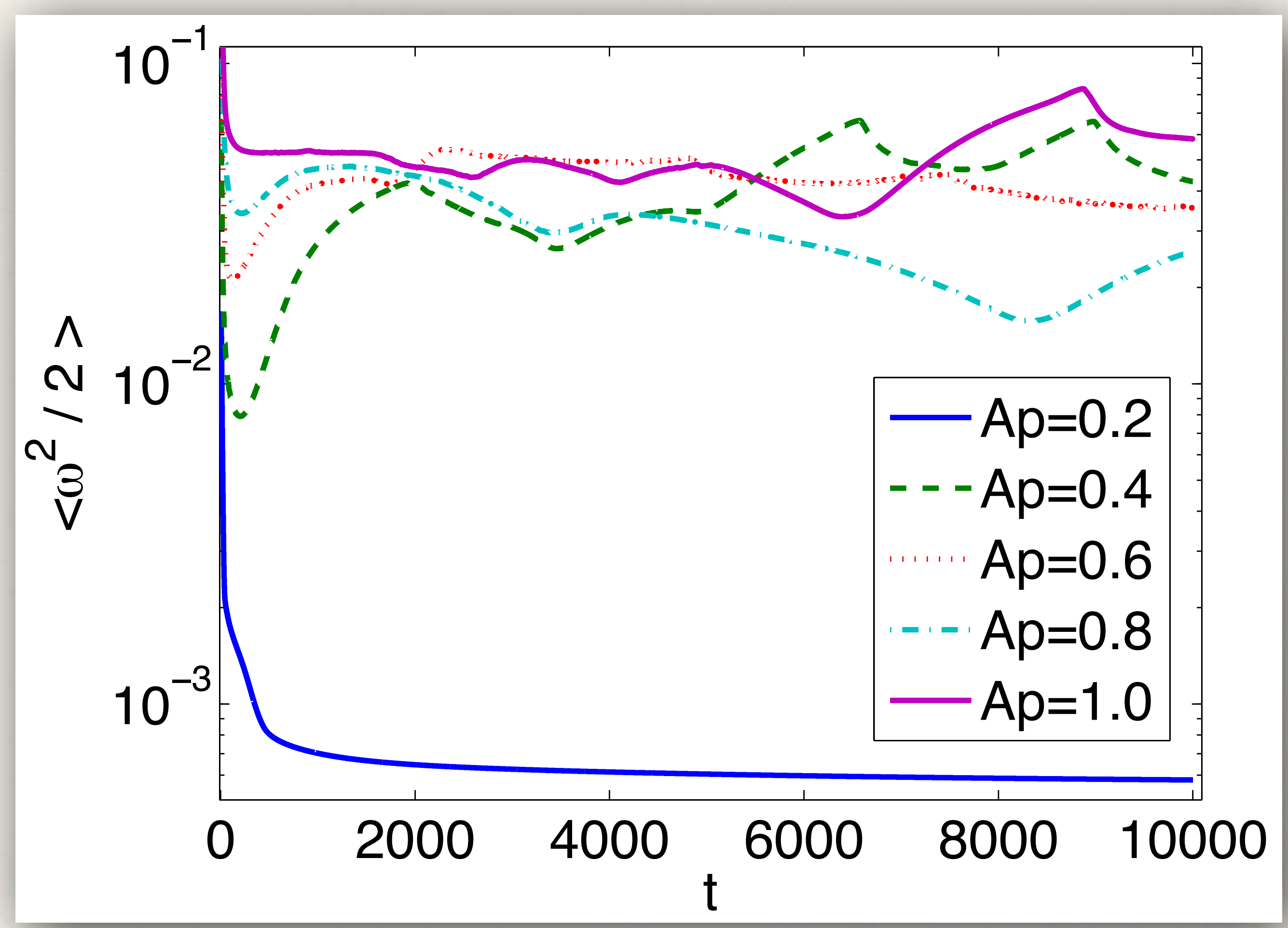
- Enstrophy $\langle|\omega|^2\rangle$ as a function of the stratification parameter $Ri = N^2/\Omega^2$



Requires $N^2<0$ (or equivalently $T \sim r^d$ with $d < -0.5$ assuming MMSN density profile)

A nonlinear instability

- Influence of the amplitude of the initial perturbation

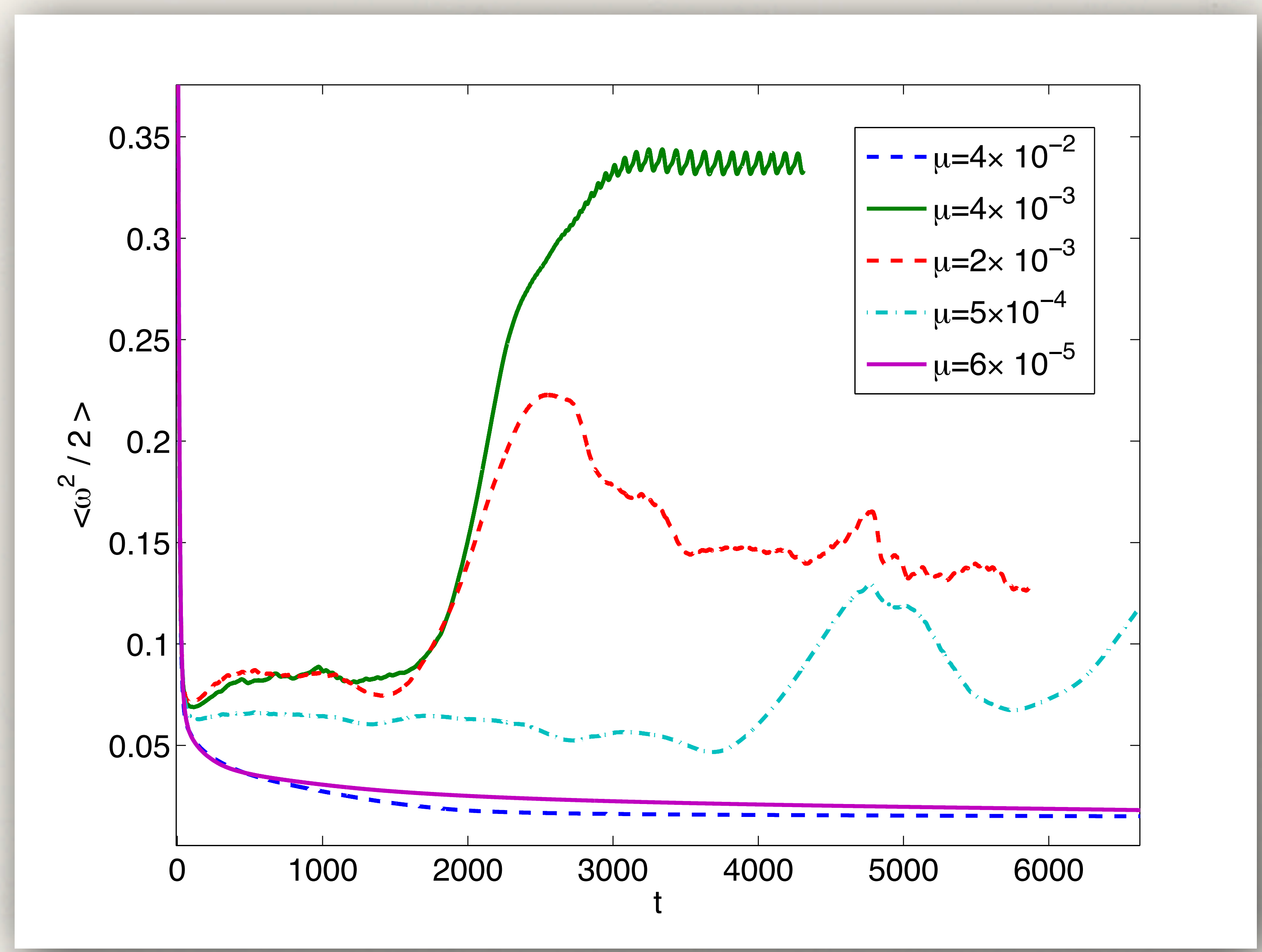


➡ The instability appears for *finite amplitude disturbances.*

➡ Explains Johnson & Gammie (2005) negative result.

