Elliptic and Magneto-Elliptic instabilities

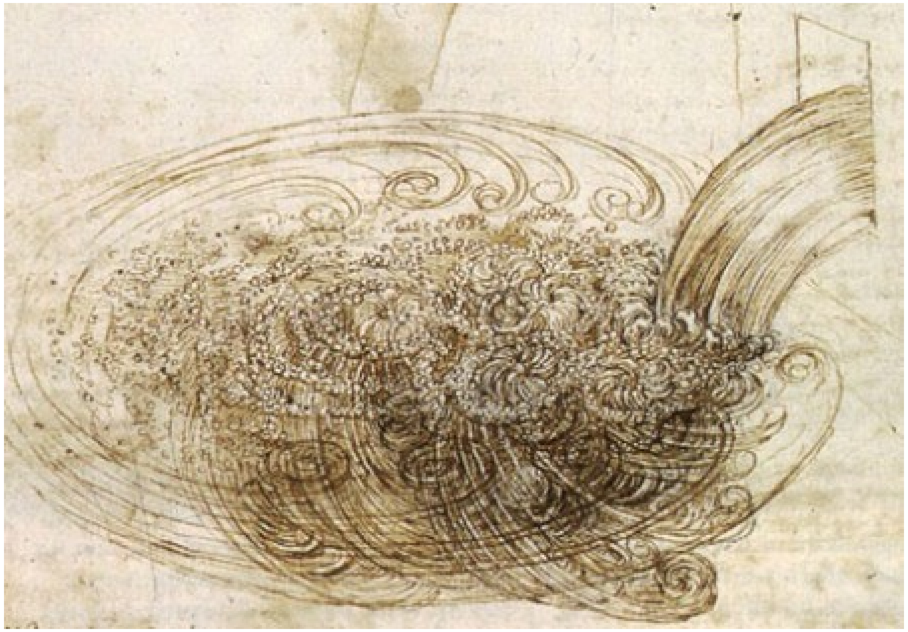
Wladimir Lyra
Carl Sagan Fellow

NASA/JPL-Caltech

Marseille, September 2012

Collaborators:
Hubert Klahr (Heidelberg), Krzysztof Mizerski (Warsaw)

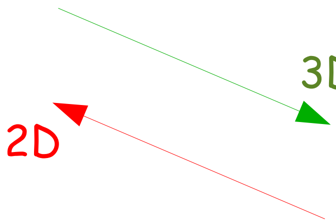
Vortices are the fundamental unit of turbulent flow.



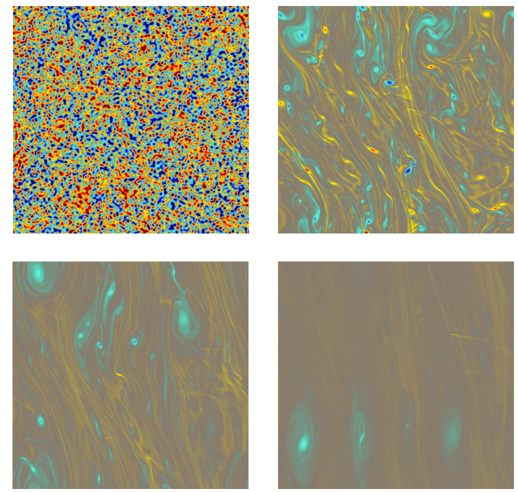
"… the smallest eddies are almost numberless,
and large things are rotated only by large eddies and not by small ones
and small things are turned by small eddies and large"

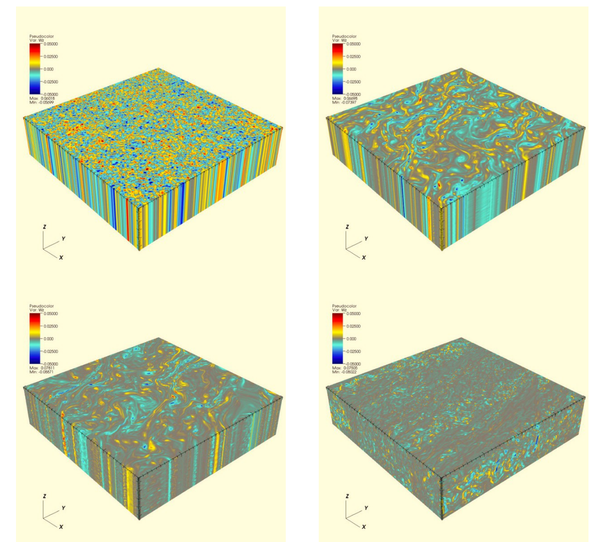
da Vinci (1500), on *torbolenza*

The energy cascade



Shen et al. (2006).
See also Batchelor (1967)





2D
Inverse Cascade
Eddies merge

3D
Direct Cascade
Eddies decay

Understanding the stability of vortices plays a fundamental role in understanding turbulence

How stable are closed elliptic streamlines?

