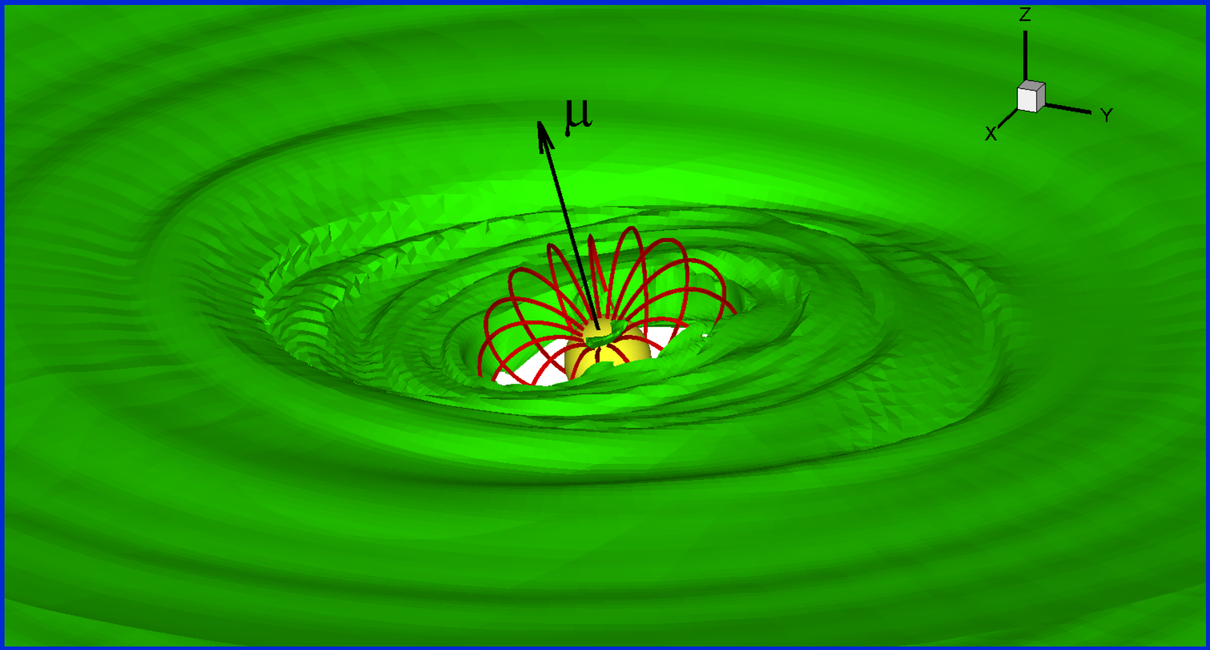
Global 3D MHD Simulations of Waves in Accretion Discs



Richard Lovelace & Marina Romanova

Instabilities and Structure in Protoplanetary Disks
September 18, 2012

Linear Theory of Waves in a Thin Disc

Perturbations of the hydrodynamical values $Q = p, \rho, , ..$ are expanded as (e.g., Kato et al. 1998)

$$Q(r, z, \phi, t) = \sum_{m,n,\omega} Q_{m,n,\omega}(r) H_n\left(\frac{z}{h}\right) \exp(im\phi - i\omega t) \ .$$

Here, ω is the angular frequency of the wave, $m = 0,\ 1,\ 2...$ is the number of arms in the azimuthal direction, $H_n, n \geq 0$ are the Hermite polynomials (with $n = 0,\ 1,\ 2, ..$), and $h(r)$ is the half-thickness of the disc at radius r. Linearization of the equations of motion leads to equation

$$\frac{d^2\psi}{dr^2} + \frac{(\tilde{\omega}^2 - \kappa^2)(\tilde{\omega}^2 - n\Omega_\perp^2)}{c_s^2\tilde{\omega}^2}\psi = 0 \ ,$$

$\tilde{\omega} \equiv \omega - m\Omega$ is the Doppler-shifted angular frequency of the wave seen by an observer orbiting at the disc's angular velocity $\Omega(r)$, $\Omega_\perp$ is the frequency of oscillations perpendicular to the disc, $\kappa^2 = r^{-3}d(r^4\Omega^2)/dr$ is the square of the radial epicyclic frequency, and c_s is the sound speed in the disc. The local free wave solution in the WKBJ approximation

$$\psi \propto k_r^{-1/2} \exp\left[i \int^r dr k_r(r)\right] \ , \qquad d^2\psi/dr^2 = -k_r^2\psi$$

where k_r is the radial wavenumber.

Linear Theory of Waves in a Thin Disk

The dispersion relation:

$$\frac{(\tilde{\omega}^2 - \kappa^2)(\tilde{\omega}^2 - n\Omega_\perp^2)}{\tilde{\omega}^2} = k_r^2 c_s^2 \ .$$

$\tilde{\omega} = \pm\kappa$ - an outer (+) or an inner (-) Lindblad resonances
$\tilde{\omega} = \pm n\Omega_\perp$ - vertical resonances
$\tilde{\omega} = 0$ - corotation resonance.

In our model, waves are 'driven' by the rotating tilted magnetosphere of the star.

$$f = f(r, z, \phi - \Omega_* t) = \sum_m f_m(r, z) \exp[im(\phi - \Omega_* t)] \ , \qquad (1)$$

where Ω_* is the angular frequency of the star. In linear approximation, we suggest that the $m-$harmonic of this force excites the $m-$armed wave in the disc. The frequency of this force is $m\Omega_*$. For a Keplerian disc, the condition for the Lindblad resonances will be $m\Omega_* - m\Omega = \pm\Omega$, or

$$r_{LR} = r_{cr}\Big(1 \pm \frac{1}{m}\Big)^{2/3} \ , \qquad r_{cr} = (GM/\Omega_*^2)^{1/3}$$

Waves in Keplerian Disc

One-armed Bending Wave (n=1, m=1)

In Keplerian disc $\kappa = \Omega$ and $\Omega_\perp = \Omega$

$$\left\{ \left(\frac{k_r c_s}{\Omega} \right)^2 - \left[\left(\frac{\tilde{\omega}}{\Omega} \right)^2 - 1 \right] \left[1 - \left(\frac{\Omega}{\tilde{\omega}} \right)^2 \right] \right\} z_w = 0,$$

The disc has a finite thickness: $h/r = c_s/(r\Omega) = 0.05, \quad \omega = \Omega(r_{cr})$.

Two-armed Bending Waves (n=1, m=2)

$$|k_r r_{cr}| = \frac{r}{h} \frac{1}{\bar{r}} \left[[(\bar{r}^{3/2} - 1)^2 - 1/4][4 - (\bar{r}^{3/2} - 1)^{-2}] \right] ,$$

Density Waves (n=0, m=1,2)

$$\tilde{\omega}^2 = \Omega^2 + k_r^2 c_s^2 ,$$

$$|k_r r_{cr}| = (r/h)(1/\bar{r}) \left[m^2 (\bar{r}^{3/2} - 1)^2 - 1 \right]^{1/2} ,$$

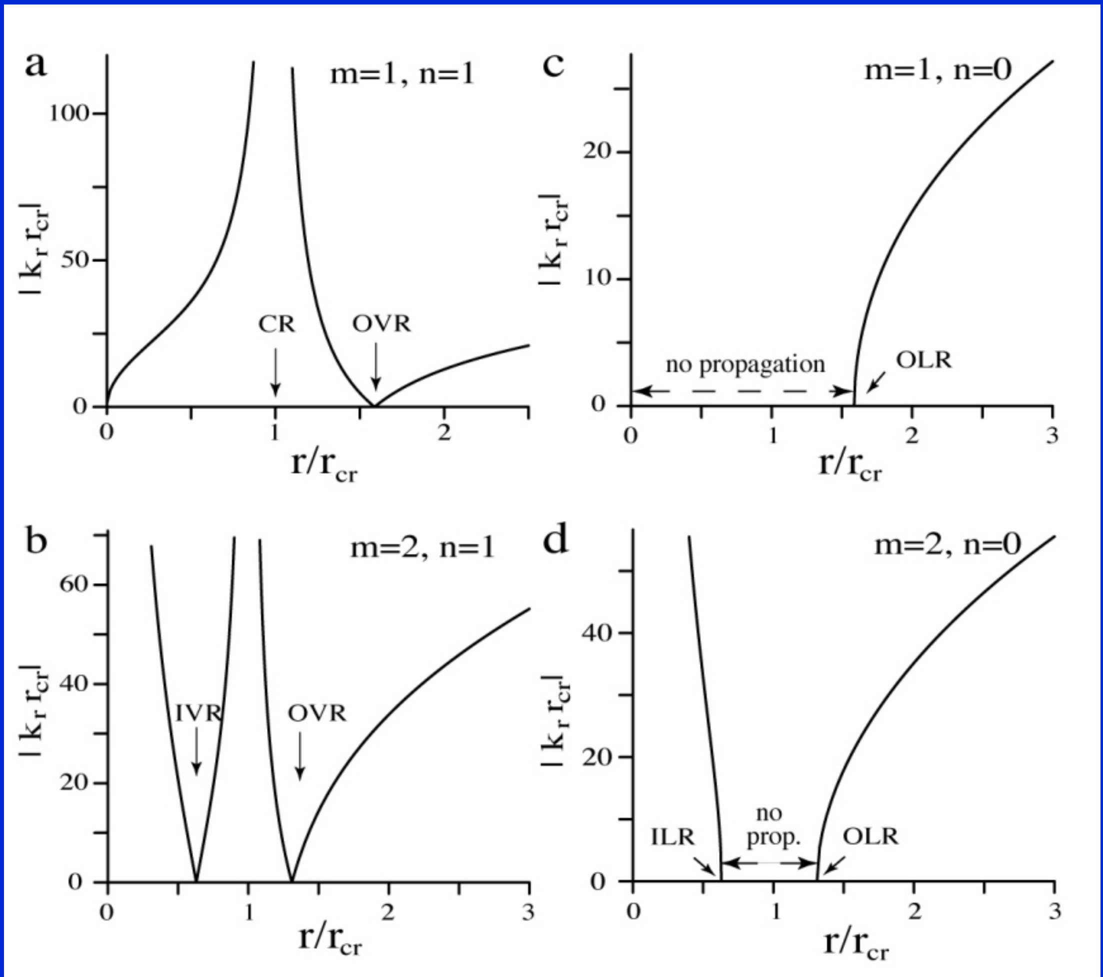
where $\bar{r} \equiv r/r_{cr}$ with r_{cr} is the corotation radius