The role of thermal diffusion

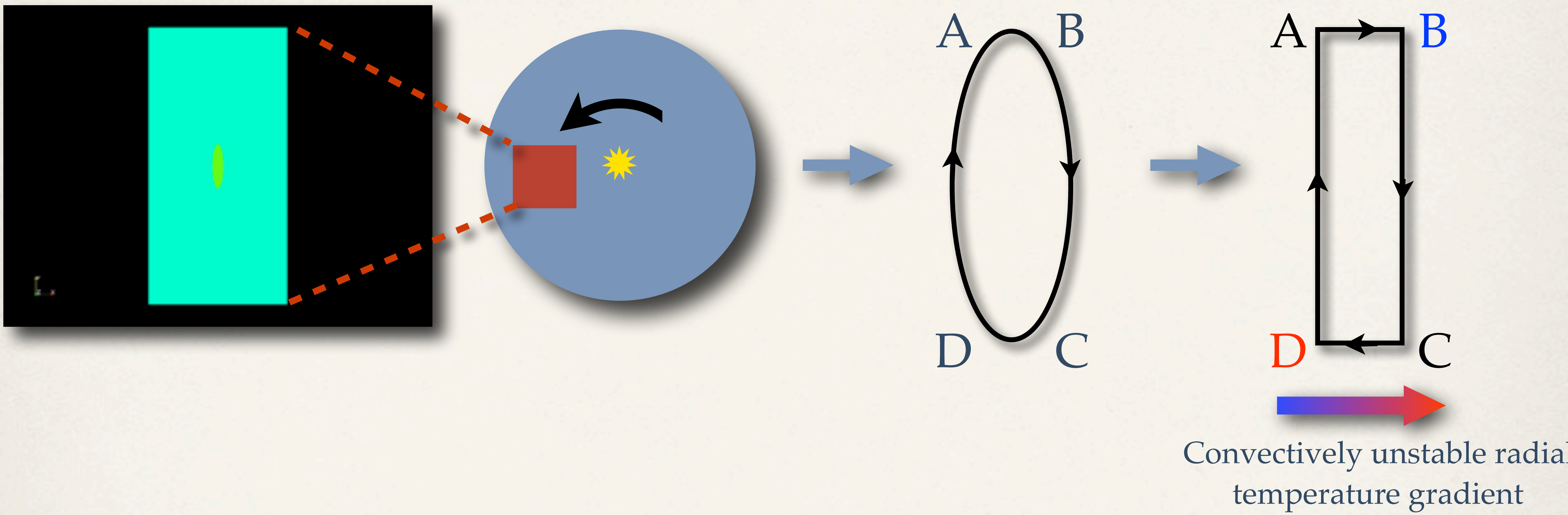
- Enstrophy evolution as a function of the thermal diffusion parameter



Lesur & Papaloizou 2010

Thermal diffusion (not too strong!) *is required* to get the instability.
See also Petersen et al. (2007)

Phenomenological description



- A to B: The fluid particle is cooler and heavier than the surrounding gas. It is accelerated by gravity toward the star.
- B to C: Background temperature is constant. The particle is reheated by thermal diffusion.
- C to D: Fluid particle hotter and lighter than the background: outward acceleration.
- D to A: Particle cooled by thermal diffusion.

➡ Fluid motion is amplified on the AB and CD branches.

Summary

- This instability:
  - produces and amplifies vortices in local disc models (shearing boxes)
  - appears when $N^2<0$ (when the temperature profile is steep enough)
  - requires explicit thermal diffusion
  - is nonlinear (or subcritical)

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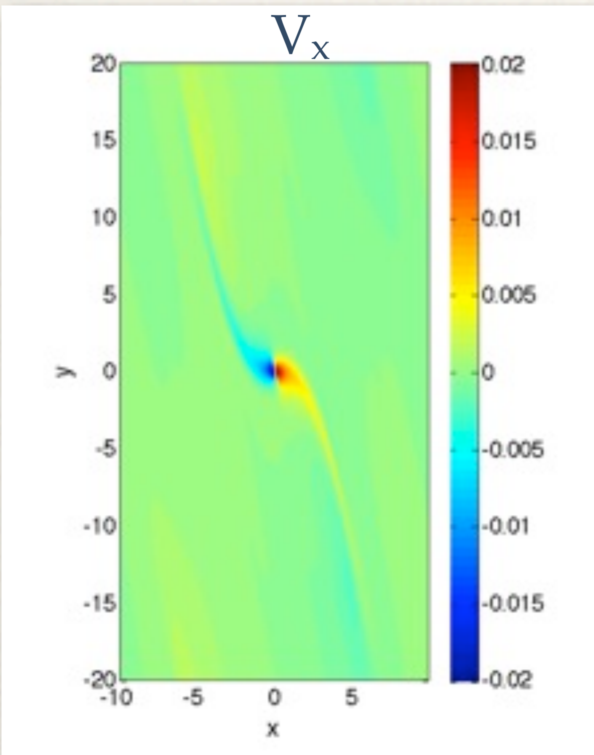
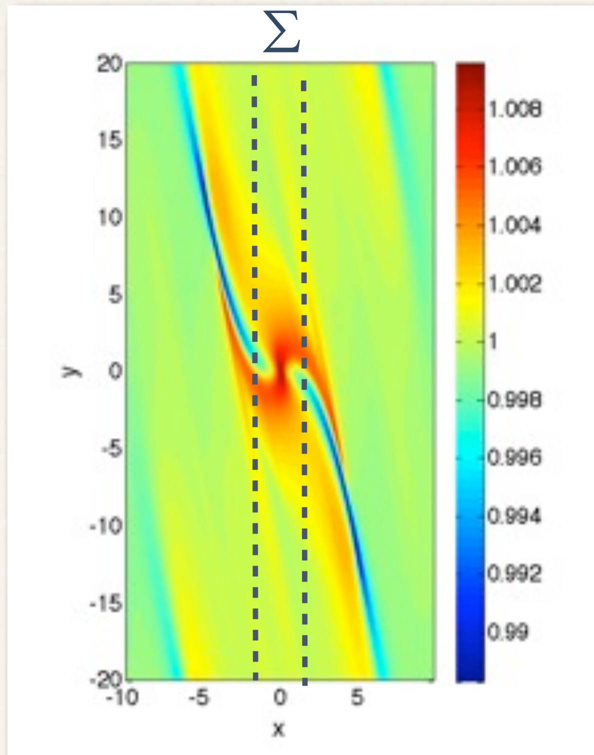
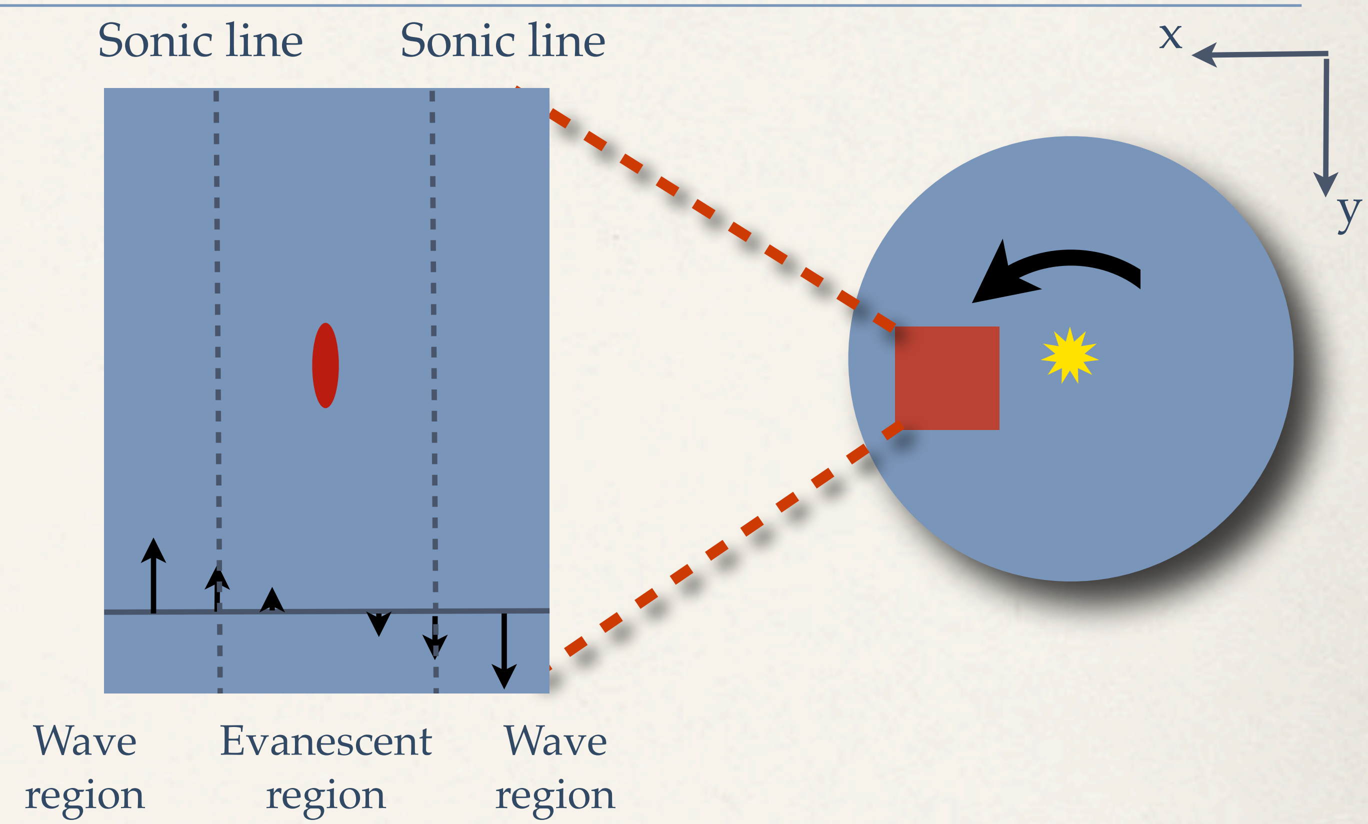
Generation of density waves (single vortex)