Elliptic flow

$$U_x = -\chi \, \Omega_V \, y$$

$$U_y = \chi^{-1} \, \Omega_V \, x$$

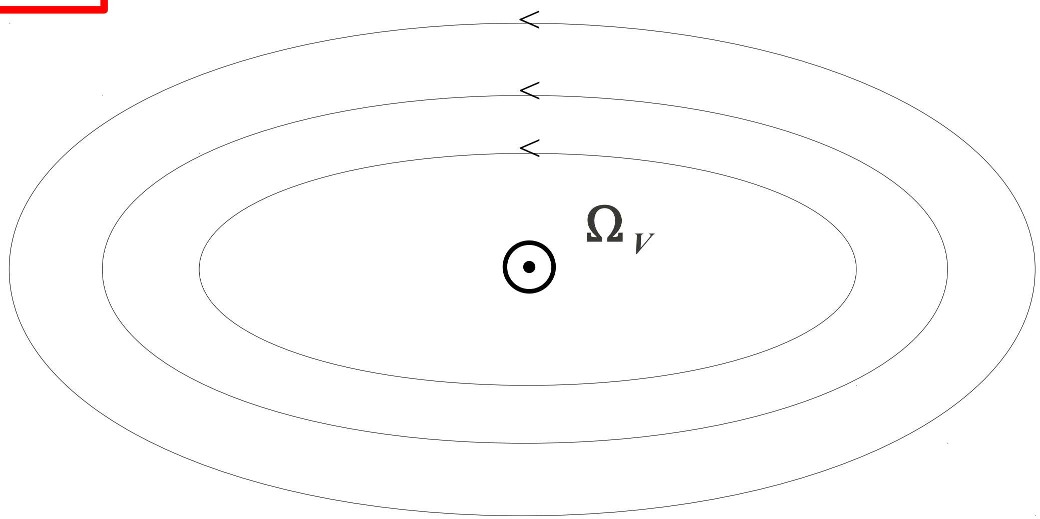
χ = aspect ratio

Ω_v = turnover frequency

Solve Euler equations for this flow

$$\partial_t u_i = -u_j \partial_j u_i - \rho^{-1} \partial_i p$$

$$\partial_i u_i = 0$$

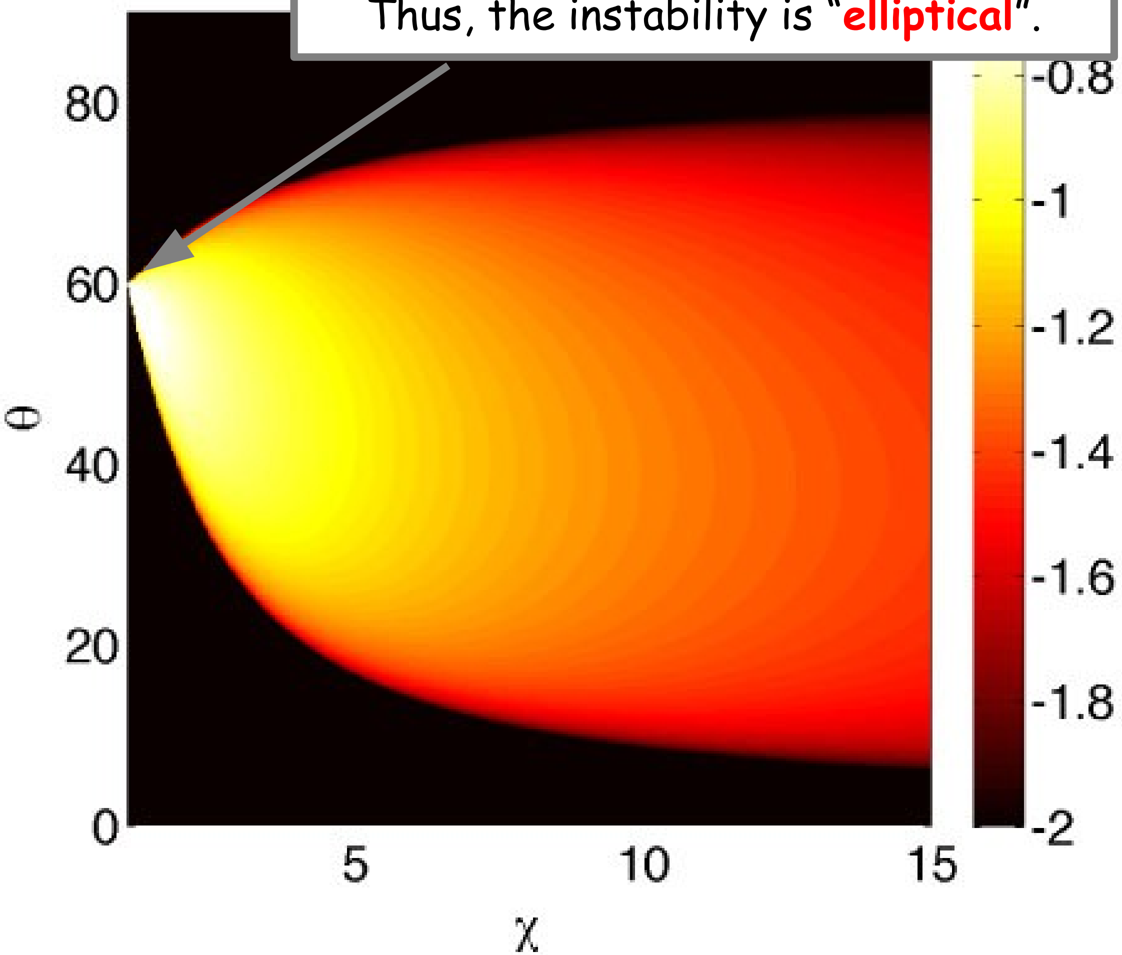


Inertial Waves

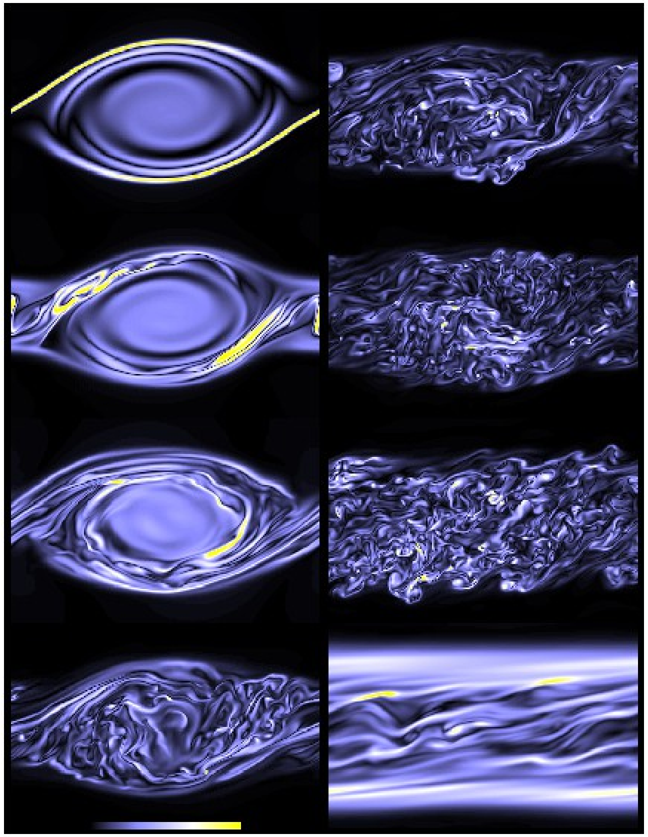
Inertial Waves

Growth rates

No instability for circular streamlines.
Thus, the instability is "**elliptical**".



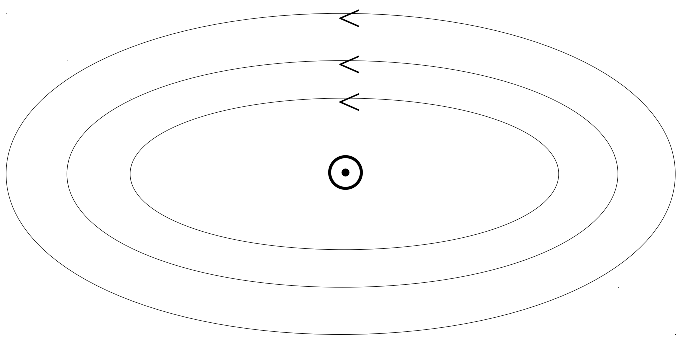
Lesur & Papaloizou (2009)
After Bayly (1986)



Vortex coherence is destroyed
Energy cascades forward and dissipates
The flow relaminarizes

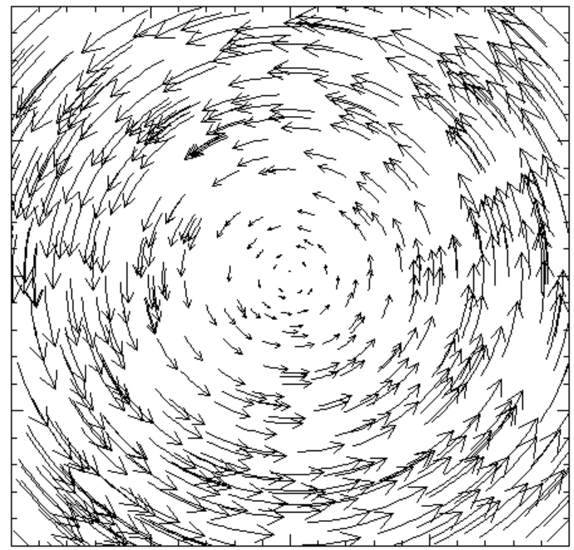
McWilliams (2010)