Resonances in the Keplerian disc

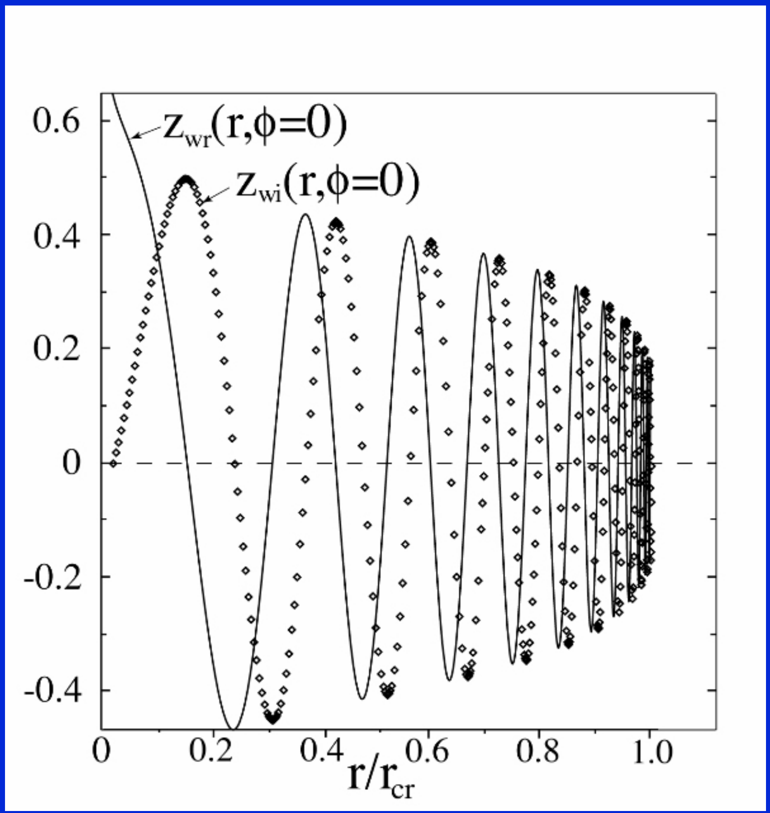
Bending waves | Density waves

m=1, n=1 | m=1, n=0

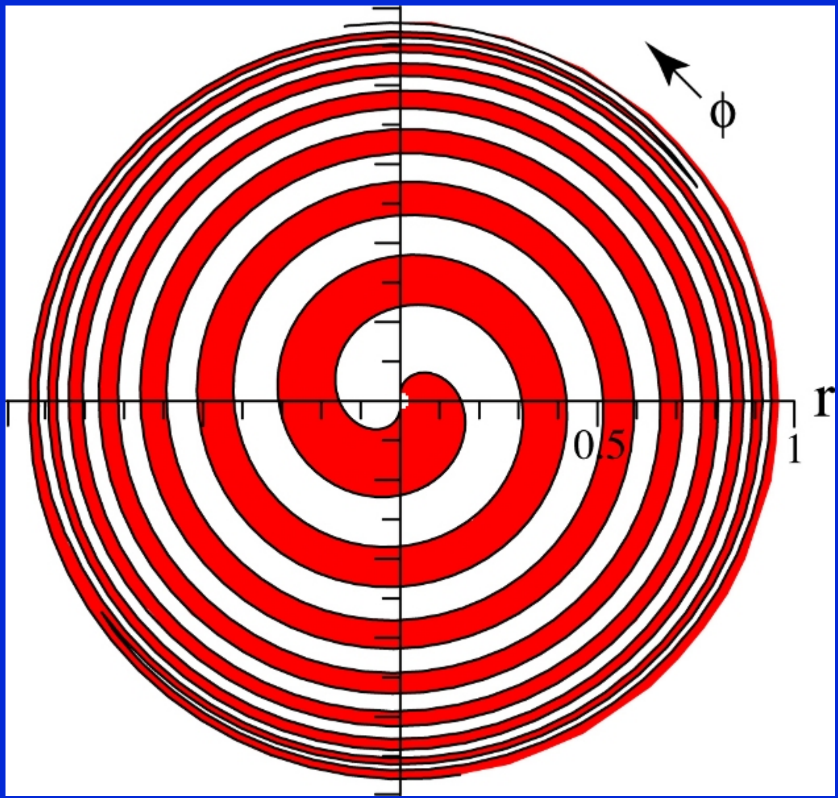
m=2, n=1 | m=2, n=0



Bending Wave ($r < r_{cr}$)

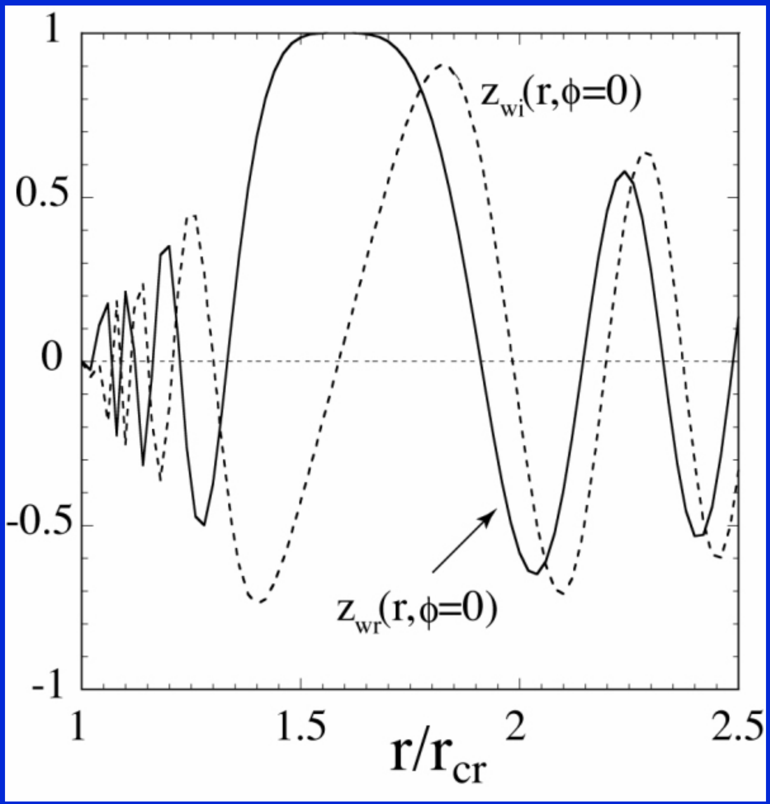


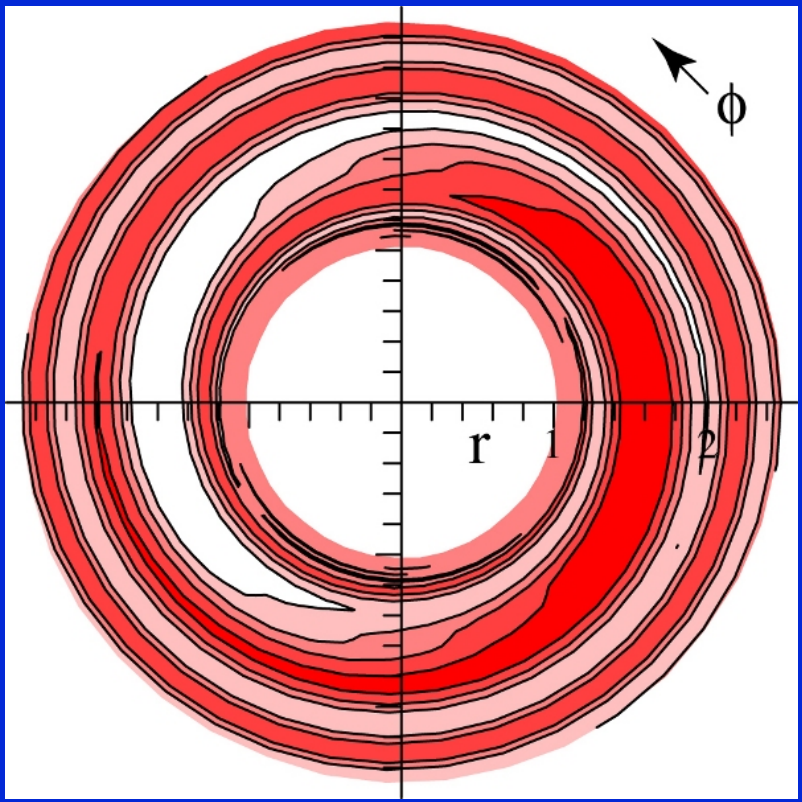
Real and imaginary parts



Amplitude of the wave
Tightly wound spiral

Bending Wave ($r > r_{cr}$)





Real and imaginary parts

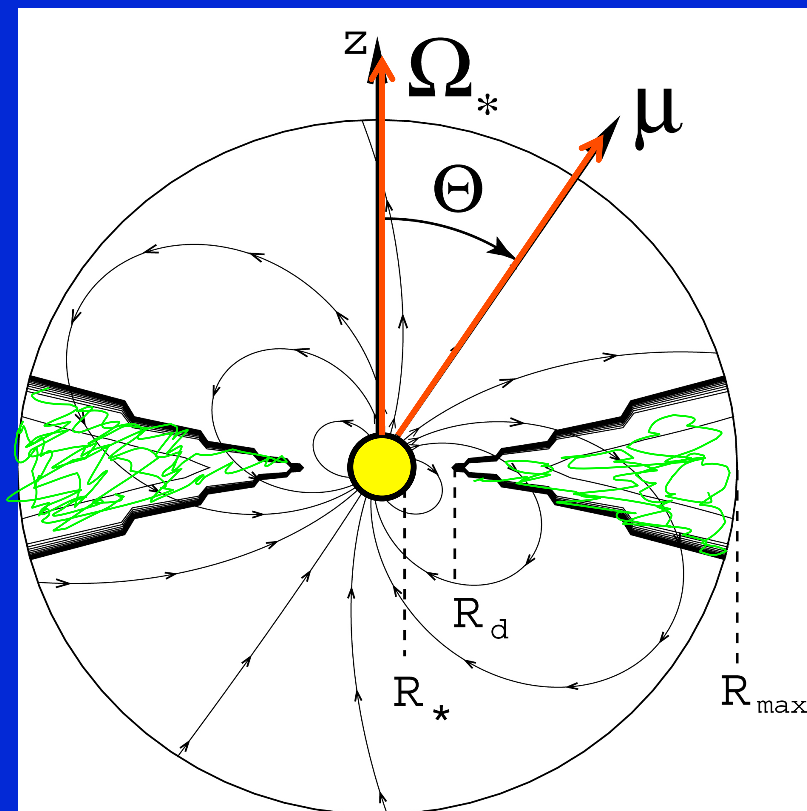
Amplitude of the wave
More relaxed spiral
Maximum at OVR

3D MHD Simulations of Waves

- Waves are excited by tilted rotating magnetosphere with the dipole field
- Disc is dense, corona has 100 times lower density
- Disc and corona are in initial equilibrium *(Romanova et al. 2002)*
- Use Godunov-type code
- Viscosity is incorporated into the code, α - type viscosity *(Shakura & Sunyaev 1973).* It helps to support slow accretion during long time
- "Cubed Sphere" grid 61x61x140 *(Koldoba et al. 2002, Romanova et al. 2003)*