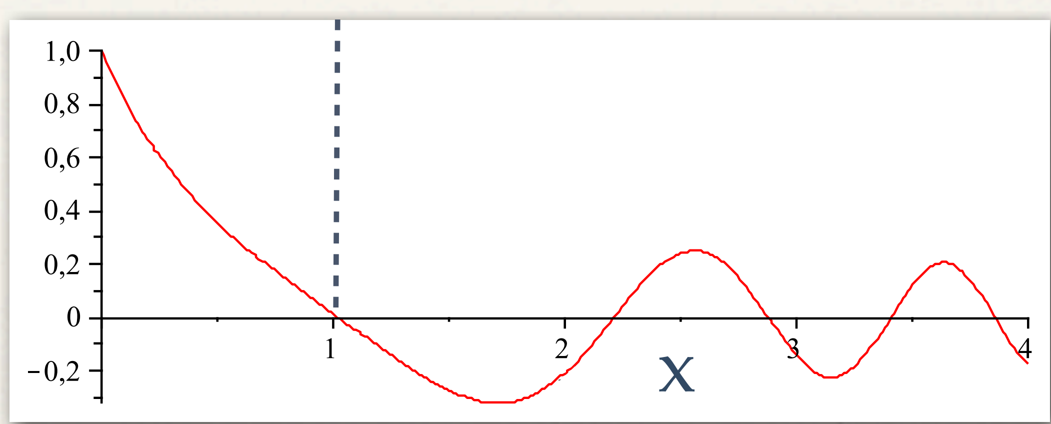




- Linear stationary waves are described by a parabolic cylinder equation

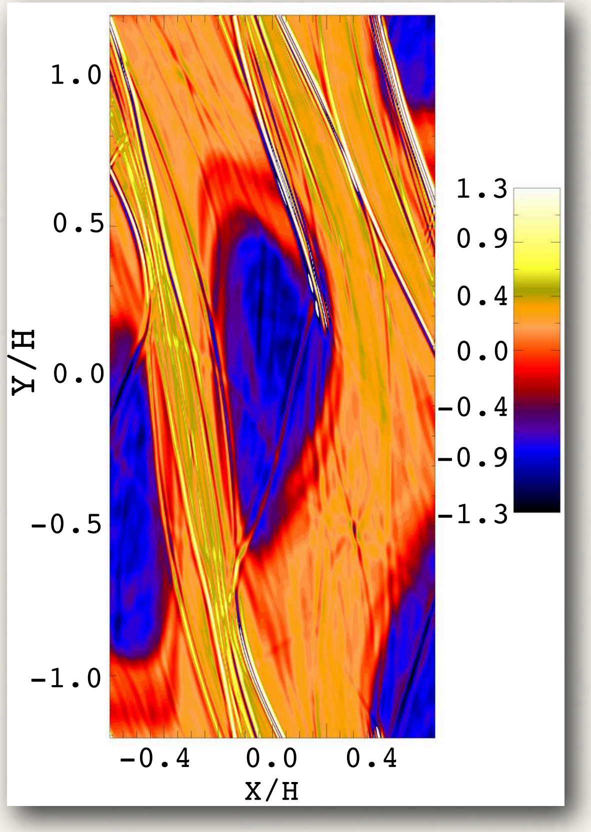
$$\frac{d^2v}{dx^2}+\left[\frac{\sigma^2-\kappa^2}{c^2}-k^2\right]v=0 \quad \text{where} \quad \sigma=\frac{3}{2}\Omega x k$$

- The vortex produces a «tail», connected to the wave region through the sonic line

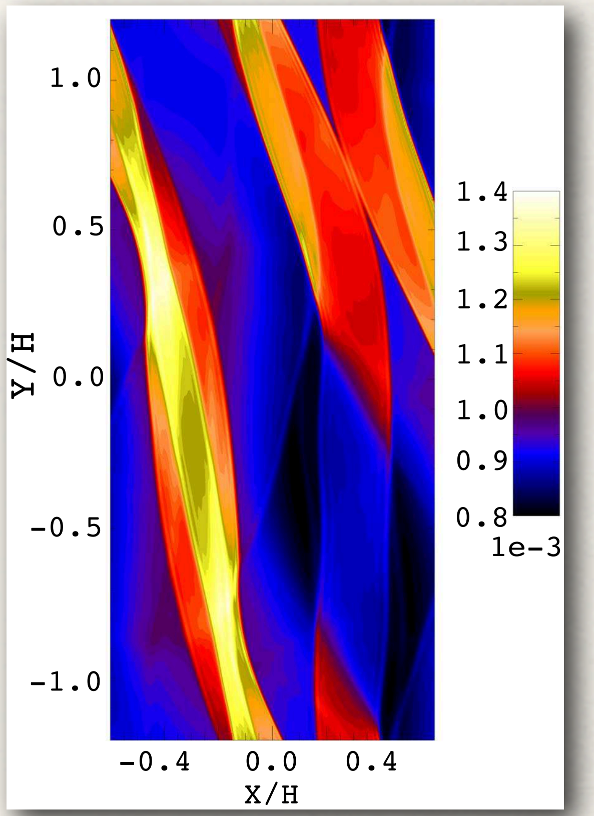


Compressibility, waves and transport

- In fully compressible simulations, vortices produce density waves *(see also Johnson & Gammie 2005 ; Bodo et al 2005, 2007 ; Heinemann & Papaloizou 2009a,b).*