Decomposing the motion



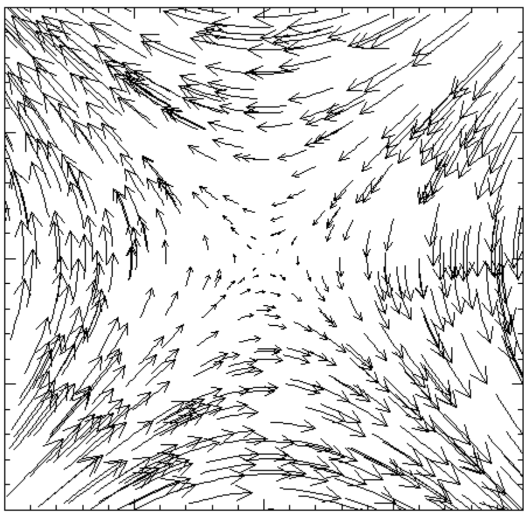
$=$

Rigid rotation



$+$

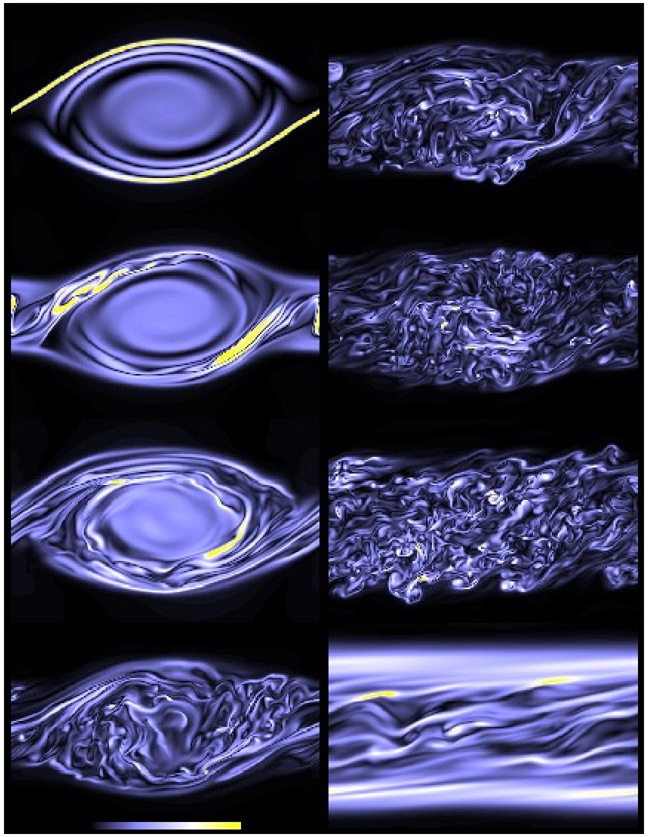
Strain field



$$U = [-(1-\epsilon)y, (1-\epsilon)x] \qquad [-y, x] \qquad -\epsilon[y, x]$$

The helical oscillations
are de-stabilized by the **strain field**.

End Result



McWilliams (2010)

The instability is 3D

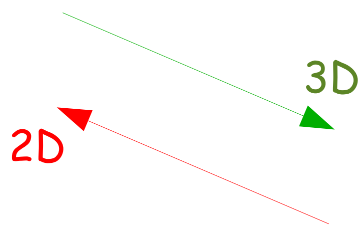
Generates 3D turbulence out of 2D motion

secondary instability

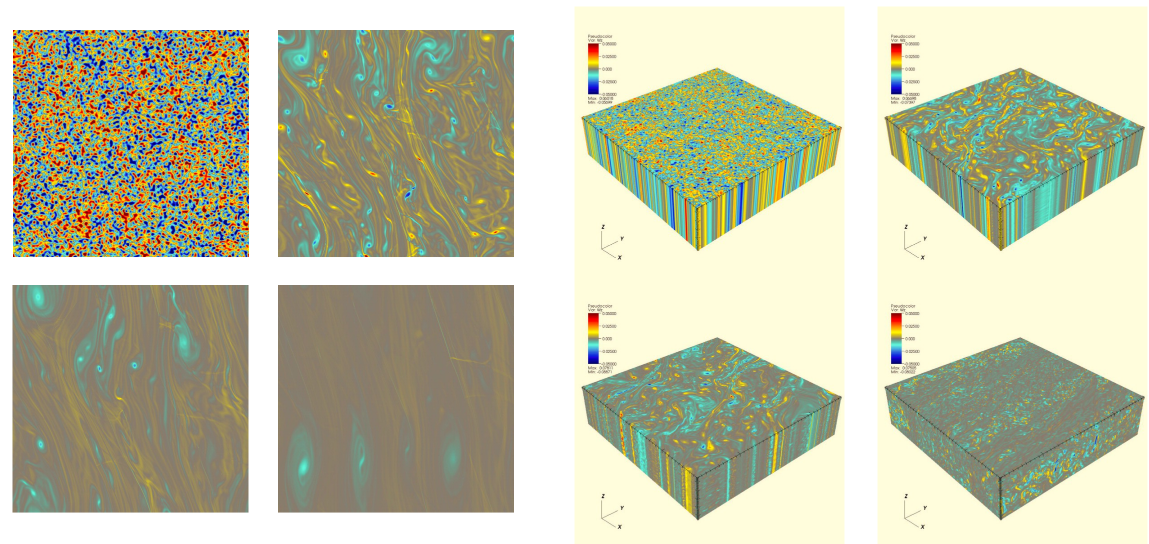
A stirring or primary instability (RT, KH) generates the first eddies.

The elliptic instability breaks them, leading to the direct cascade.

The energy cascade



Shen et al. (2006).
See also Batchelor (1967)



2D

3D

Inverse Cascade

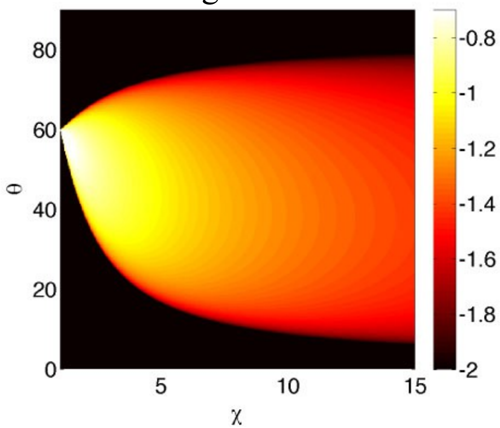
No elliptic instability
Eddies merge viscously

Direct Cascade

Elliptic destruction occurs faster (turnover frequency) than viscous merging

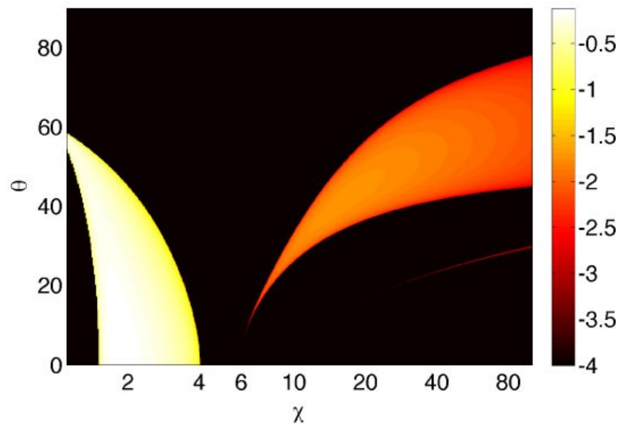
Elliptic-Rotational Instability

No background rotation

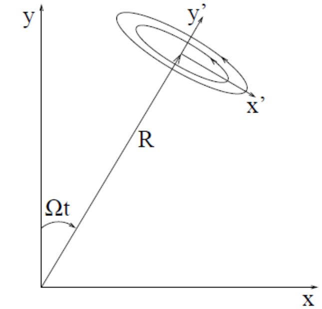


Lesur & Papaloizou (2009)
Bayly (1986)

Including Coriolis



Lesur & Papaloizou (2009)
Miyakazi (1992)



Instability of elliptic streamlines

* In the **non-rotating** case:

- **Resonance** between

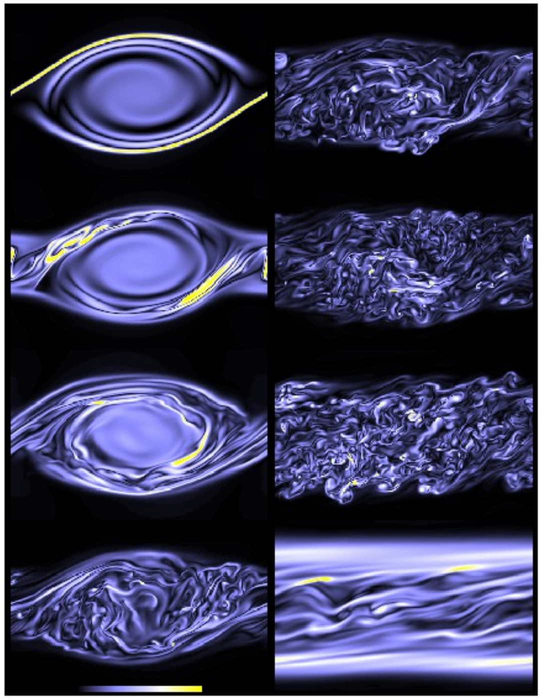
  Strain field and Inertial waves

* In the **rotating** case:

- Strong "horizontal" (theta=0) unstable mode:

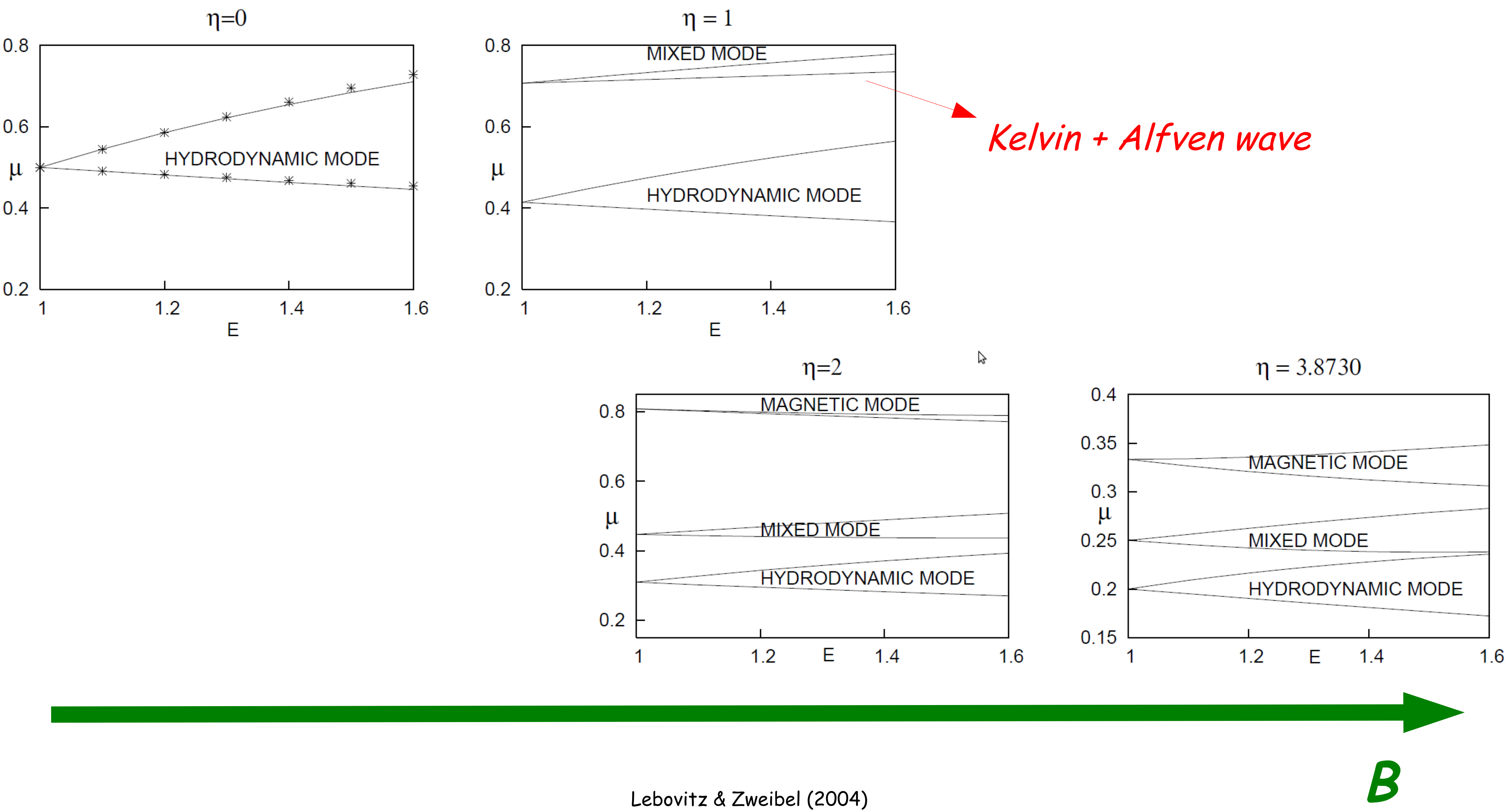
  Exponential growth of epicyclic disturbances

Vortex coherence is destroyed
Energy cascades forward and dissipates

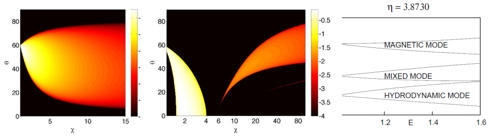


McWilliams (2010)

Magneto-elliptic instability



Lebovitz & Zweibel (2004)



Magneto-elliptic instability with rotation

Rv = Inverse Rossby
h = Magnetic field

Pure elliptic

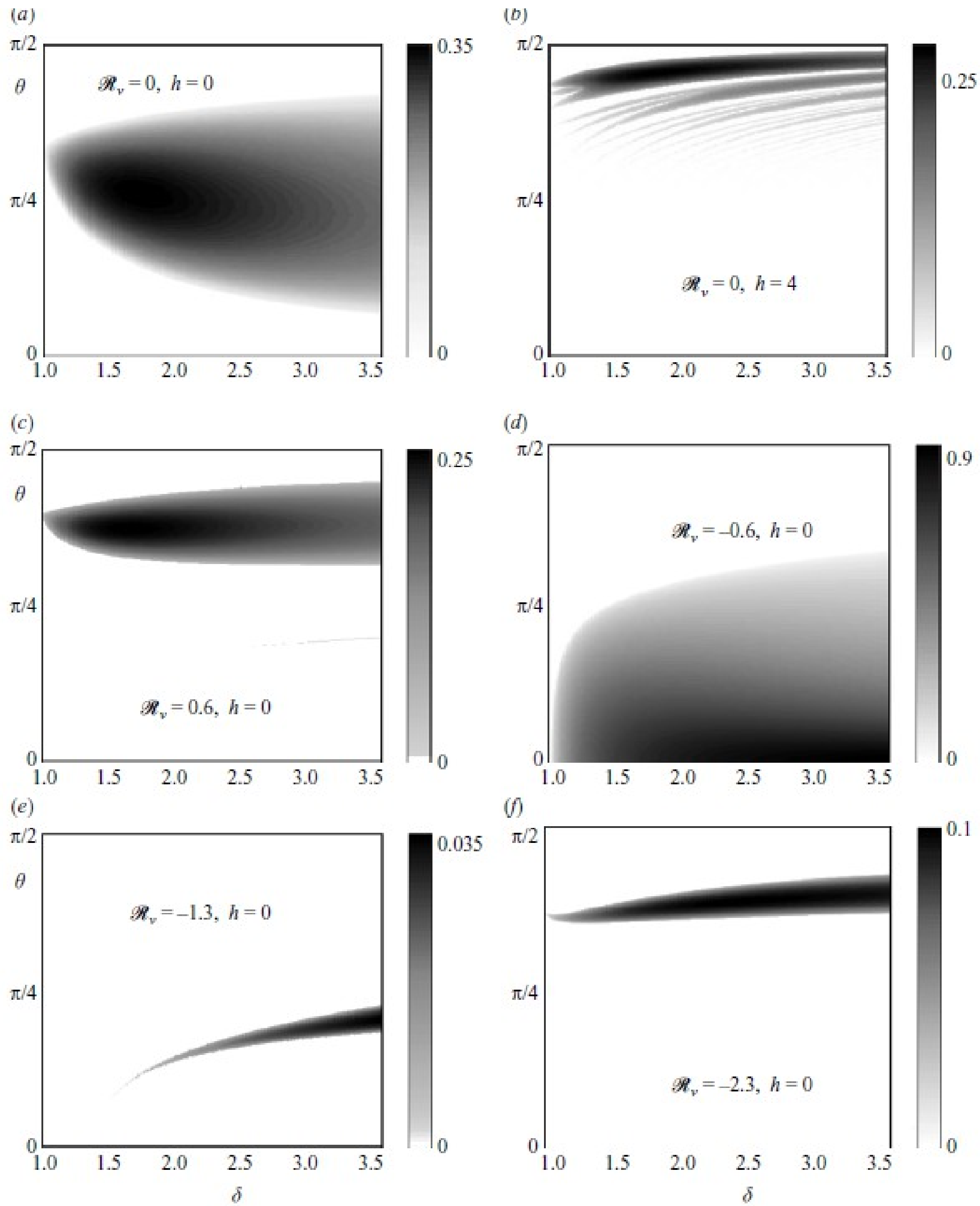
Pure magneto-elliptic

Elliptic-rotational cyclonic

Elliptic-rotational anticyclonic

Elliptic-rotational Anticyclonic (slow rotation)