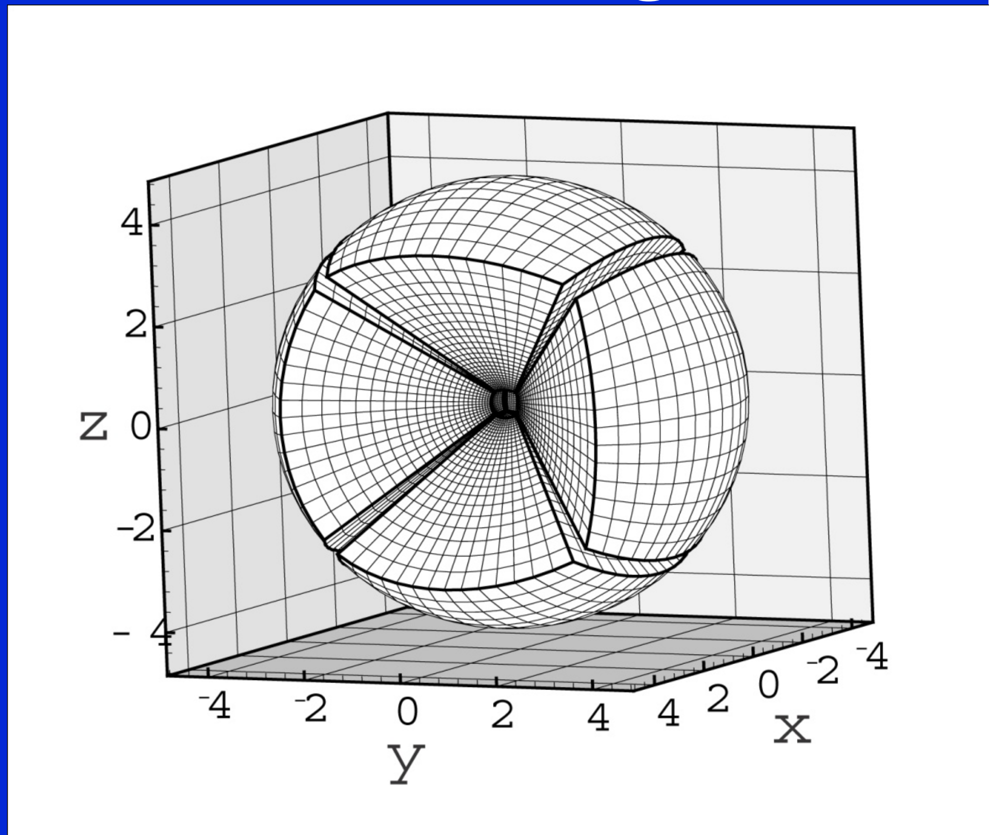
3D MHD Simulations: Model

Initial setup:



Inner part of the simulation region; but R_{max} =30-50R_*

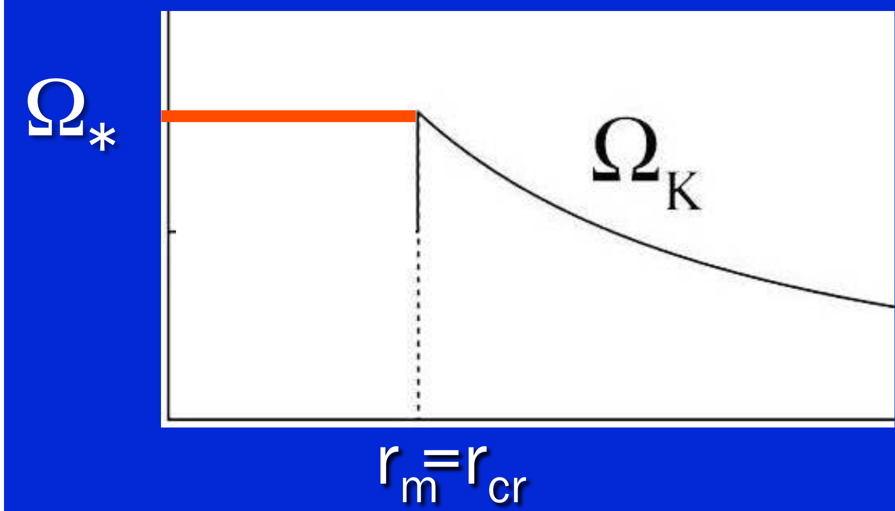
Cubed sphere grid



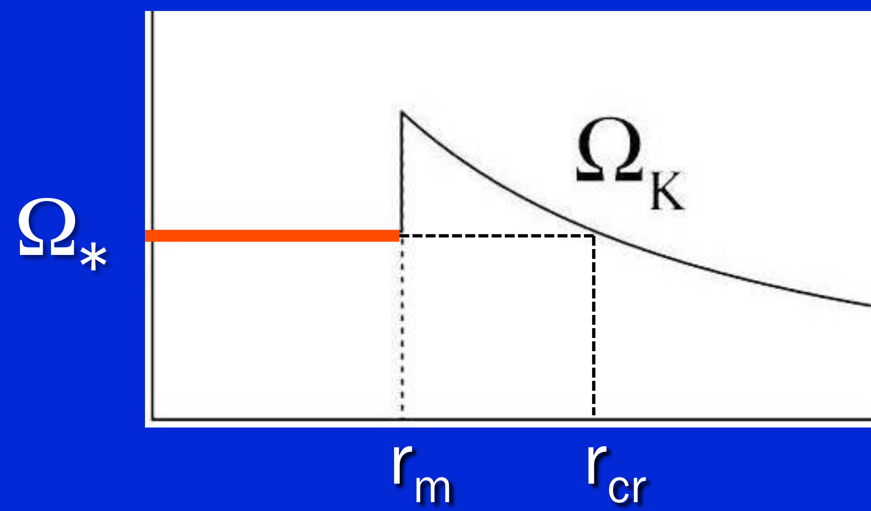
"CS" has advantages over Cartesian and spherical grids

Two Cases: Fast or Slow Rotation

Fast rotation



Slow rotation



I. Magnetosphere corotates with the inner disc

II. Magnetosphere rotates slower than the inner disc

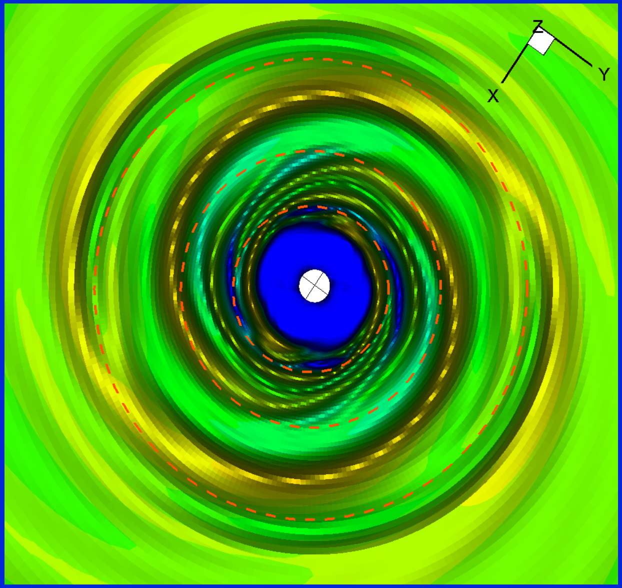
r_{cr} - corotation radius, where $\Omega_K = \Omega_{star}$

r_m - corotation radius, where $B^2/8\pi = \rho\, v^2 + p$

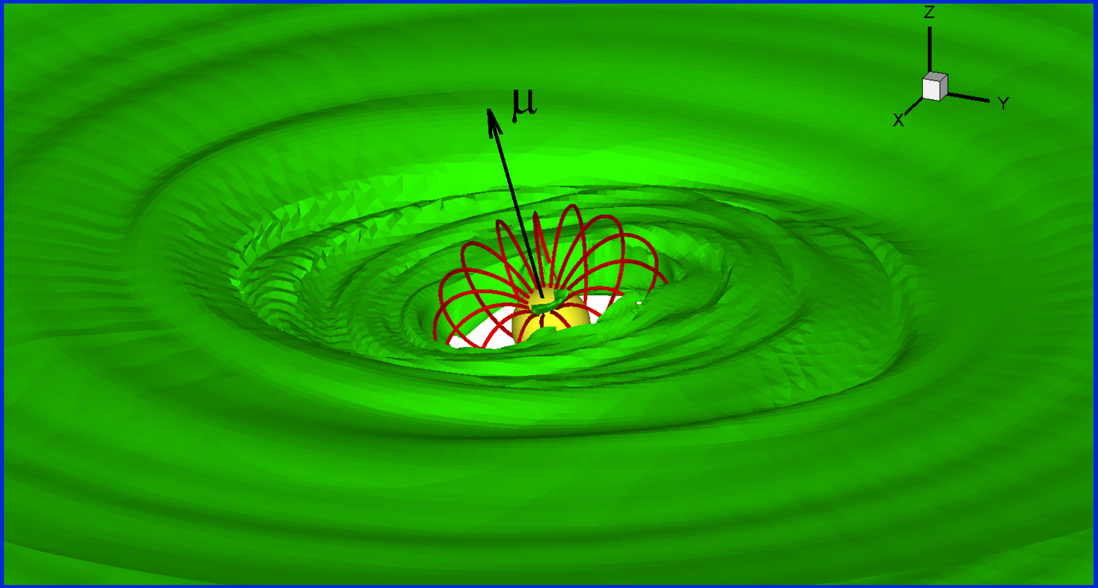
Waves in the Disc

$$\sigma = \sigma(t, r, \phi) = \int \rho dz \ , \qquad z_w(t, r, \phi) = \frac{\int \rho z dz}{\int \rho dz}$$