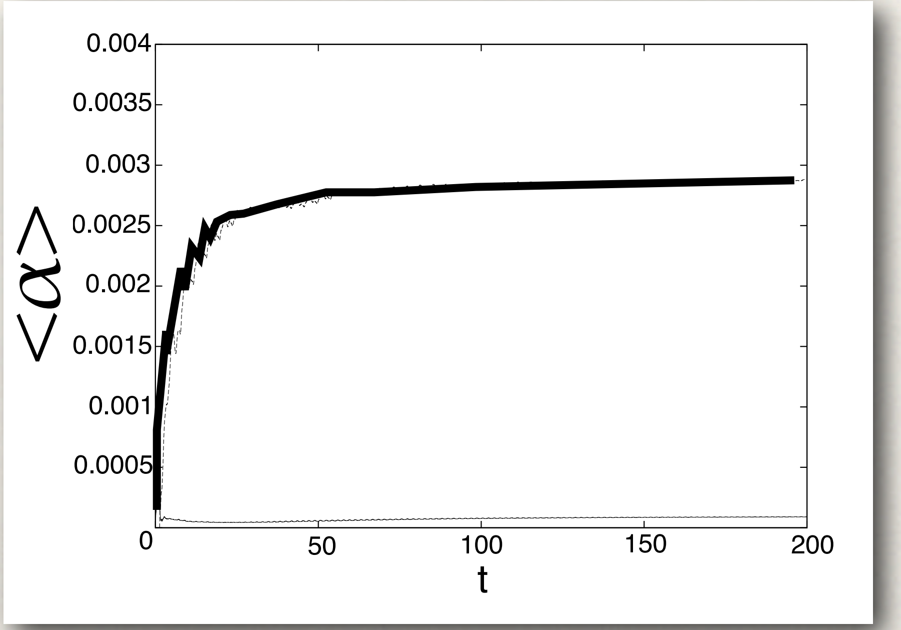


vorticity



density

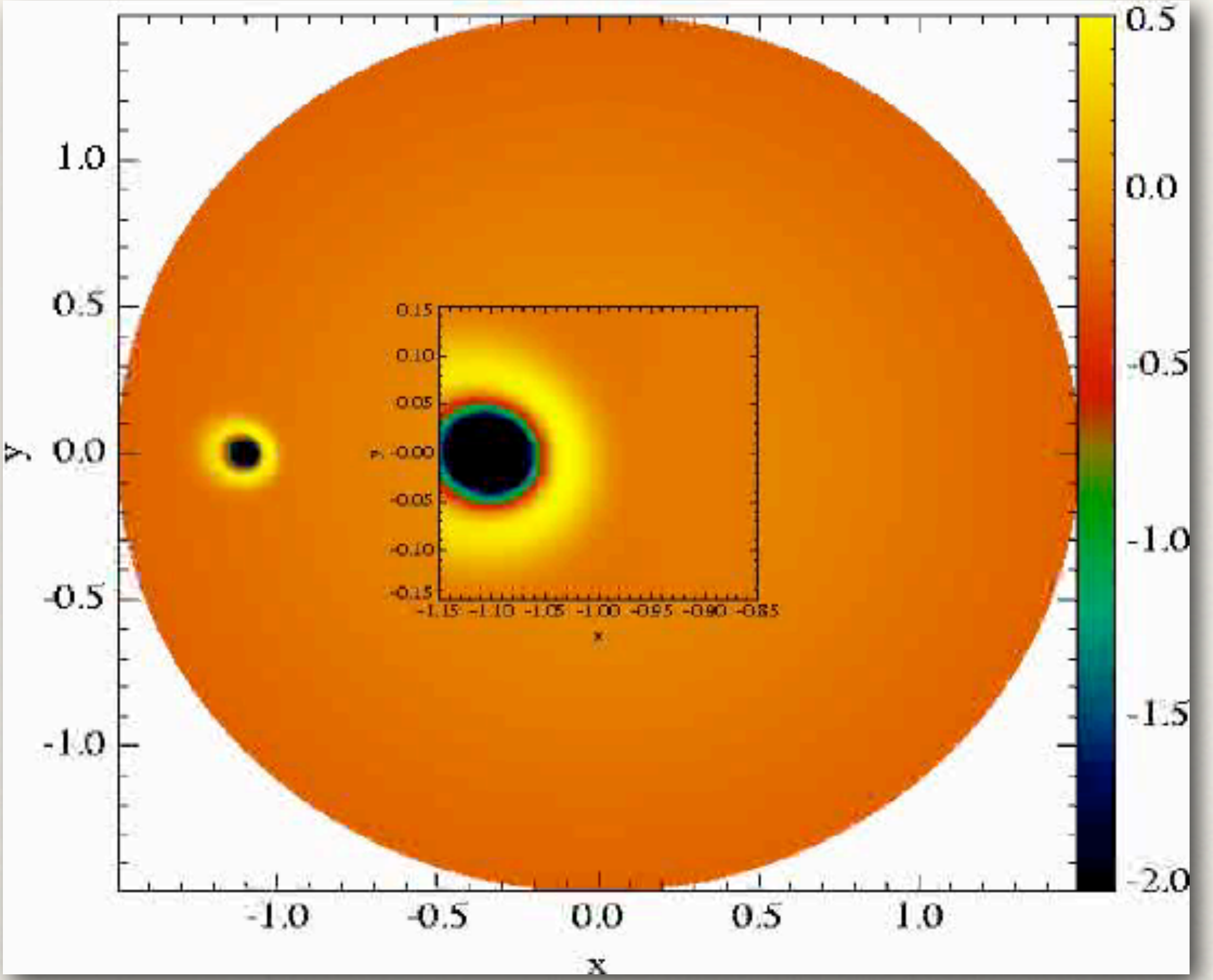
- Density waves transport angular momentum outward with $\alpha \sim 10^{-3}$



Waves and vortex migration

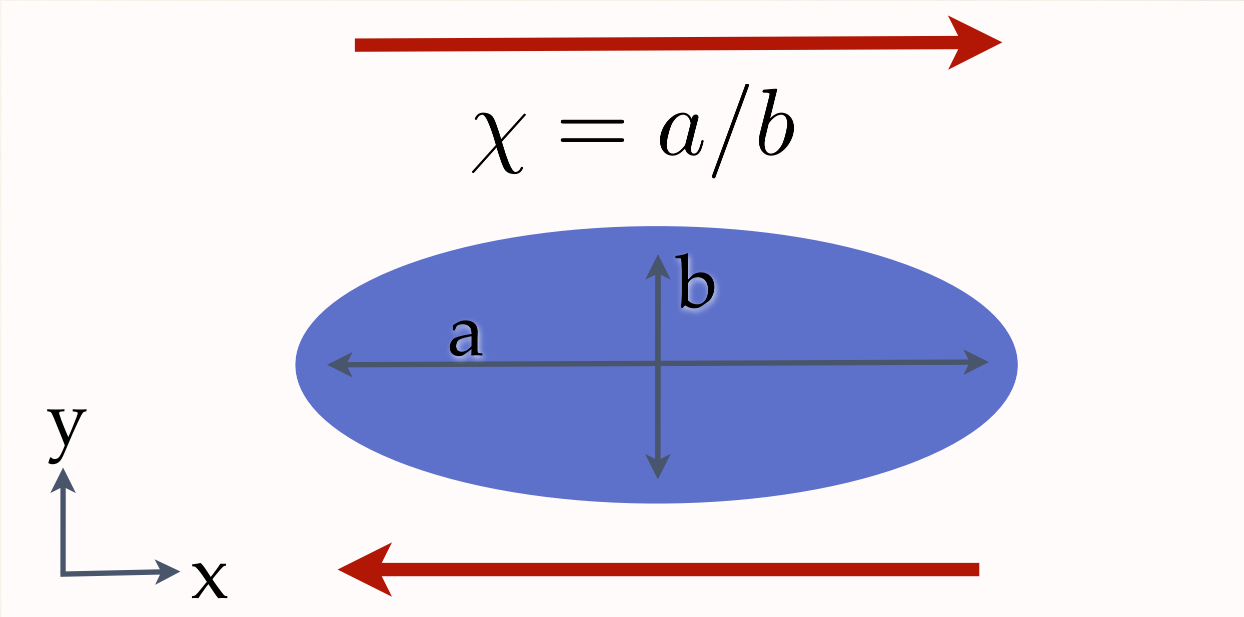
- The Baroclinic instability still work in global simulations
- Asymmetric wave excitation

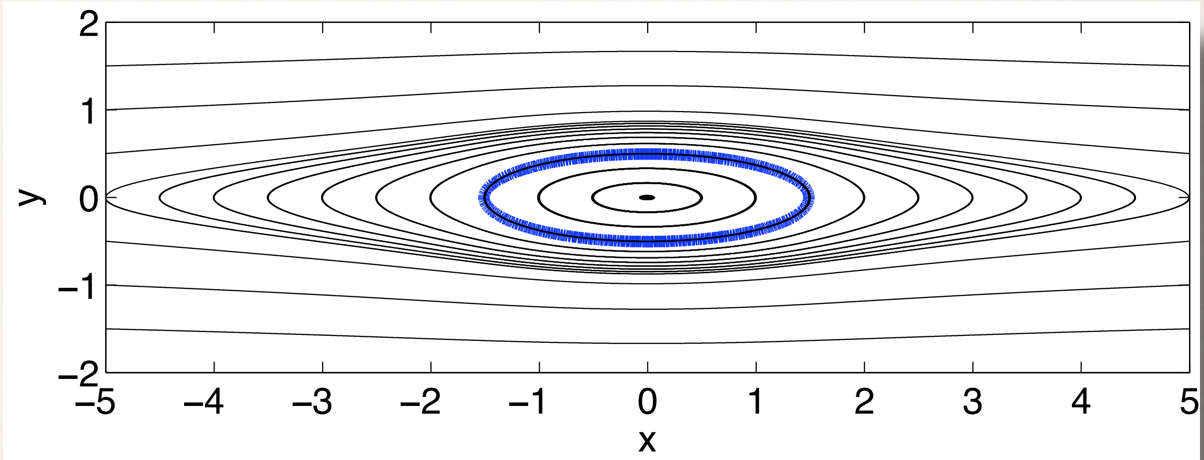
➡ Vortex migration ! *(See Pardekooper's talk)*



Vortex stability: the Kida vortex

- Kida vortex defined my a vorticity patch ω_V.
- Inside the vortex core, streamlines are elliptical
- Outside of the core, streamlines are closed, but not elliptical





Kida Vortex (cont'd)

- Inside the vortex core, the 2D velocity reads

$$u_x^0 = S\frac{1}{\chi-1}\chi y, \qquad u_y^0 = -S\frac{1}{(\chi-1)}\frac{1}{\chi}x.$$

- Or in simplified version

$$u_i^0 = SA_{ij}x_j \qquad \boldsymbol{A} = \frac{1}{\chi-1}\begin{pmatrix} 0 & \chi & 0 \\ -\chi^{-1} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- Describe 3D perturbations inside the core:

$$\begin{aligned} \partial_t \mathbf{v} &= -\nabla P - \mathbf{u^0}\cdot\nabla\mathbf{v} - \mathbf{v}\cdot\nabla\mathbf{u^0} - 2\boldsymbol{\Omega}\times\mathbf{v} \\ \nabla\cdot\mathbf{v} &= 0 \end{aligned}$$

- Analysis valid only inside the vortex core!