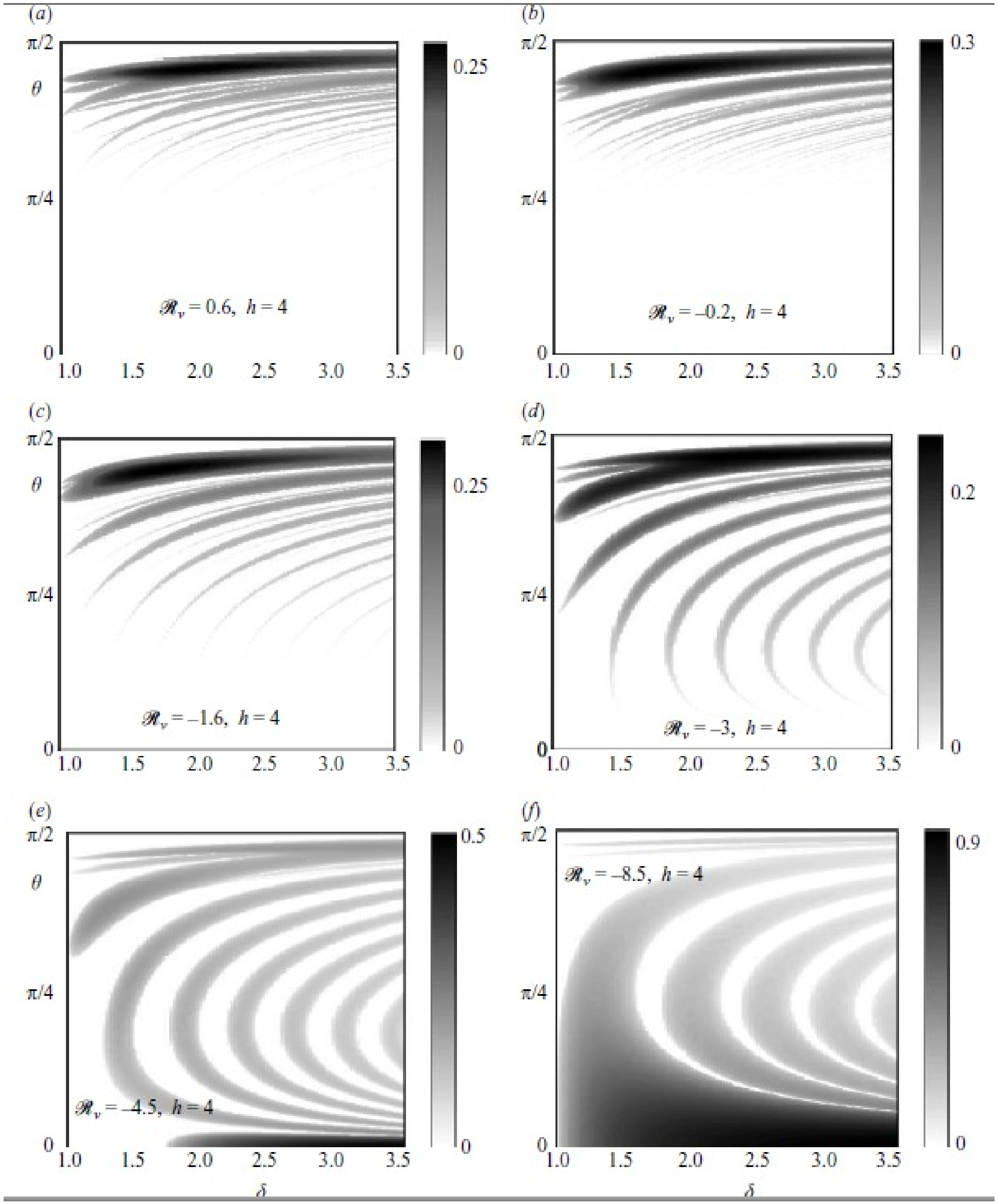
Elliptic-rotational Anticyclonic (slower rotation)



Mizerski & Bajer (2009)

Horizontal magneto-elliptic instability

Rv = Inverse Rossby
h = Magnetic field



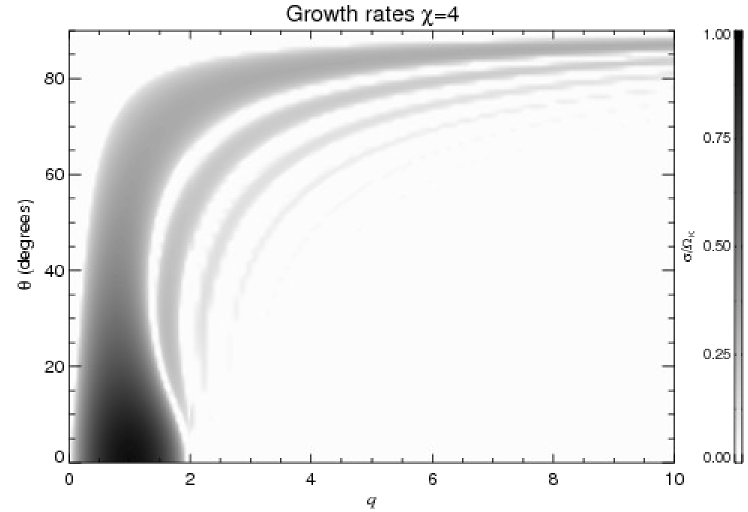
When rotation and field satisfy

$$Ro^{-1} < -\frac{h^2}{4}$$

a **strong horizontal mode** appears

Mizerski & Bajer (2009)

Magneto-elliptic instability



Lyra & Klahr 2011
(after Mizerski & Bajer 2009)

Mizerski & Bajer (2009, Journal of Fluid Mechanics)

"The presence of magnetic fields widens the range of existence of the horizontal instability to an unbounded interval of aspect ratios when

$$Ro^{-1} < -\frac{h^2}{4}$$

$$0 < k/k_{BH} < 2|Ro|^{1/2}$$

$h = q/Ro$

$q = k/k_{BH}$

$Ro = \frac{\Omega_V \delta}{\Omega_K}$

$k_{BH} = \frac{\Omega_K}{v_A}$

$\delta = \frac{1}{2}(\chi + \chi^{-1})$

Magneto-elliptic instability

$$0 < k/k_{BH} < 2|Ro|^{1/2}$$

$$Ro = \frac{\Omega_V \delta}{\Omega_K}$$

$$\delta = \frac{1}{2}(\chi + \chi^{-1})$$

χ Aspect ratio

Write the criterion in terms of vorticity instead of angular frequency:

Vortex flow

$$u_x = -\Omega_V y/\chi$$

$$u_y = \Omega_V x \chi$$

$\nabla \times$ →

Vorticity

$$\omega_T = \Omega_V(\chi + \chi^{-1}) = 2\Omega_V \delta = 2\, Ro\, \Omega_K$$

$$= \omega_V + \omega_{box}$$

$$= \omega_V - 3/2\, \Omega_K$$

$$Ro = \frac{\omega_V}{2\Omega_K} - \frac{3}{4}$$

In the no-vortex limit (ω_v=0) , Ro=-3/4

$$0 < k/k_{BH} < \sqrt{3}$$

Magneto-elliptic instability → No vortex limit

$$0 < k / k_{BH} < \sqrt{3}$$

Consistency

$$Ro = \frac{\Omega_V \delta}{\Omega_K}$$

$$\delta = \frac{1}{2}(\chi + \chi^{-1})$$

χ Aspect ratio

Remember:

In the no-vortex limit (ω_v=0) , Ro=-3/4

Kida solution

$$\Omega_V = -\frac{3\Omega_K}{2(\chi-1)}$$

$$Ro = -\frac{3}{4}\frac{\chi^2-1}{\chi(\chi-1)}$$

$\Omega_V = 0$

$\chi \to \infty$

$\chi \to \infty$

$$\lim_{\chi\to\infty} Ro = -\frac{3}{4}$$

QED!