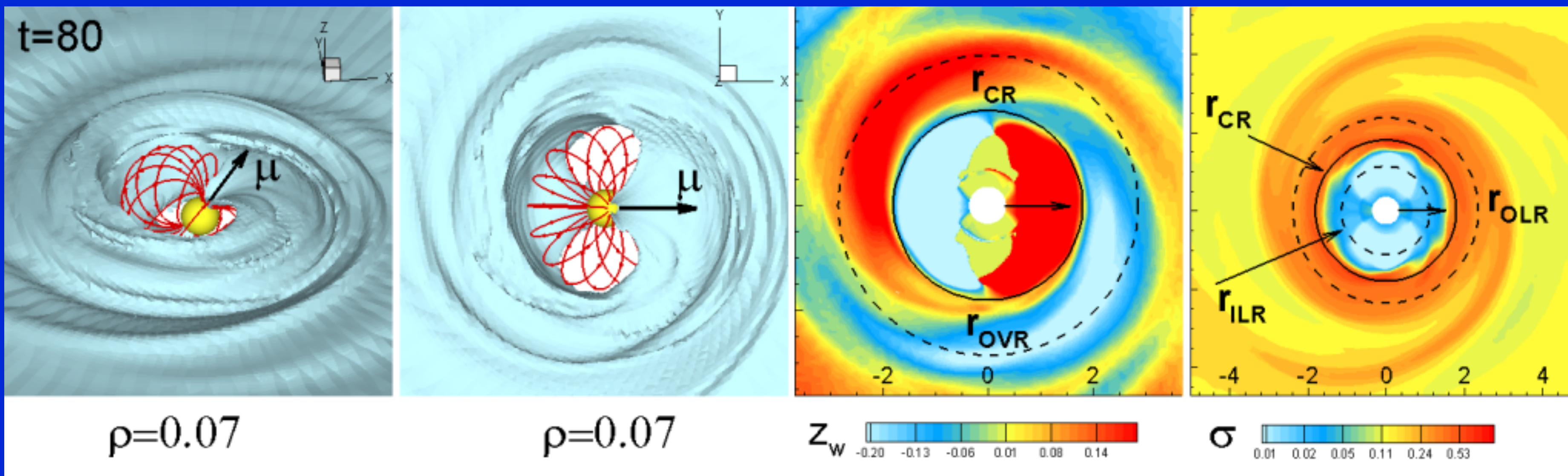
Density waves



Bending wave



I. Magnetosphere Corotates with the Disc



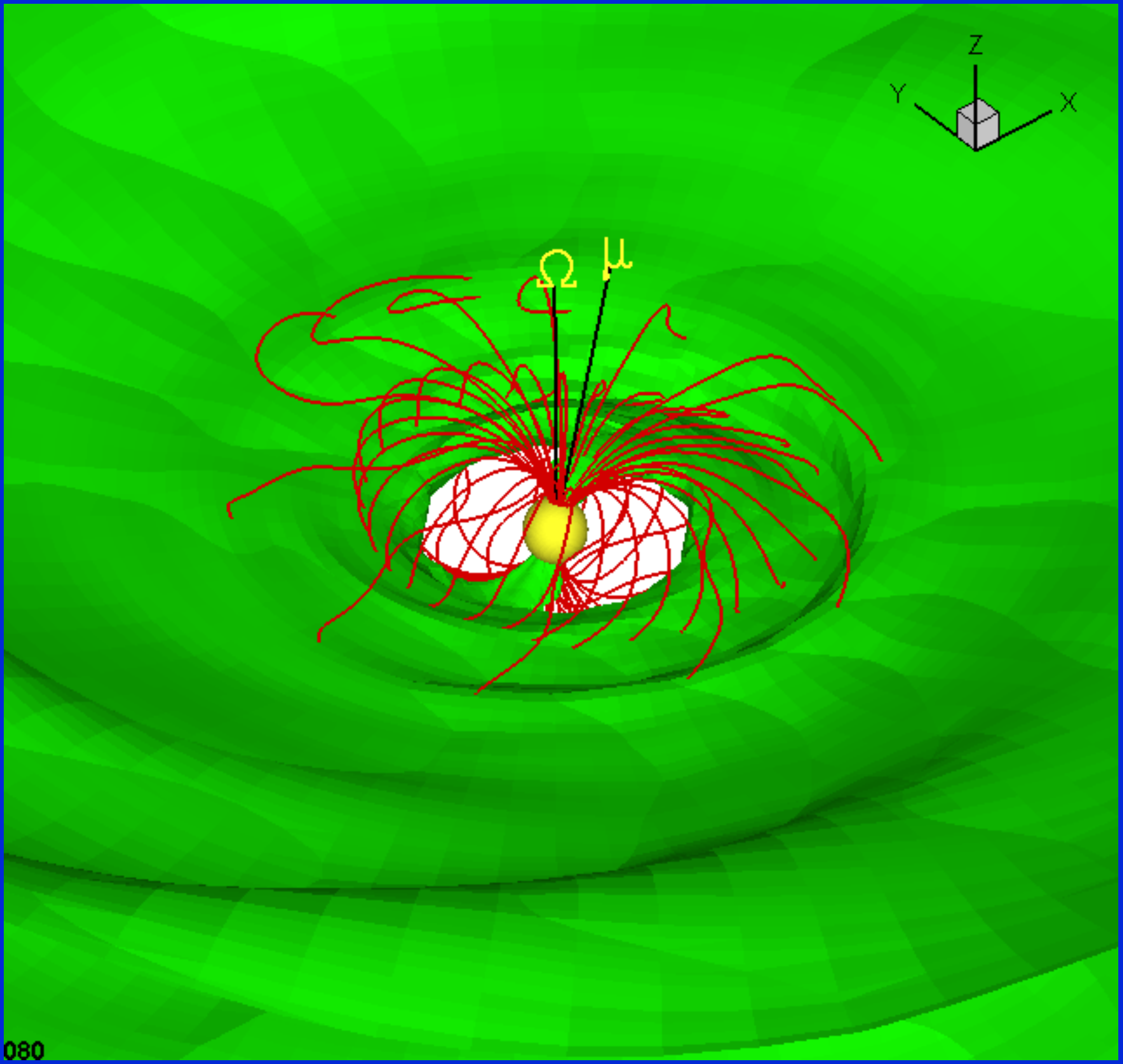
3D side view   3D face view   Bending wave   Density waves

A high-amplitude bending wave (a warp) forms and corotates with the star

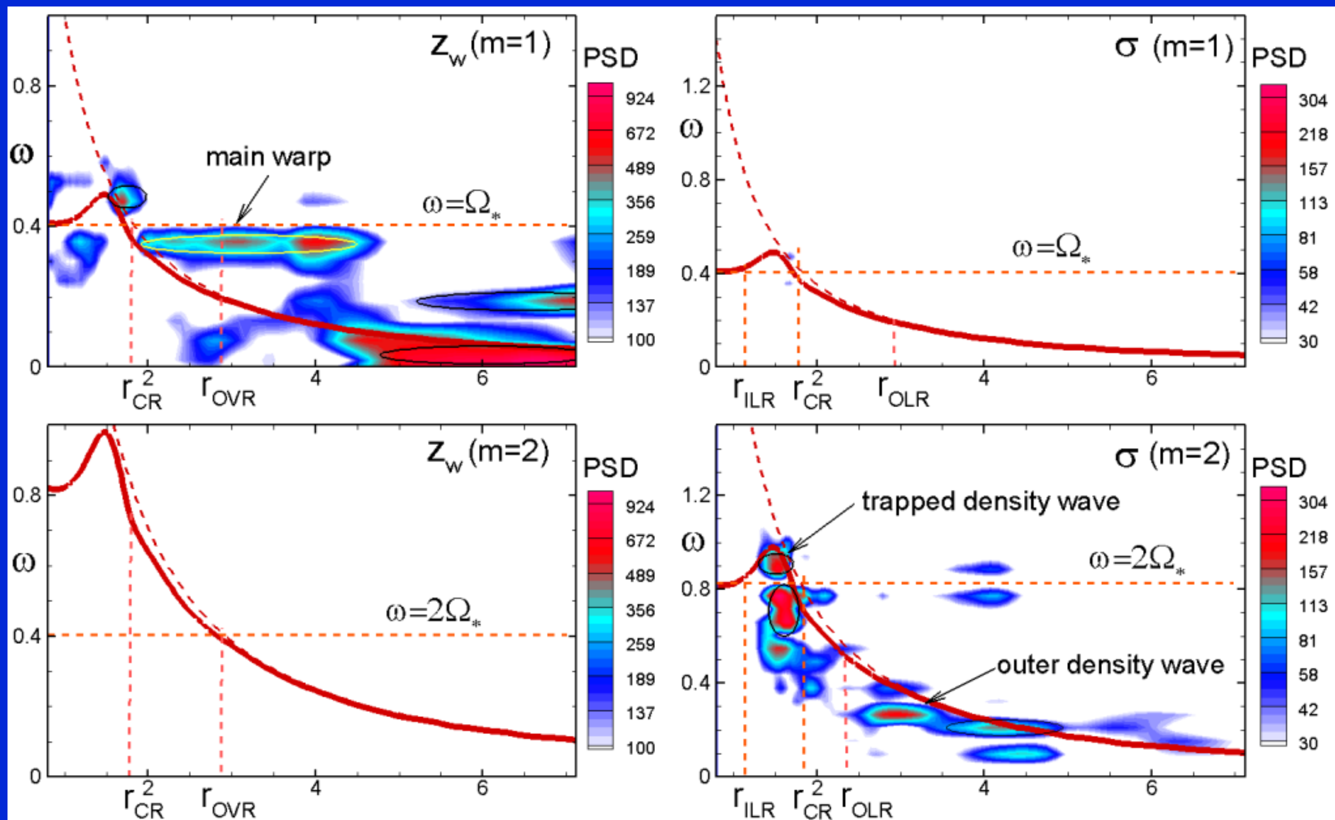
Maximum amplitude of the warp is at the OVR as predicted by the theory