Evolution of 3D perturbations

- Fourier decomposition using time dependent wave-vectors:

$$\mathbf{v} = \mathbf{v}(t)\exp(i\mathbf{k}(t)\cdot\mathbf{x})$$

- Leading to:

$$\begin{aligned} \dot{v}_i + i x_k v_i(\dot{k}_k + S k_j A_{jk}) &= -i k_i \Pi - S v_j A_{ij} \\ &\quad -2\epsilon_{ijk}\Omega_j v_k \\ k_j v_j &= 0 \end{aligned}$$

- Solution for **k**(t):

$$\mathbf{k}(t) = k_0\Big(\sin(\theta)\cos[\phi(t)]\mathbf{e_x} - \chi\sin(\theta)\sin[\phi(t)]\mathbf{e_y} + \cos\theta\mathbf{e_z}\Big)$$

with a turnover angle $\phi(t) = \frac{S}{\chi - 1}(t - t_0)$

- Final equation (similar to Bayly 1986):

$$\frac{dv_i}{d\phi} = \Big[\Big(\frac{2k_ik_j}{k^2} - \delta_{ij}\Big)\bar{A}_{jm} + 2\Big(\frac{k_ik_j}{k^2} - \delta_{ij}\Big)\bar{R}_{jm}\Big]v_m.$$

with $\bar{\boldsymbol{A}} = (\chi - 1)\boldsymbol{A}$ and $\bar{R}_{jm} = (\chi - 1)\epsilon_{jlm}\Omega_l/S$

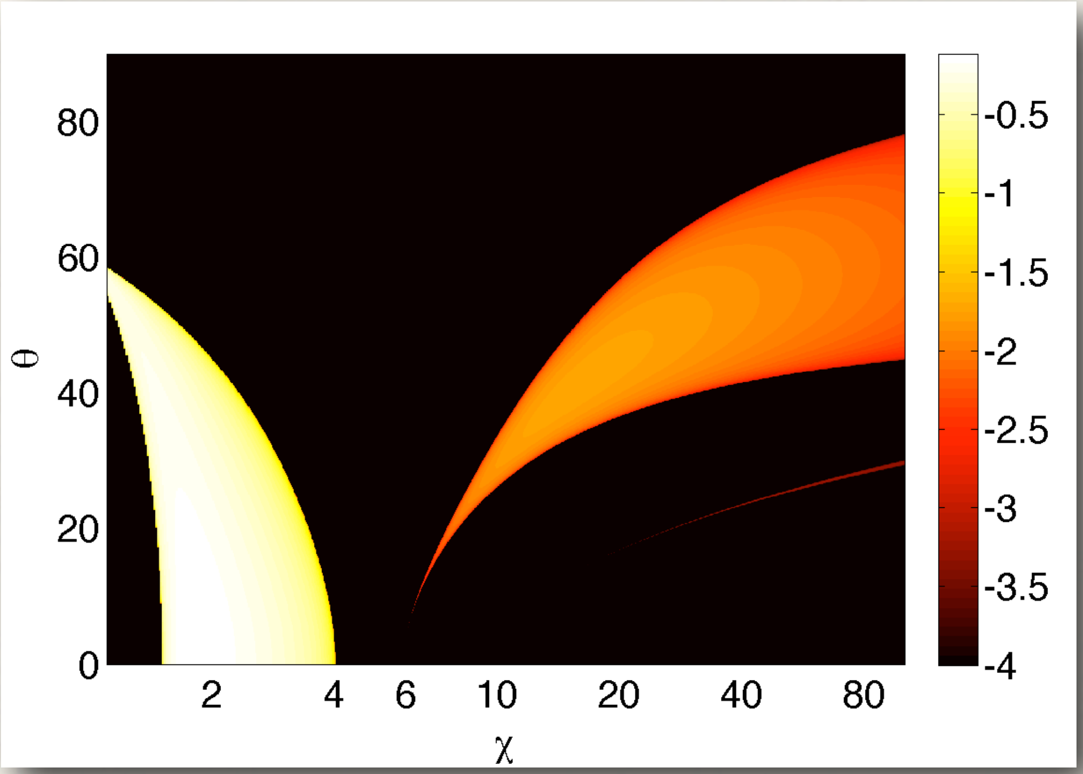
- Stability properties do not depend on |**k**|, but just on its direction !

Stability analysis

log(growth rate)

Vertical perturbations

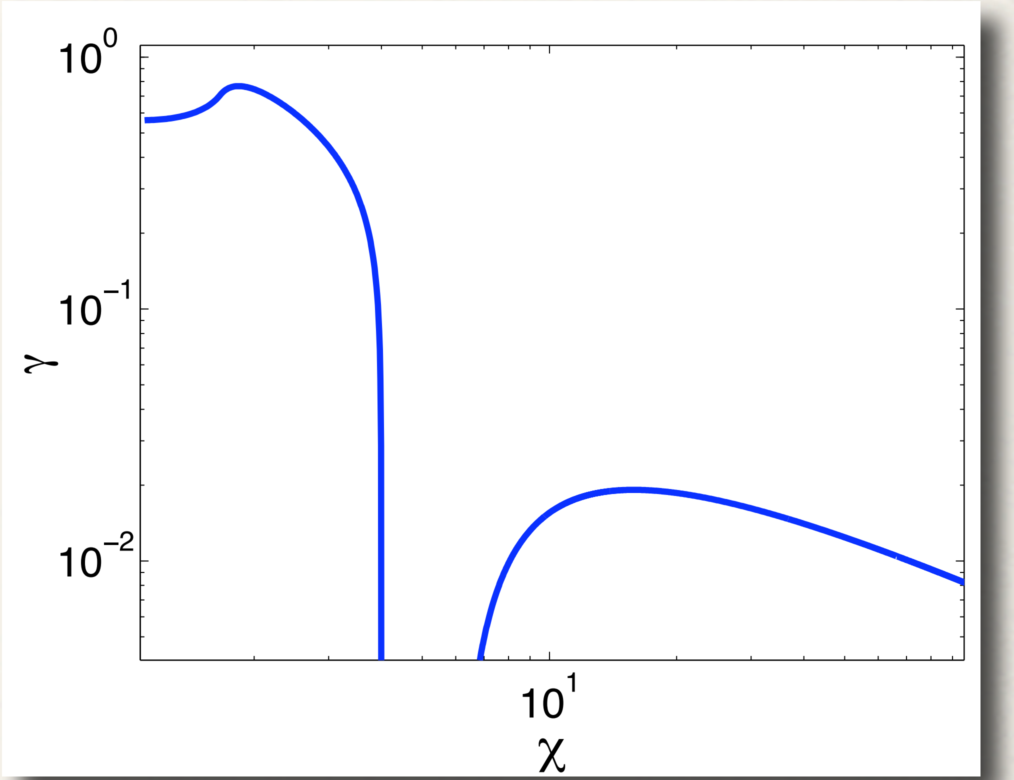
Horizontal perturbations



strong vortices (nearly circular)