Vortex flow

$$u_x = -\Omega_V y/\chi$$

$$u_y = \Omega_V x \chi$$

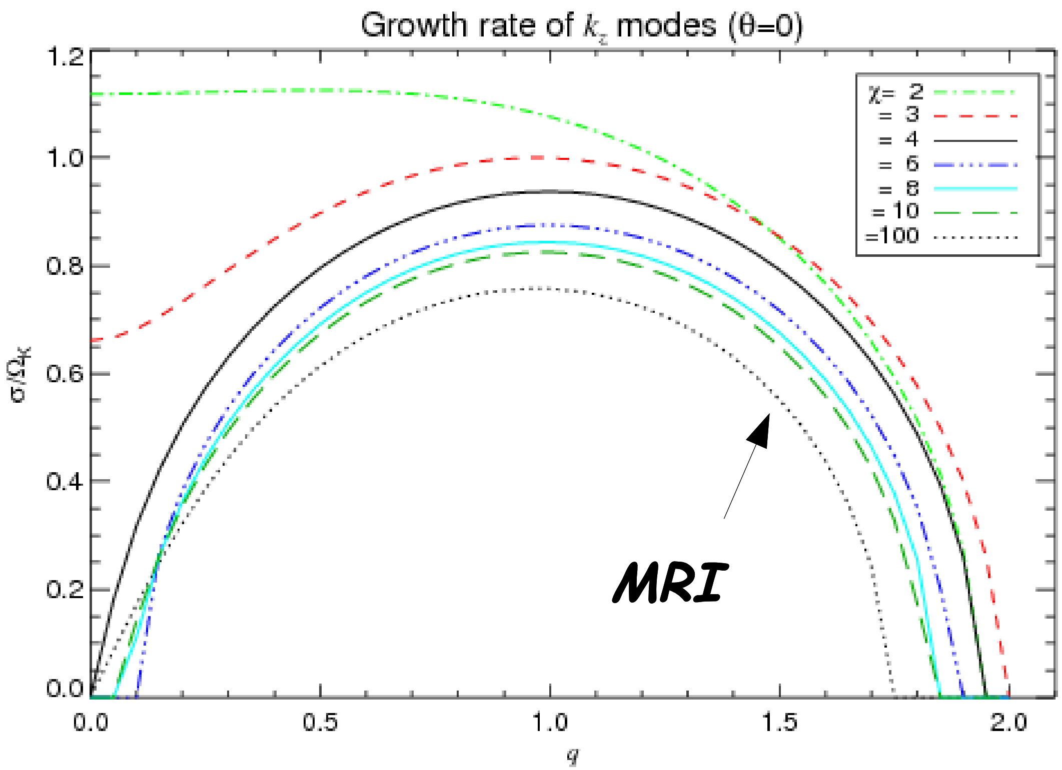
$\chi \to \infty$

$$\lim_{\chi\to\infty} u_x = 0$$

$$\lim_{\chi\to\infty} u_y = -3/2\,\Omega_K x$$

A vortex of infinite aspect ratio is equivalent to a shear flow

Growth rates of the Magneto-Elliptic-Rotational Instability



… we explain both
the MEI and the MRI
as different manifestations
of the same
Magneto-Elliptic-Rotational Instability

Mizerski & Lyra (2012)

On the connection between the magneto-elliptic and magneto-rotational instabilities

KRZYSZTOF A. MIZERSKI[1]†, AND WLADIMIR LYRA[2,3,4]

[1]Department of Mechanics and Physics of Fluids, Institute of Fundamental and Technological Research, Polish Academy of Sciences, Pawinskiego 5B, 02-106, Poland

[2]Department of Astrophysics, American Museum of Natural History, 79th Street at Central Park West, New York NY 10024-5192, USA

[3]Jet Propulsion Laboratory, 4800 Oak Grove Drive, Pasadena CA 91109, USA

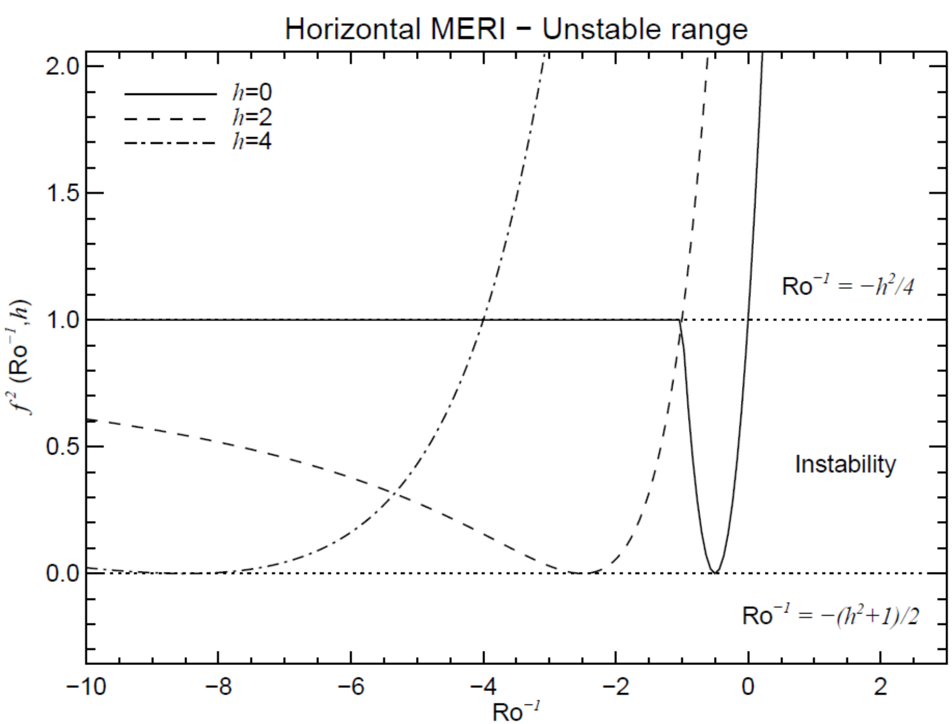
[4]NASA Carl Sagan Fellow

(Received 11 July 2011; Accepted 15 February 2012.)

Dispersion Relation

$$\begin{bmatrix} \dot{\hat{v}}_x \\ \dot{\hat{v}}_y \\ \dot{\hat{b}}_x \\ \dot{\hat{b}}_y \end{bmatrix} = \begin{bmatrix} 0 & 1+\varepsilon+2\mathrm{Ro}^{-1} & ih & 0 \\ -(1-\varepsilon)-2\mathrm{Ro}^{-1} & 0 & 0 & ih \\ ih & 0 & 0 & -(1+\varepsilon) \\ 0 & ih & 1-\varepsilon & 0 \end{bmatrix} \begin{bmatrix} \hat{v}_x \\ \hat{v}_y \\ \hat{b}_x \\ \hat{b}_y \end{bmatrix}$$

Mizerski & Lyra (2012)



$$f_{\pm} = Ro^{-1} \pm \sqrt{(Ro^{-1}+1)^2 + h^2}$$

Horizontal MEI

$$\frac{\sigma^2}{\gamma^2} = \epsilon^2 - Ro^{-2}\left[\sqrt{(Ro^{-1}+1)+q^2} - 1\right]^2$$

$\epsilon = 1$ $Ro = -3/4$

Horiz. MEI at shear flow lim.

$$\frac{\sigma^2}{\gamma^2} = \frac{8}{9}\left(\sqrt{16q^2+1} - 2q^2 - 1\right)$$