Romanova, Ustyugova, Koldoba, Lovelace 2012, MNRAS, submitted

An Example of Bending Waves

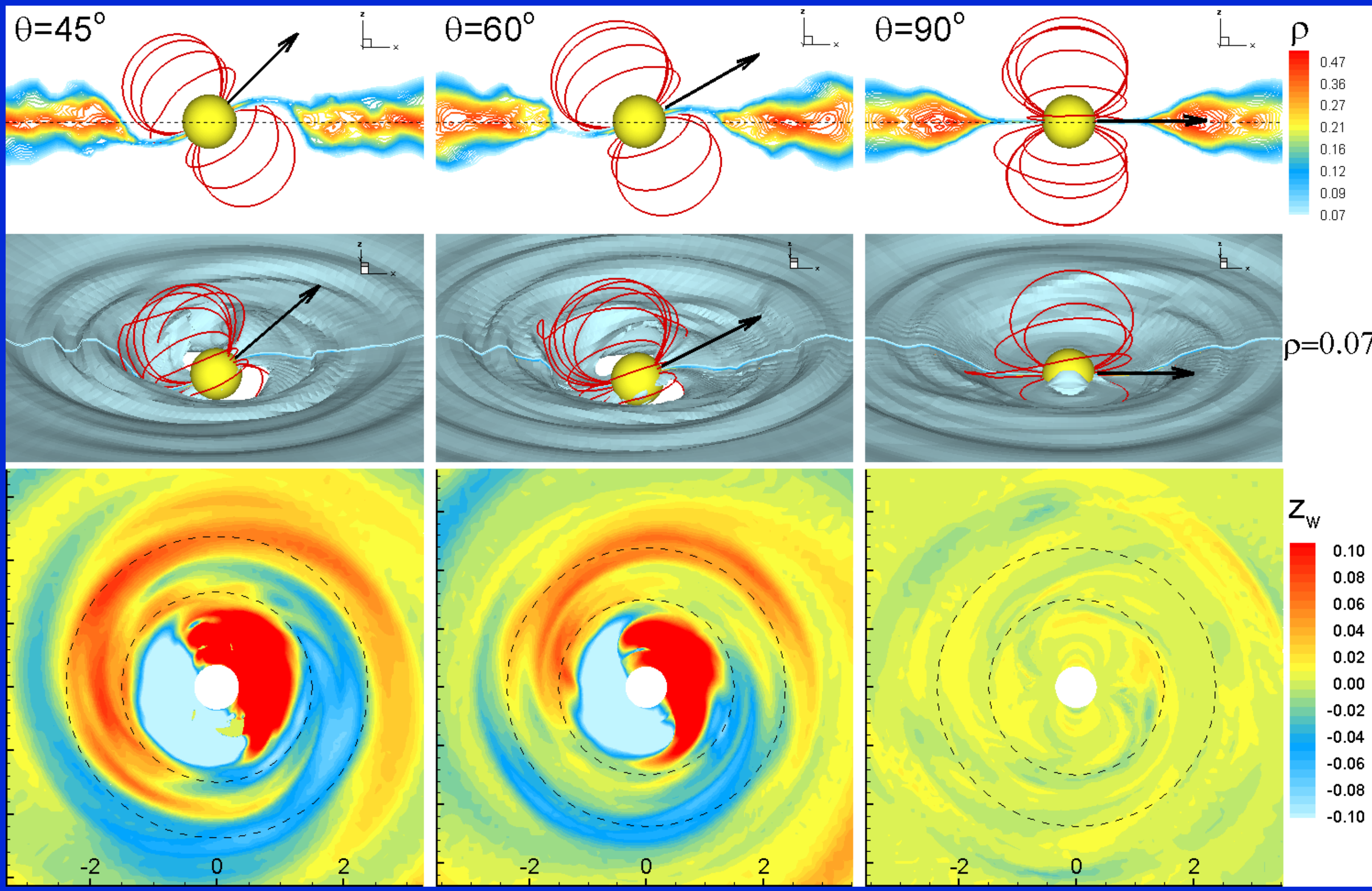


Power Spectral Density of Waves

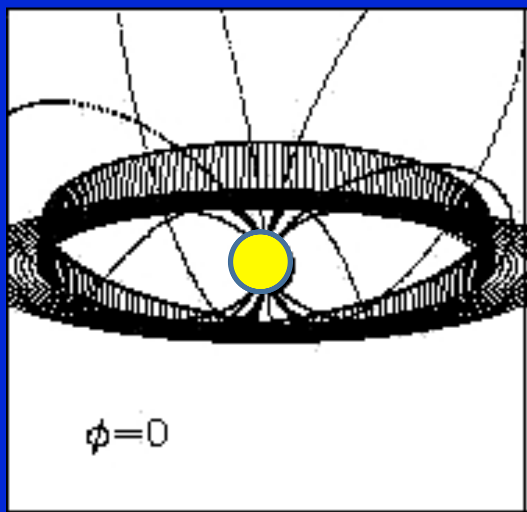


- One-armed bending wave corotates with the star, max. at OVR
- Two-armed density waves
- Corotating warp has been predicted by *Terquem & Papaloizou 2000*

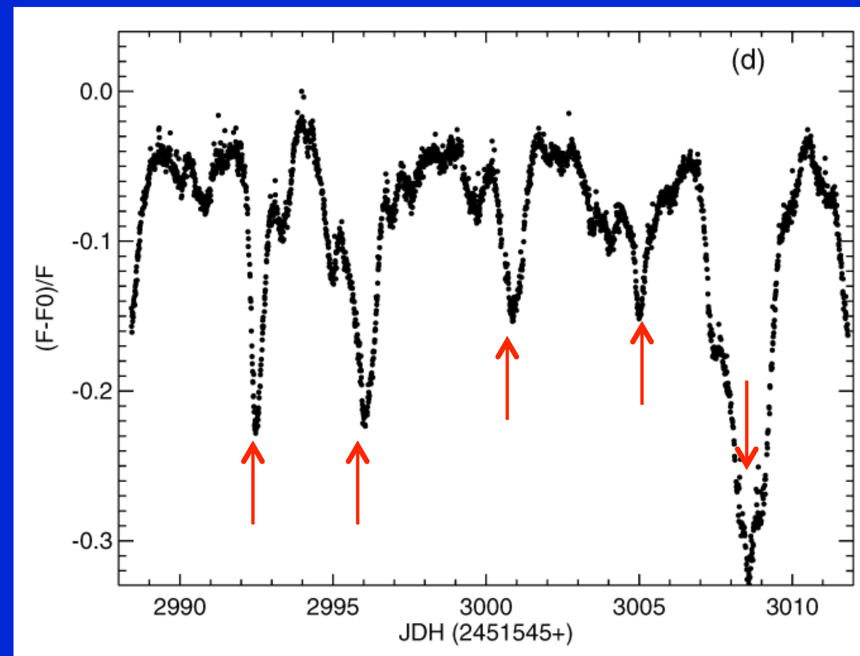
Warps are Present at Different Tilts



Dips in light curves of young stars



• Empirical model of obscuration of stellar light by the tilted disc (*Bouvier et al. 2007*)

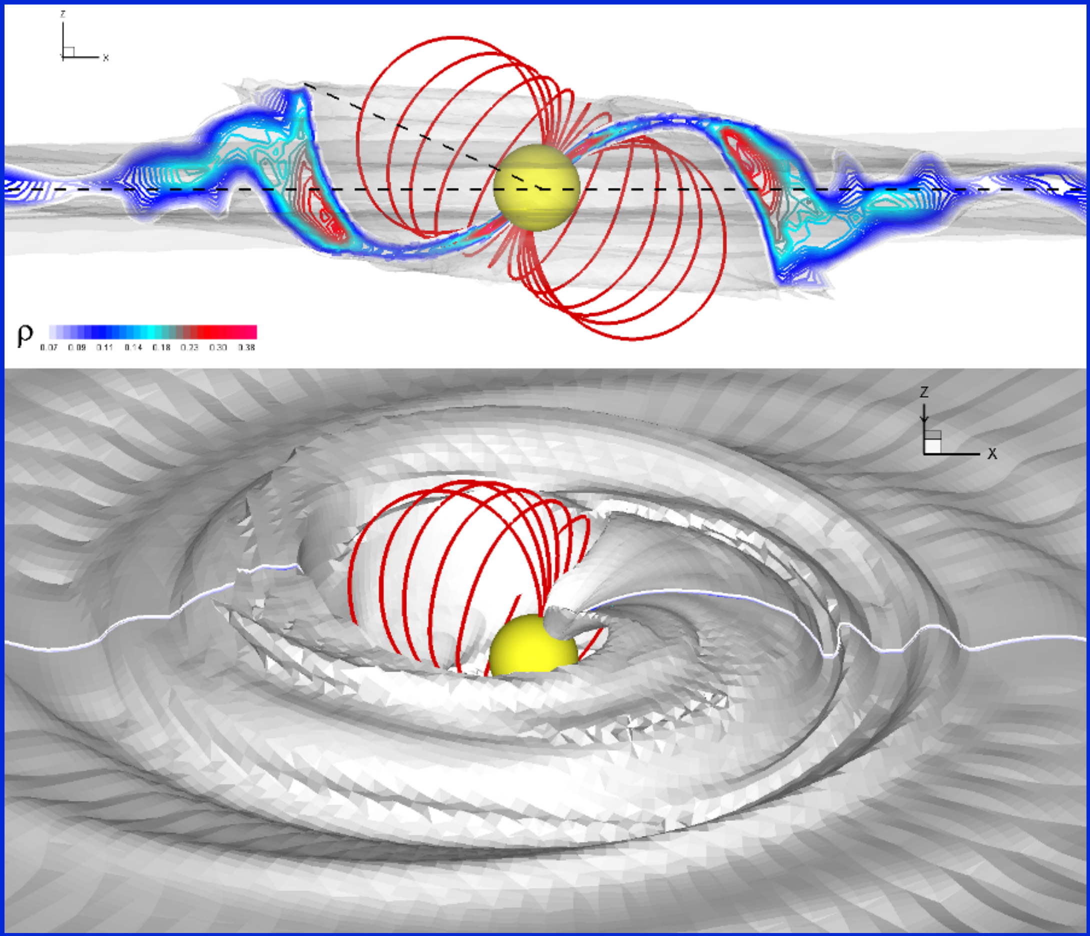


•Dips in light curve of young star AA Tau

• Time interval – about period of the star (*Alencar et al. 2010*)

About 28% of classical T Tauri stars stars have dips (*Alencar et al. 2010*)

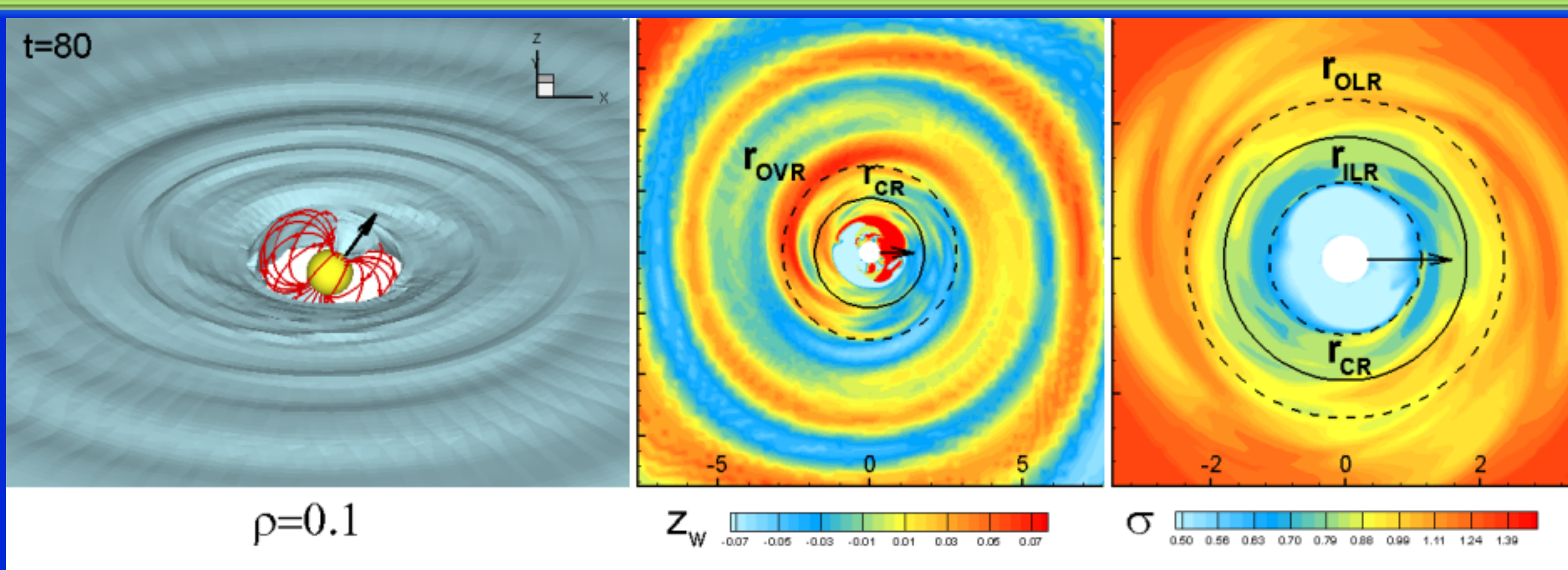
Warp can block stellar light



- Amplitude of the warp can be large, 30%
- Observations show possible diminishing of light in T Tauri stars