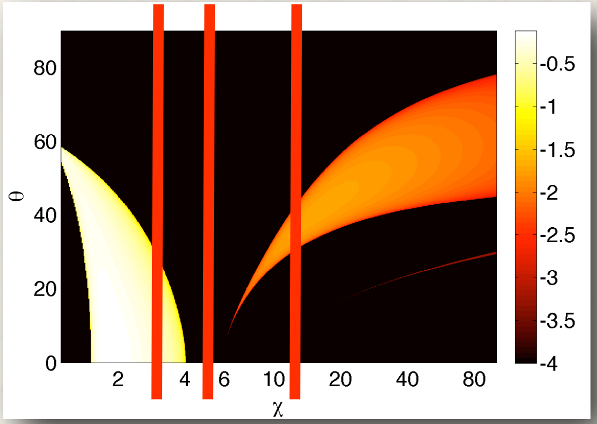
weak vortices (elongated)

Maximum growth rate

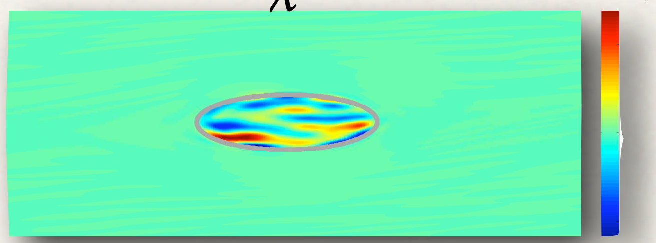


- Weak vortices always unstable with small growth rates
- Strong horizontal instability for $3/2<\chi<4$
- No instability (?) for $4<\chi<5.9$

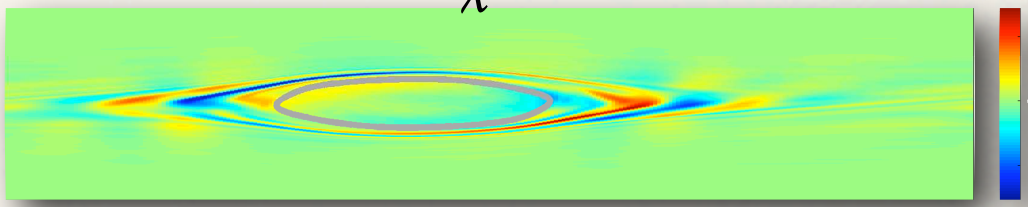
Elliptical instability simulations



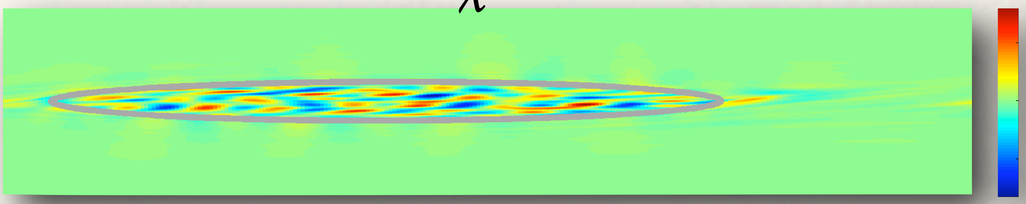
$\chi=3$



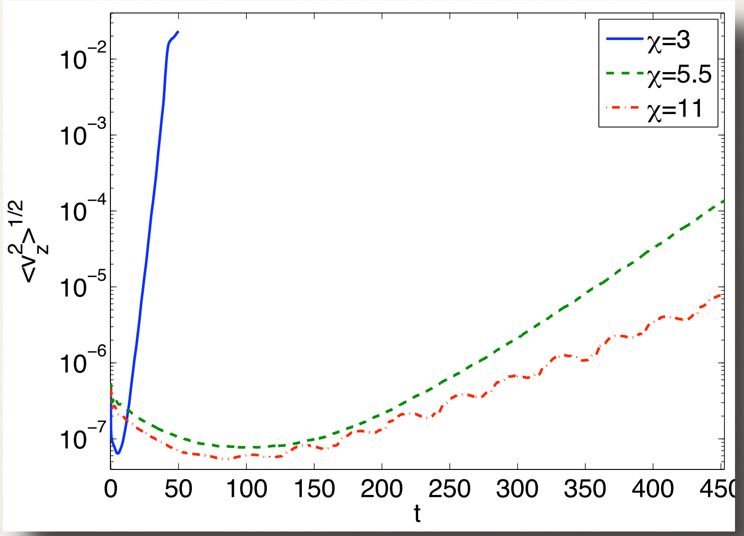
$\chi=5.5$



$\chi=11$



Perturbation localisation



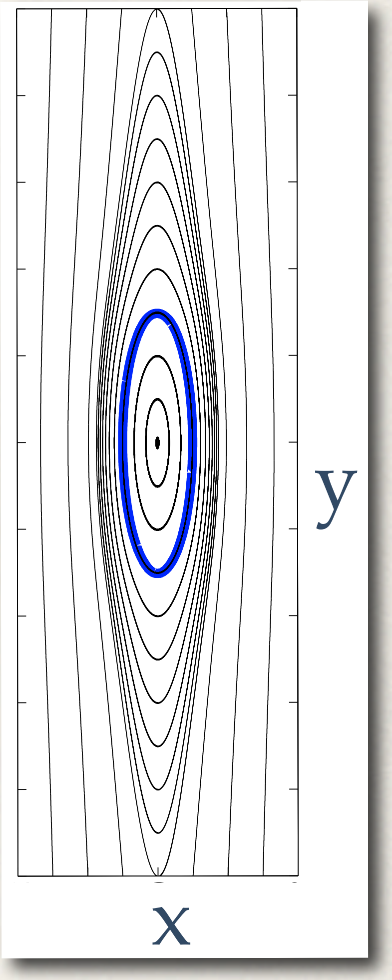
Temporal evolution

- Instability localised in the core for $\chi=3$ and $\chi=11$
- Instability localised outside of the core for $\chi=5.5$
- Growth rate comparable to linear theory
- Instability for $\chi=5.5$ can be explained by a resonance outside of the vortex core

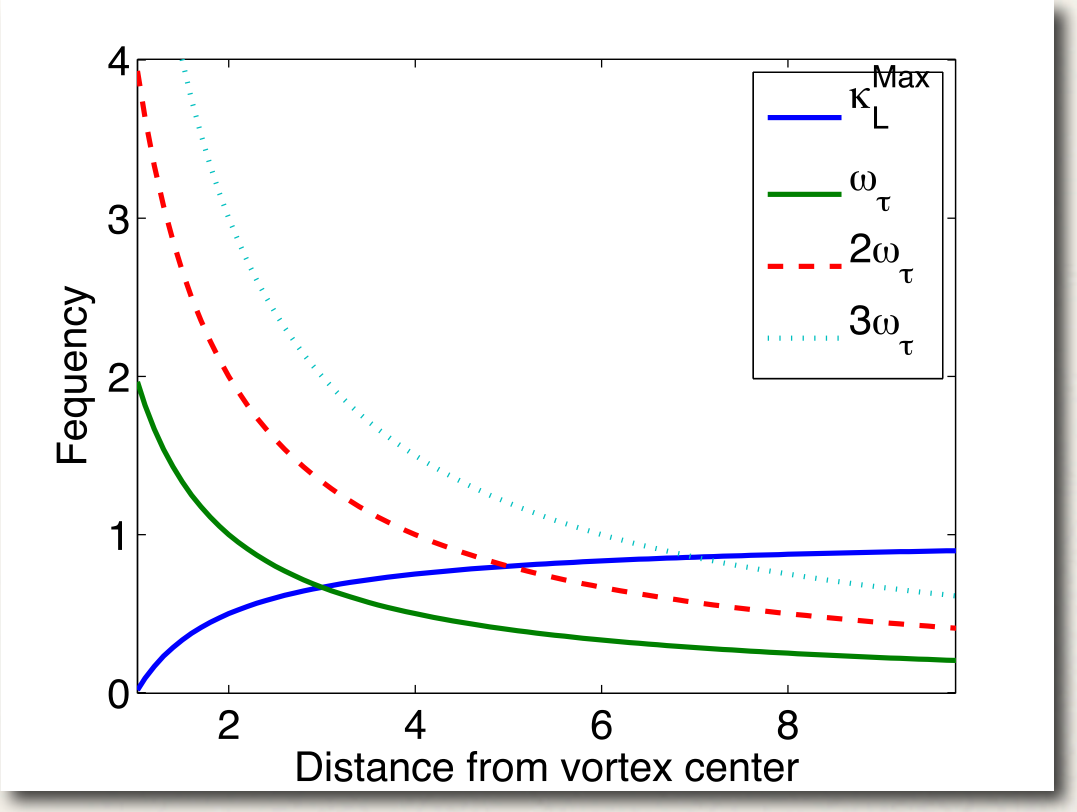
Phenomenological argument

- Two characteristic frequencies in 3D vortices:
  - Turnover frequency of *closed* streamlines ω_τ
  - Frequency of *local* inertial modes (modified epicyclic frequency) κ_L
- An elliptic instability exists when a resonance occurs on one streamline:

$$\kappa_L = n\,\omega_\tau \qquad \text{where} \qquad \kappa_L = \kappa_L^{\max}\frac{k_z}{|\mathbf{k}|}$$