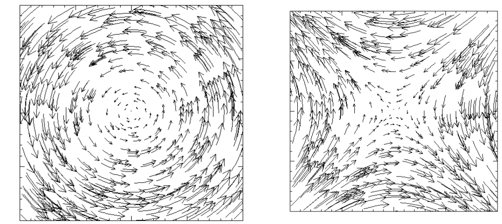
Properties

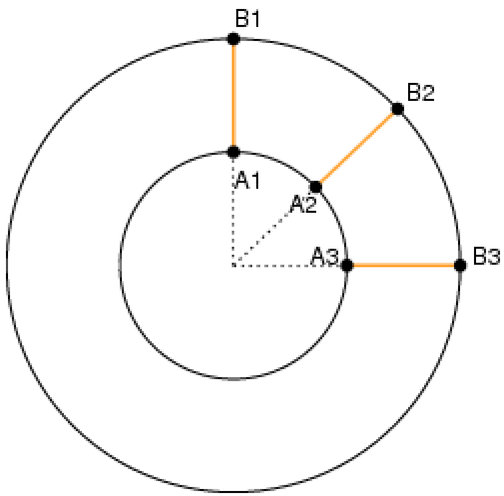
$$0 < q < \sqrt{3}$$

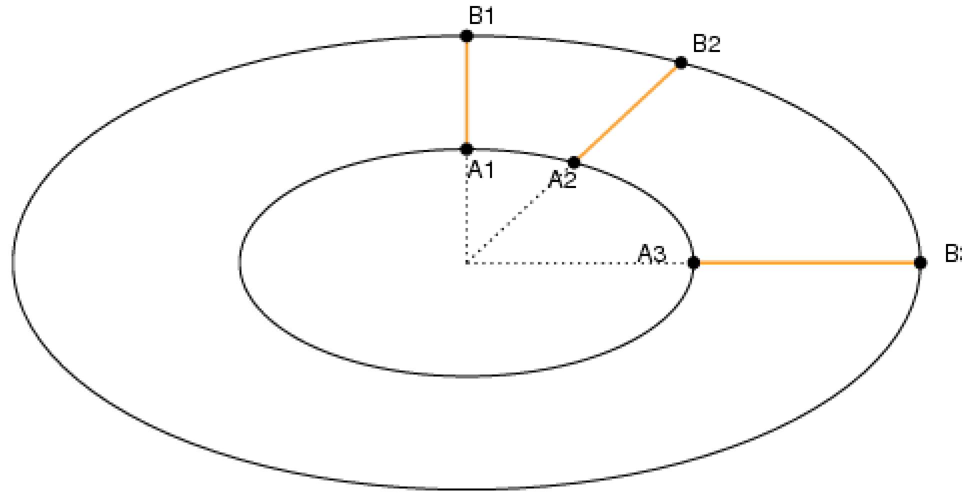
$$\sigma = \gamma = 3/4\,\Omega$$

$$q = \sqrt{15}/4 \approx 0.9682$$

Common ground between MRI and MEI



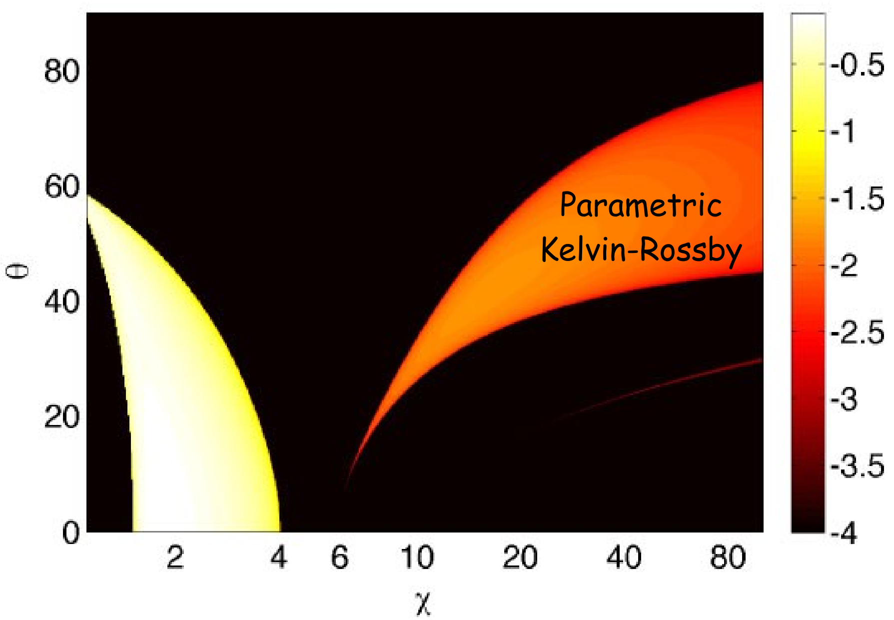




Elliptic streamlines have shear
even in uniform rotation.

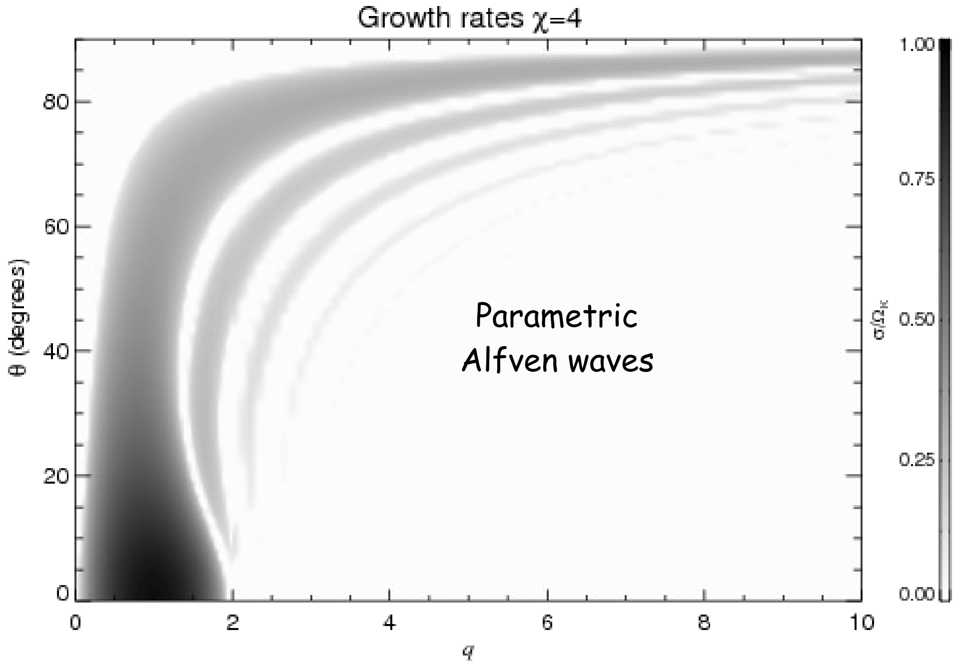
Destroying vortices with 3D instabilities

Elliptic-Rotational



Rayleigh instability

Magneto-Elliptic-Rotational



MRI

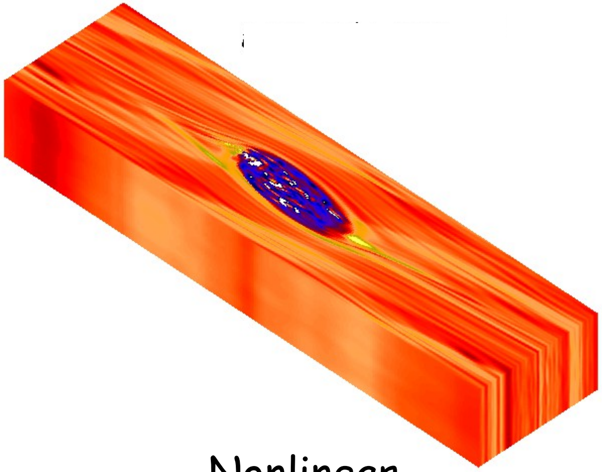
Sustaning vortices

Mechanisms to

inject vorticity

to counteract the vorticity lost in the direct cascade

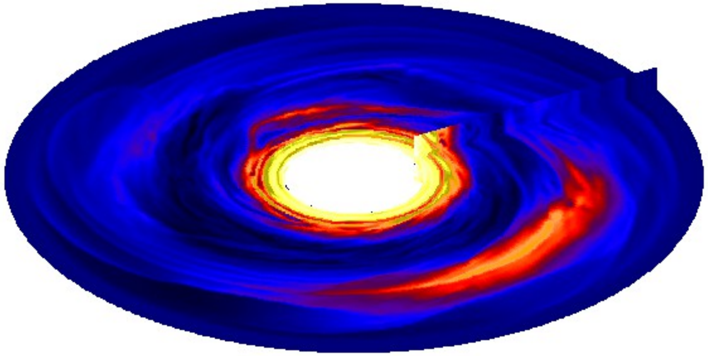
Baroclinic Instability



Nonlinear

Powered by:
buoyancy, thermal diffusion
(baroclinic source term)