Bouvier et al. 1997, 2003, 2007; Alencar et al. 2012

II. Magnetosphere Rotates Slower
than the Inner Disc

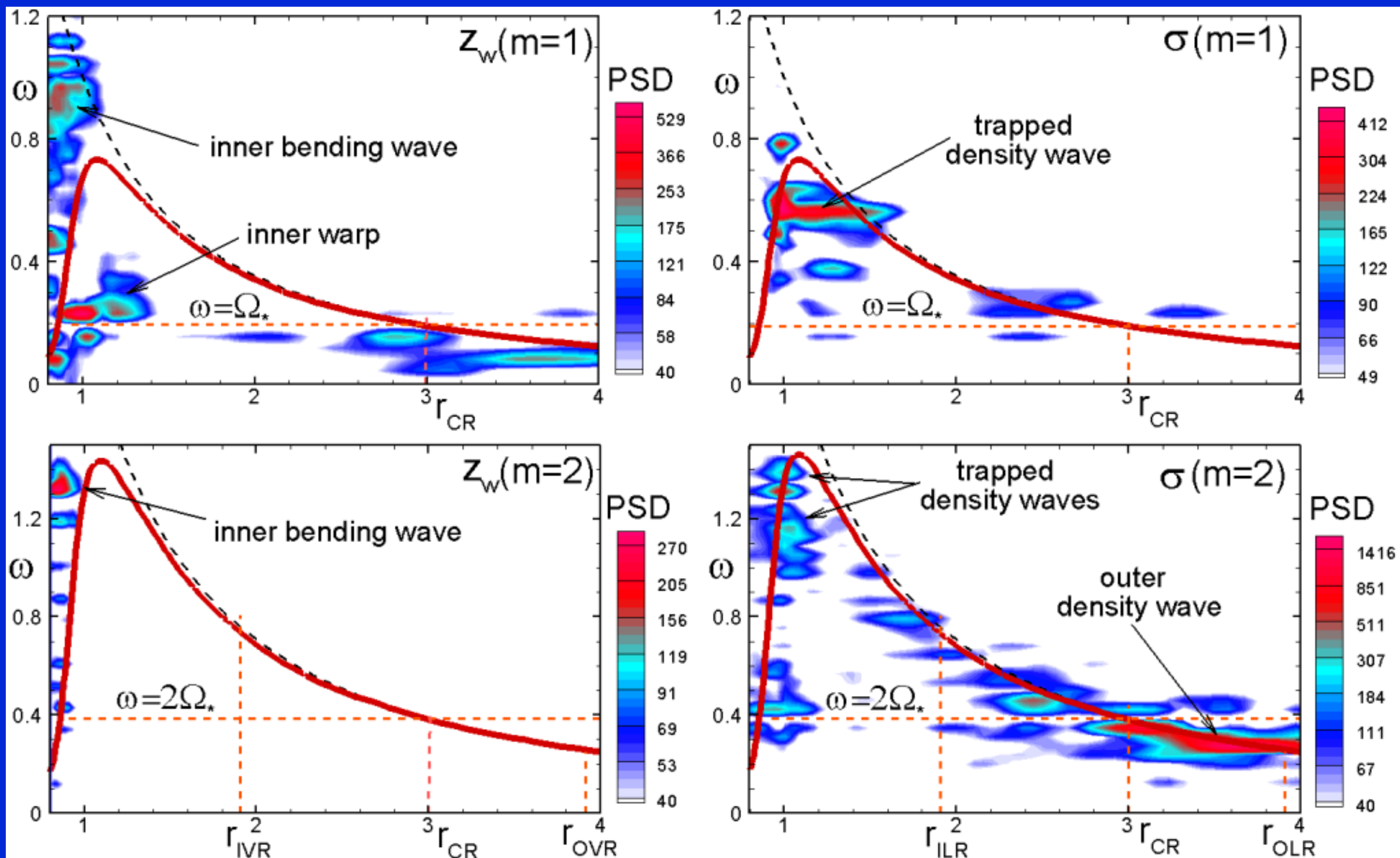


3D view　　Bending wave　　Density waves

- Bending waves propagate to large distances

Romanova, Ustyugova, Koldoba, Lovelace 2012

NO Large Warp Corotating with the Star

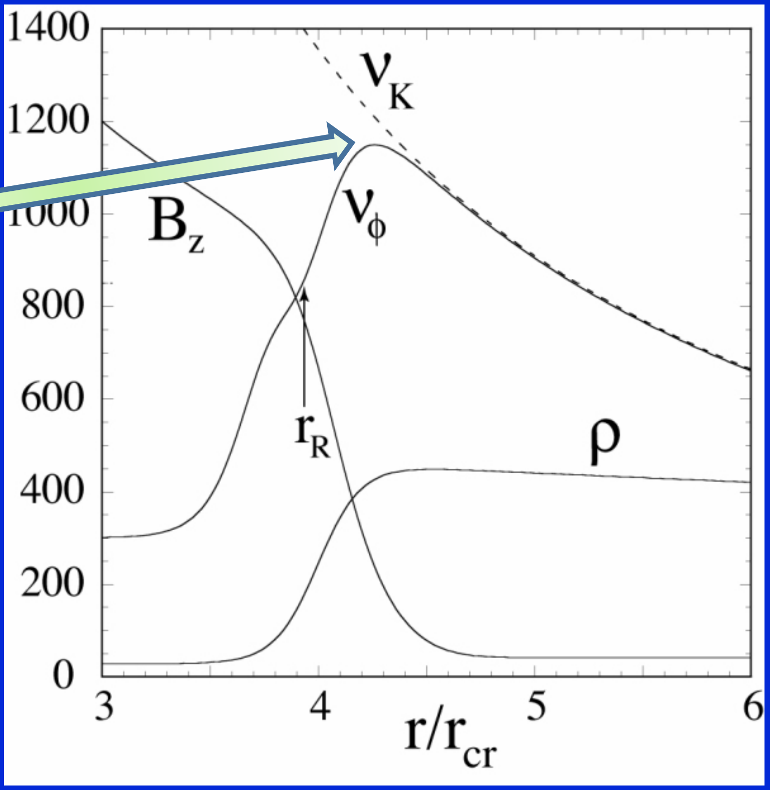


Romanova, Ustyugova, Koldoba, Lovelace 2012

High-frequency Trapped Waves: Theory

- Star rotates slower than the inner disc
- Angular velocity has a maximum (non-Keplerian disc)
- Trapped waves can form
- Similar to Rossby wave

Maximum in the angular velocity distribution

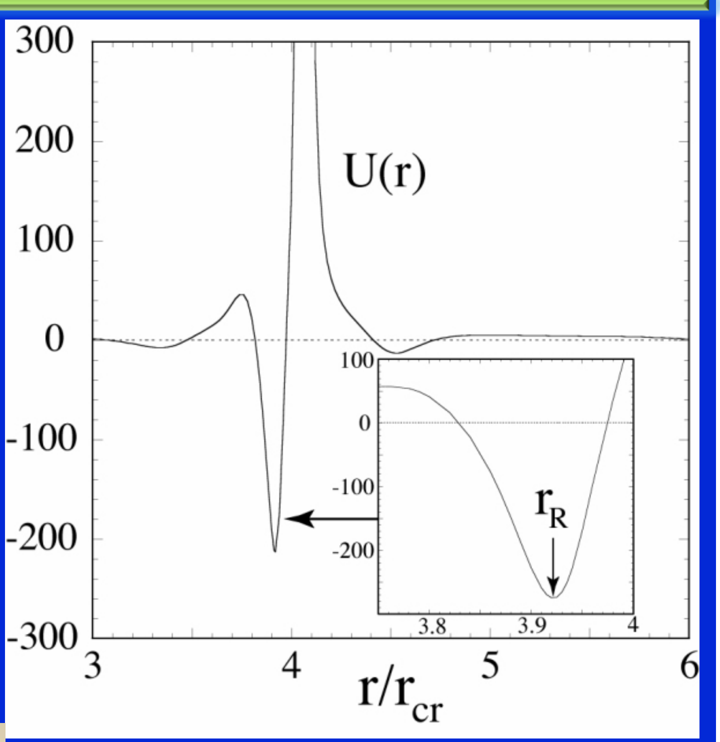


Lovelace, Turner, Romanova 2009

High-frequency Trapped Waves: Theory

$$\frac{d^2\psi}{dr^2} = U(r|\omega)\psi \ ,$$

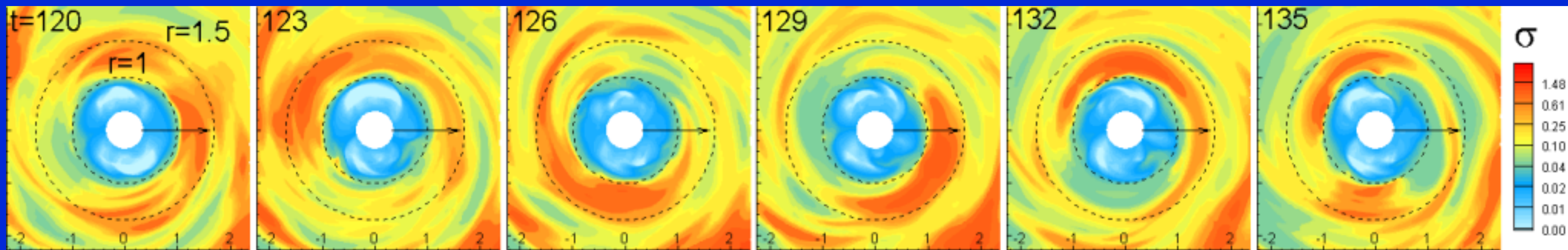
$U(r|\omega)$ – is an effective potential