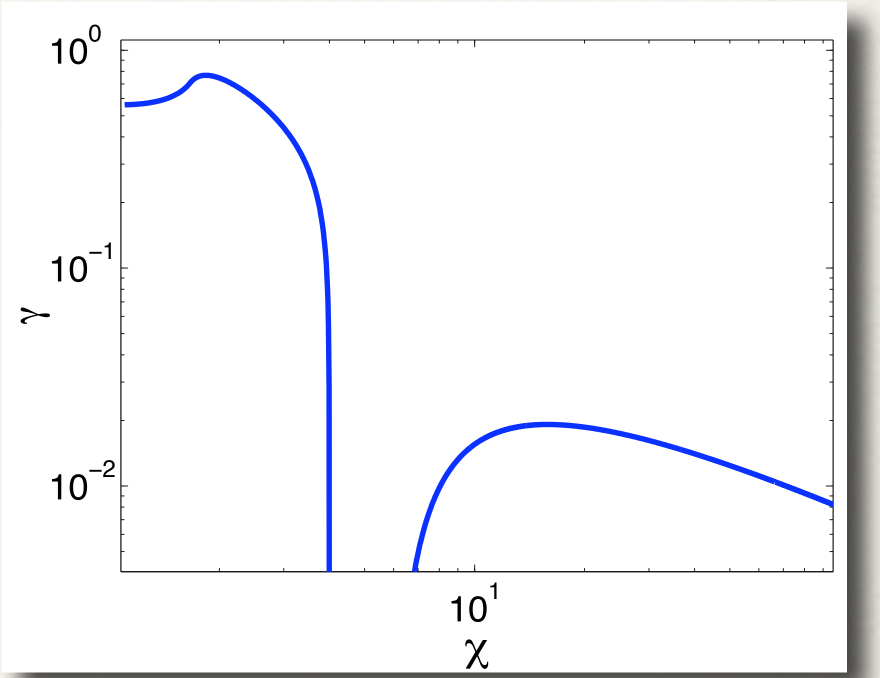


for a point vortex:

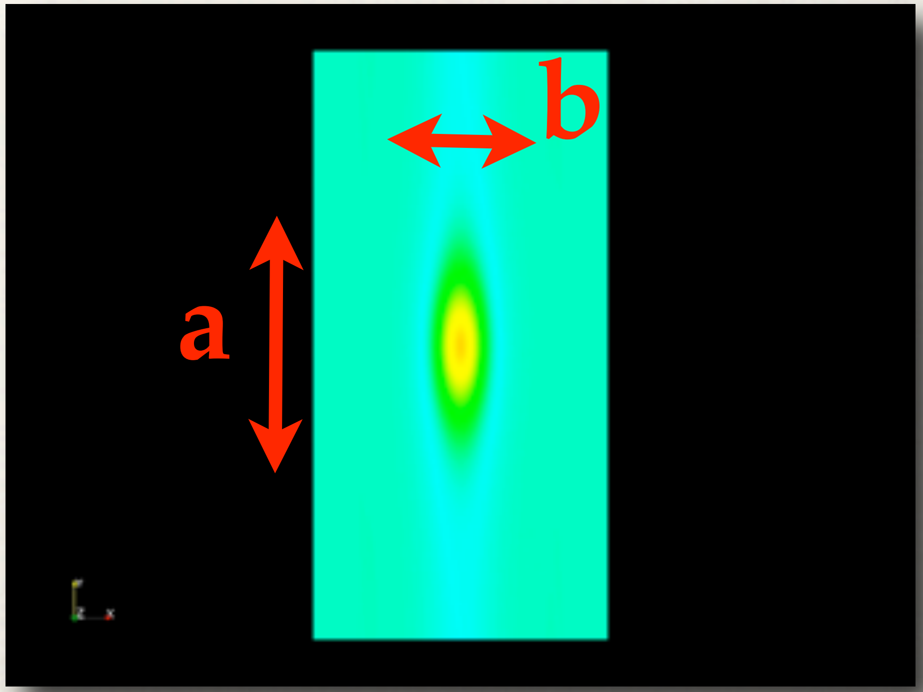


3D instabilities and the SBI

- 3D growth rates depend on the vortex aspect ratio χ=a/b.



Elliptical instability growth rate in a Kida vortex



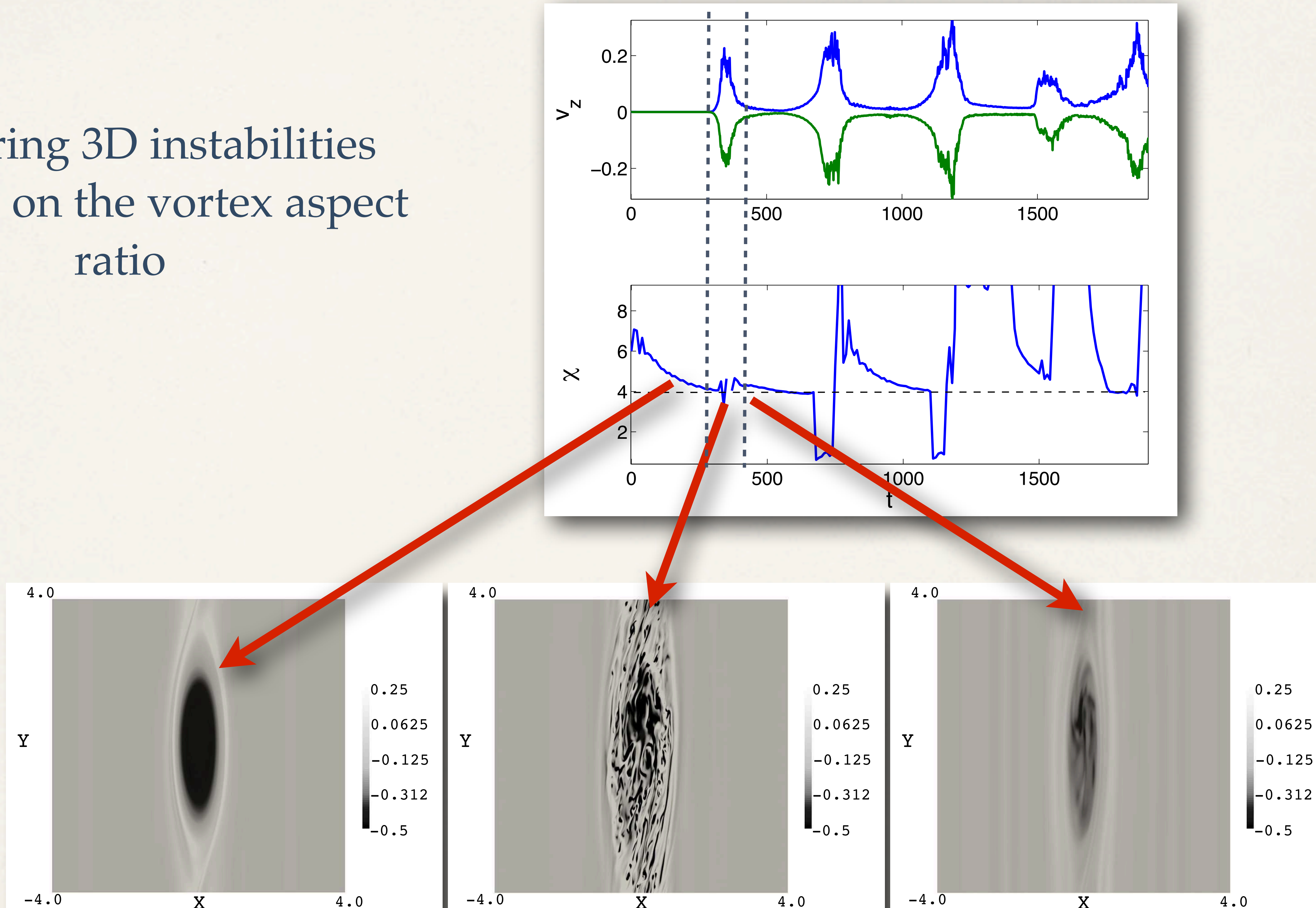
- The baroclinic instability amplifies the vortex

➡ χ decreases with time

➡ At some point, 3D instabilities will be dominant...

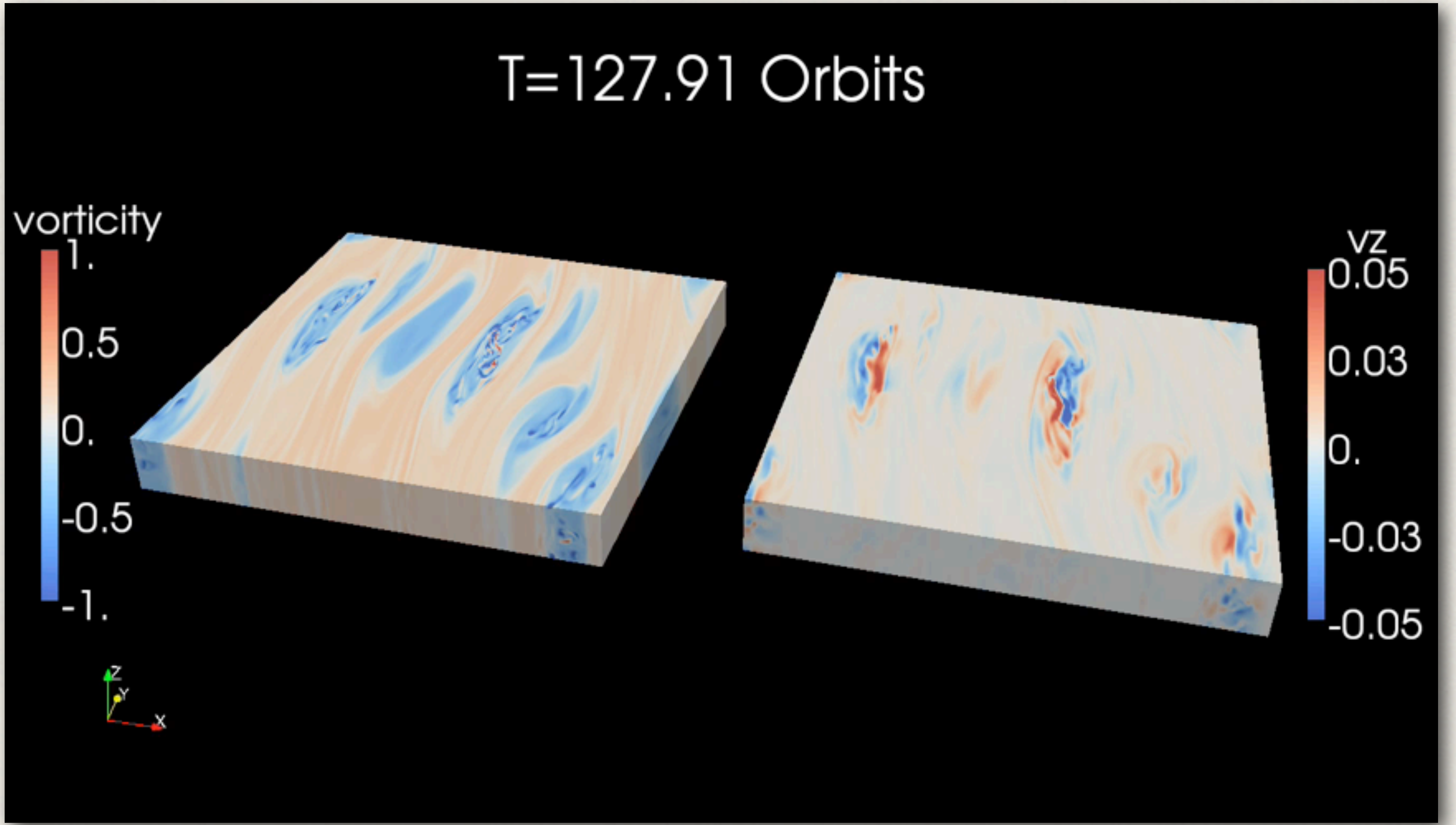
3D instabilities and the SBI

Triggering 3D instabilities depends on the vortex aspect ratio



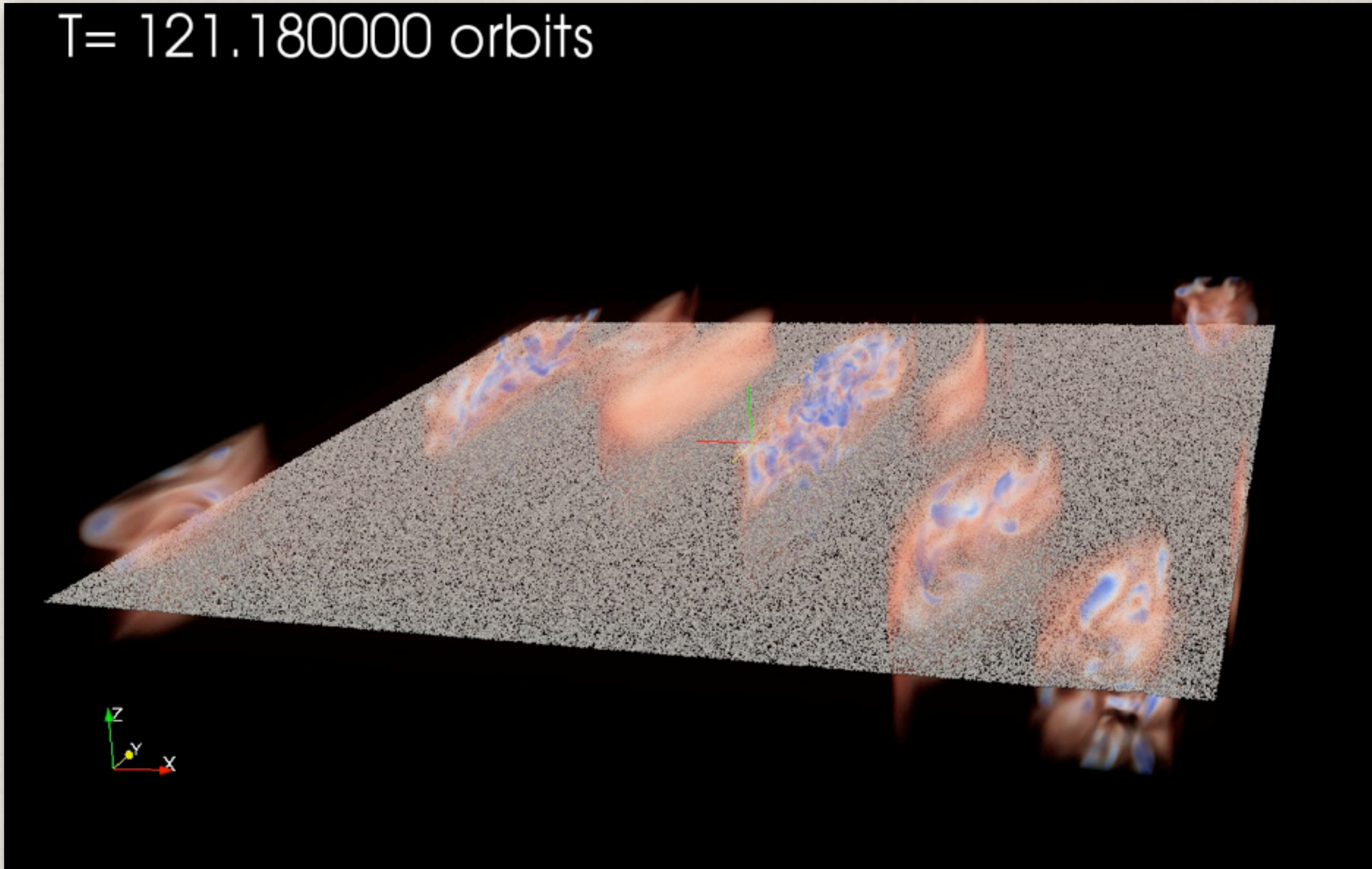
3D instabilities and the SBI (cont'd)

After some time, a quasi-equilibrium is reached...



Self-sustained turbulent vortices
SNOOPY (1024 x 512 x 128)

Dust accumulation in turbulent vortices



Conclusions

- A «steep» temperature profile will generate vortices *everywhere* in a disc.
- Vortices produce waves which
  - transport angular momentum
  - generate vortex migration

- *Open questions:*
  - Magnetic fields? (magneto-elliptic instabilities, MRI turbulence) cf W. Lyra talk
  - 3D circulation? cf H. Méheut talk
  - Temperature profile in the disc? cf H. Klahr talk

Thank you