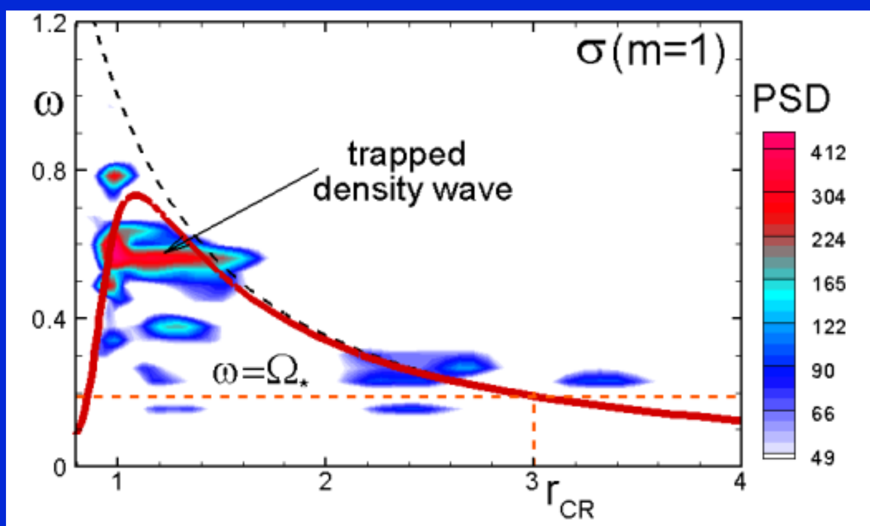
- The ground state is found by varying both the real and imaginary parts of the mode frequency ω so as to give the $\psi(r)$ most deeply trapped in the potential well.
- For typical profiles and $m = 1$, the growth rate is $\omega_i \sim 0.1(v_\phi/r)_R$ and the real part of the frequency is $\omega_r \approx (v_\phi/r)_R$
- The radial width of the mode is $\Delta r/r_R \sim 0.05$.
- The linear growth of the Rossby wave is predicted to saturate when the azimuthal frequency of trapping of a fluid particle in the trough of the wave ω_T grows to a value equal to the growth rate ω_i.
- For the $m = 1$ mode the saturation level is estimated as $|\delta\rho|/\rho \sim 1.4(\omega_i/\omega_r)^2(r/h)$.

Lovelace et al. 2009

High-frequency Trapped Waves: Simulations



Trapped density wave in the inner disc

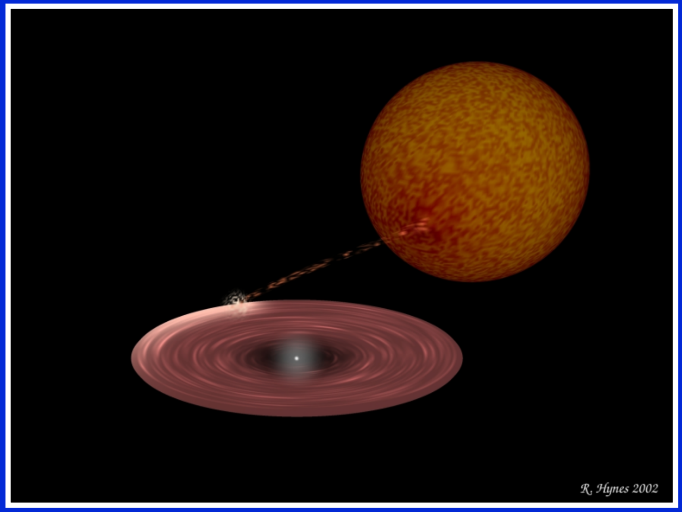


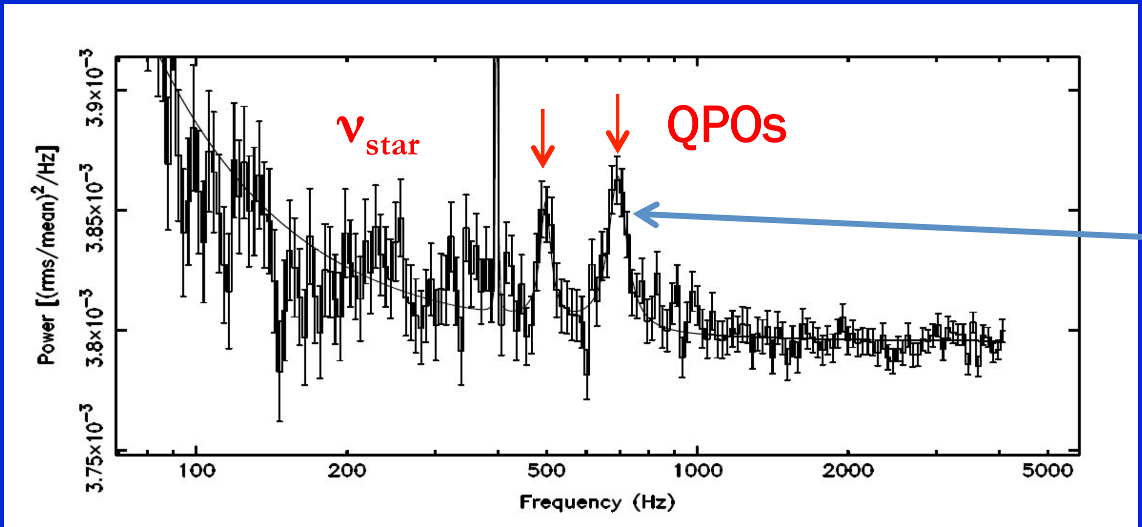
Trapped density wave predicted by the theory (*Lovelace et al. 2009*) in clearly seen in the PSD

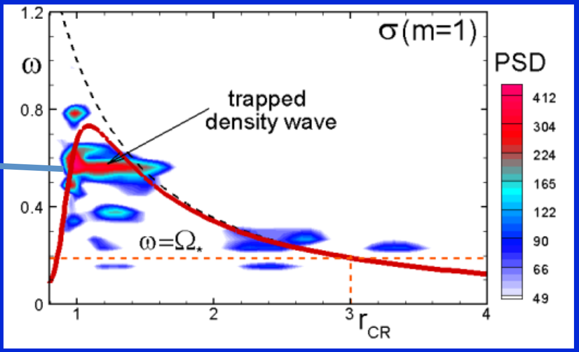
Romanova et al. 2012

Accreting Millisecond Pulsars (AMP)

- Two high-frequency peaks, the origin ?
- Suggestion: blobs at the inner dsic.
- However, the coherence is too high
- In our model – trapped density waves
- Frequency varies with the accretion rate



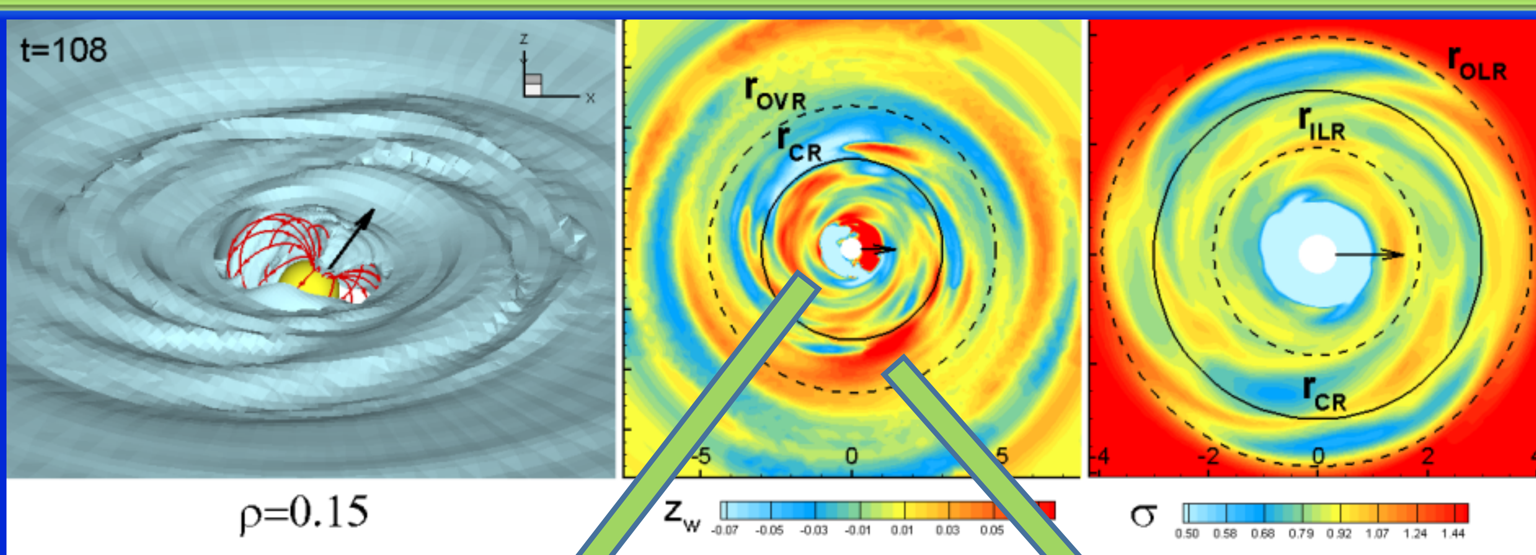




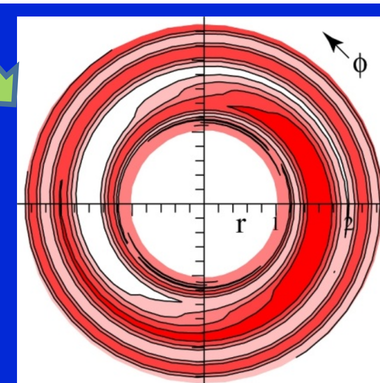
SAX J1808.4-3658 (Wijnands et al. 2003)

Comparison with Theory (case r_{cr}=3)

Now, we can compare warp structure at $r>r_{cr}$ and $r<r_{cr}$



In accord with the theory !



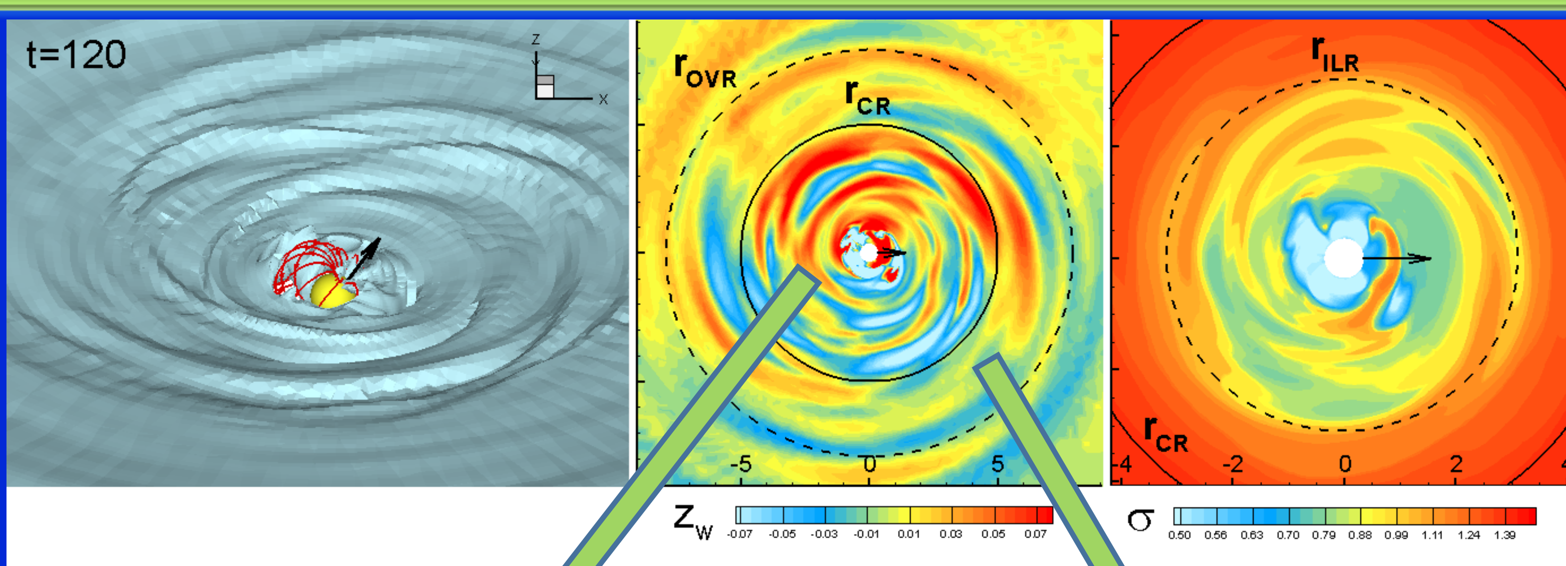
Inner warp, $r<r_{cr}$

Outer warp, $r<r_{cr}$

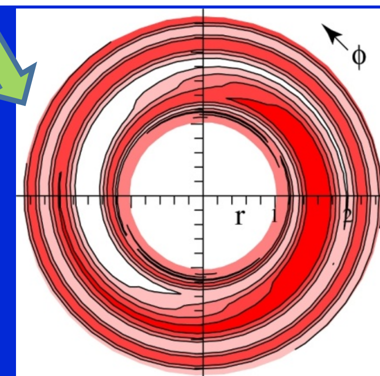
Romanova et al. 2012

Even Slower (r_{cr}=5)

Now, we can compare warp structure at $r>r_{cr}$ and $r<r_{cr}$



In accord with the theory !

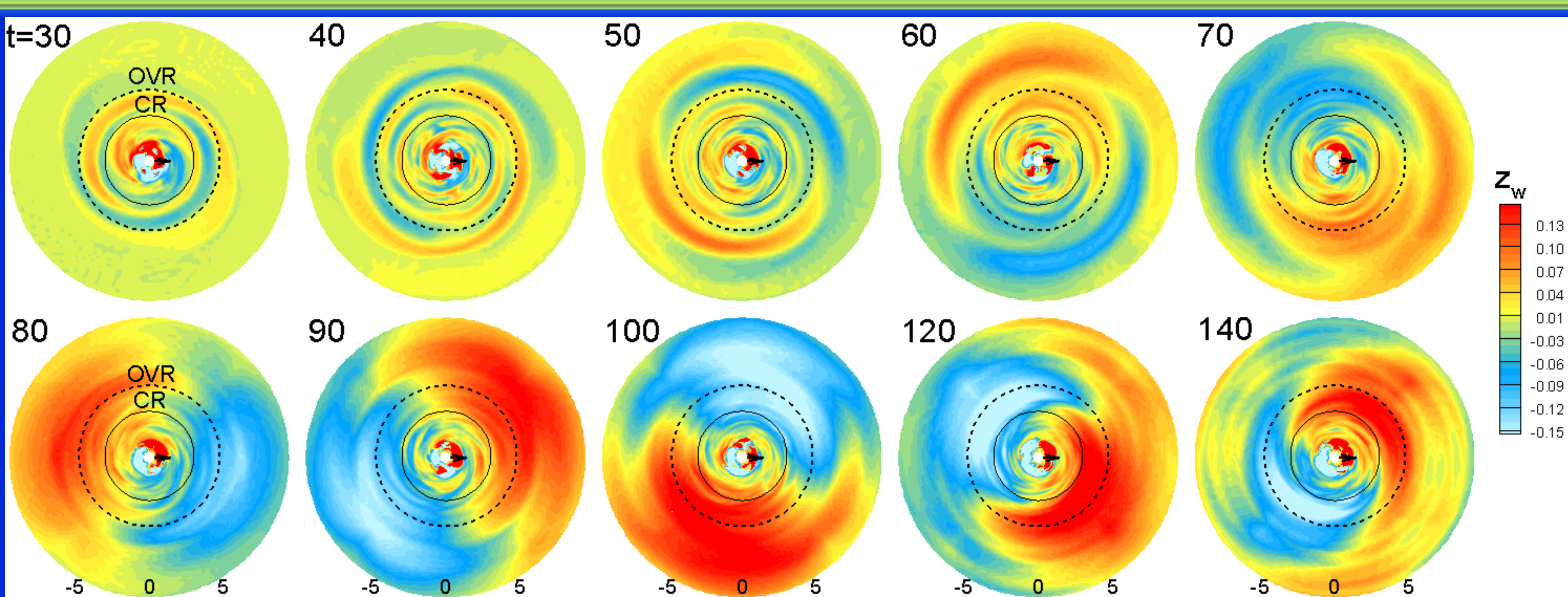


Inner warp, $r<r_{cr}$

Outer warp, $r<r_{cr}$

Romanova et al. 2012

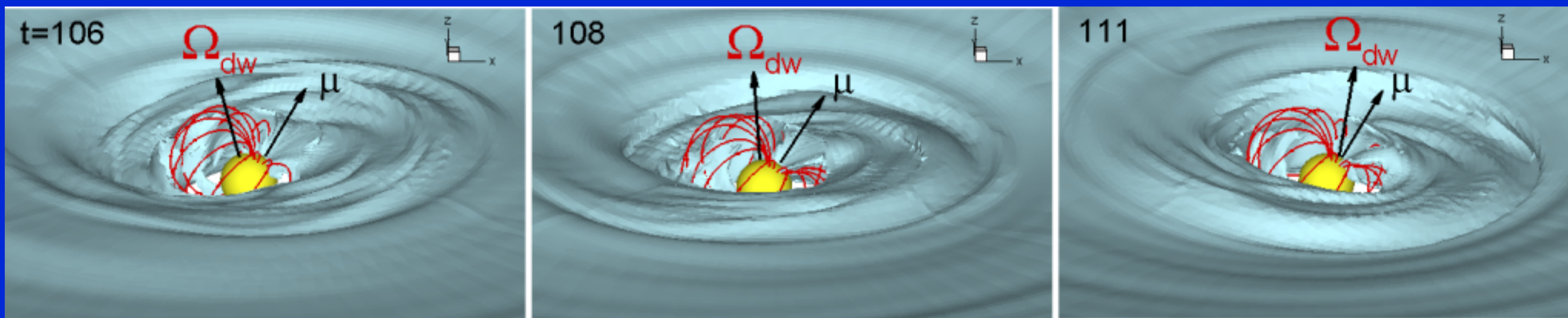
Low-frequency Oscillations of the Disc



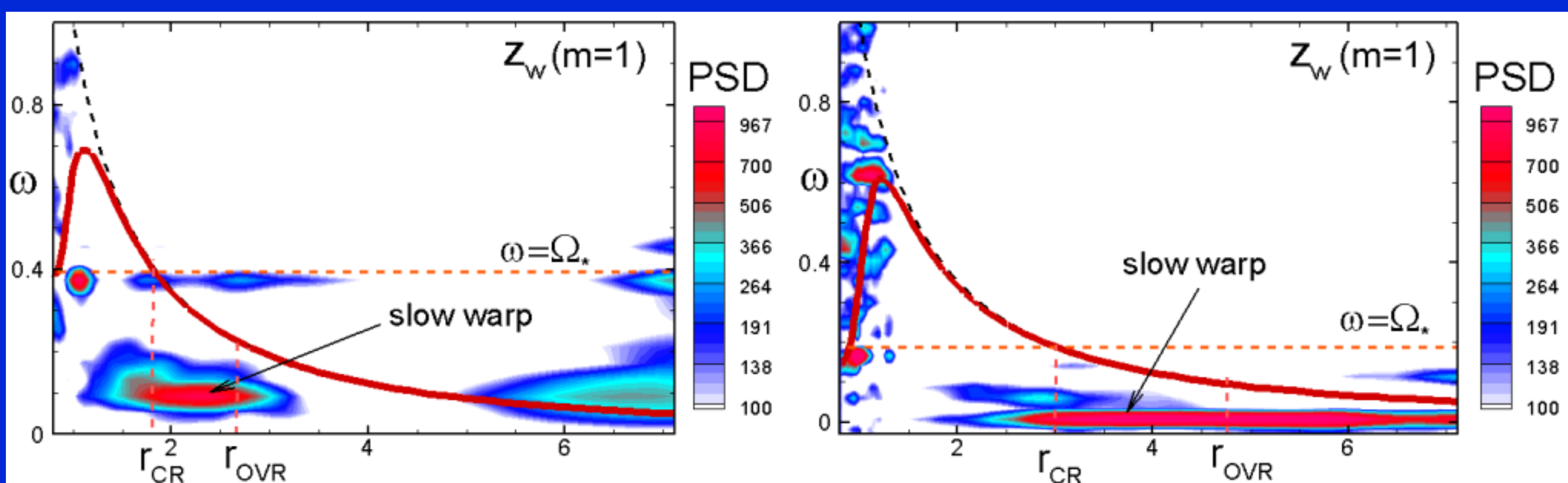
- Frequency of oscillations can be as low as Keplerian at the edge of the disc, or even lower
- It is amazing, how easy is excitation of bending waves
- May influence to formation of planets ?

Romanova et al. 2012

Slowly-Rotating Warp Can Form
(Magnetosphere Rotates Slower than the Inner Disc)



Warp forms and rotates slower than the magnetosphere



Romanova, Ustyugova, Koldoba, Lovelace 2012

Results:

1. We investigated waves in the disc excited by tilted rotator
2. In cases where magnetosphere and inner disc corotate, a large warp forms which corotates with the star
3. In case of slowly-rotating star, a large warp does not form, or it may form but rotate slowly
4. In case of slowly-rotating star, there is a maximum in the angular velocity distribution, and trapped density waves were predicted (Lovelace et al. 2009)
5. Such waves were detected in our simulation
6. The whole disc may experience low-frequency bending oscillations
7. Usually, bending oscillations are excited very easily. They may possibly influence to formation of planets.