Rossby Wave Instability



Linear

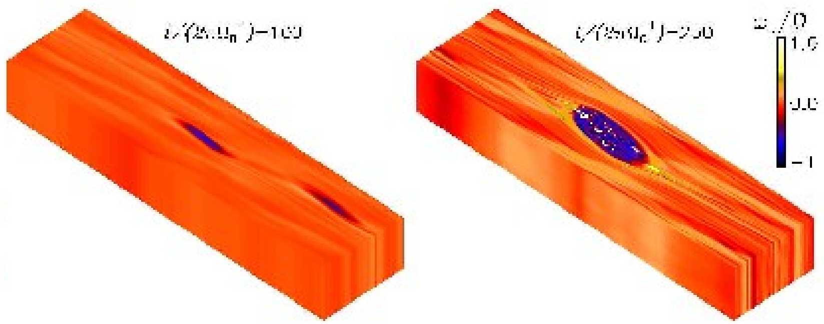
Powered by:
Modification of shear profile
(external vortensity reservoir)

Baroclinic Instability and Elliptic Instability

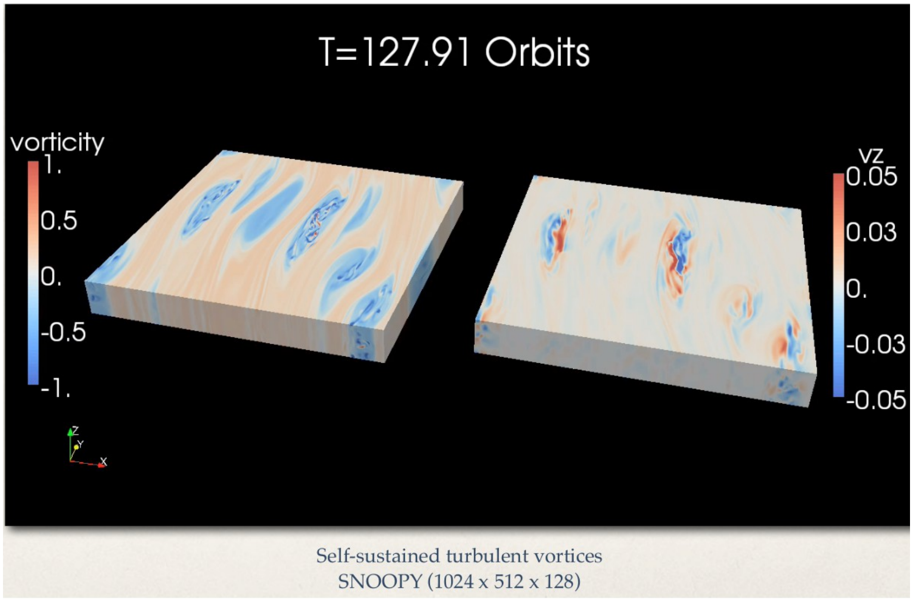
Despite the elliptical instability,
baroclinity keeps the vortex coherent.

The result is "core turbulence" only

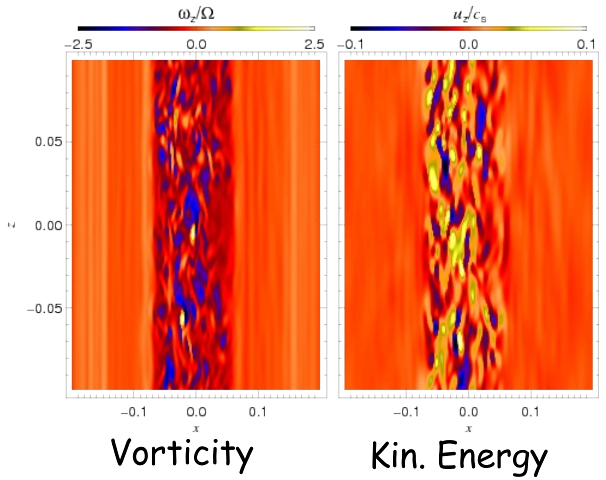
Vorticity



Lyra & Klahr (2011)

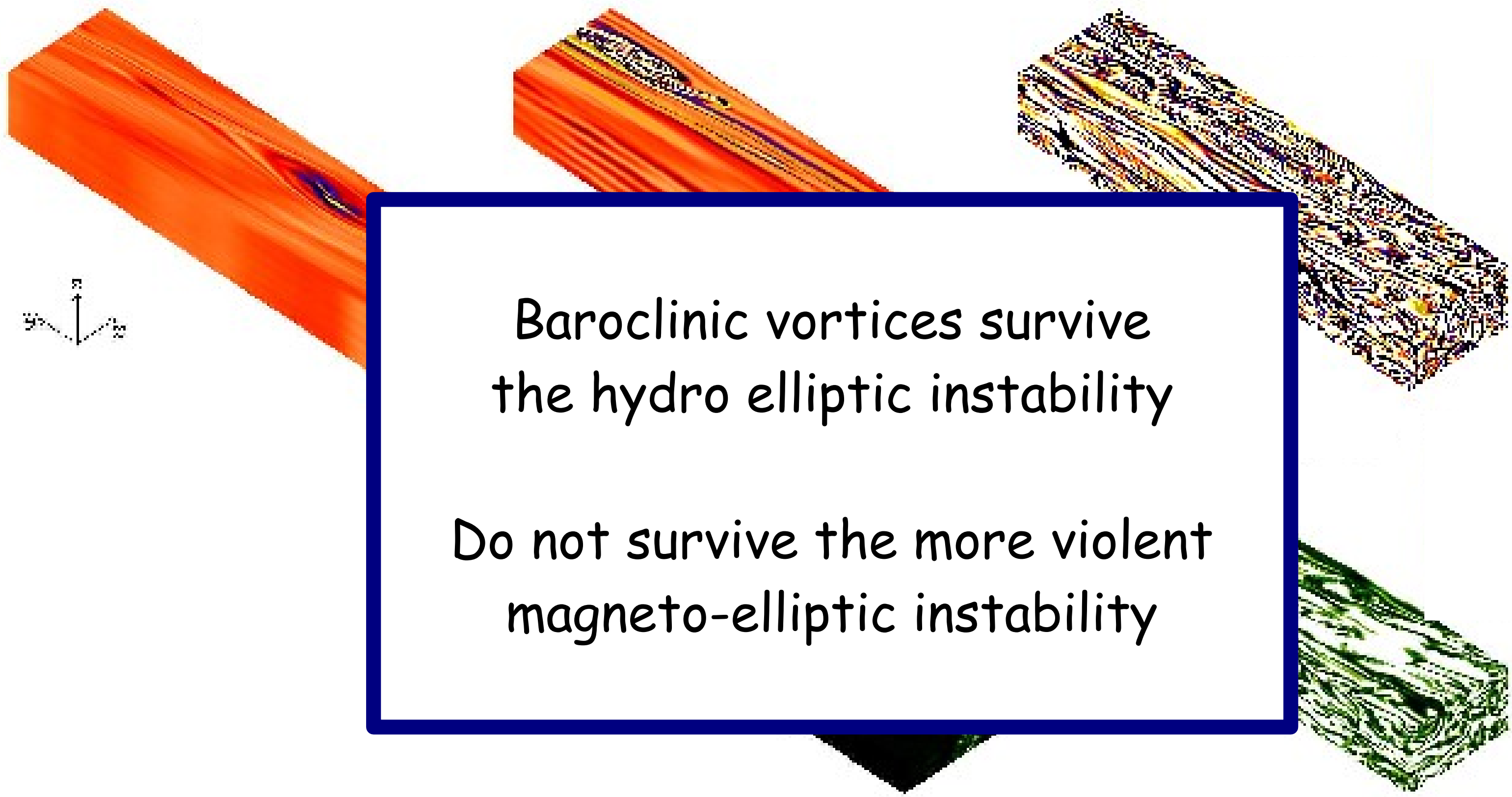


Lesur & Papaloizou (2010)



Baroclinic Instability and Magneto-Elliptic Instability

What happens when the vortex is magnetized?



Vorticity

Magnetic Energy

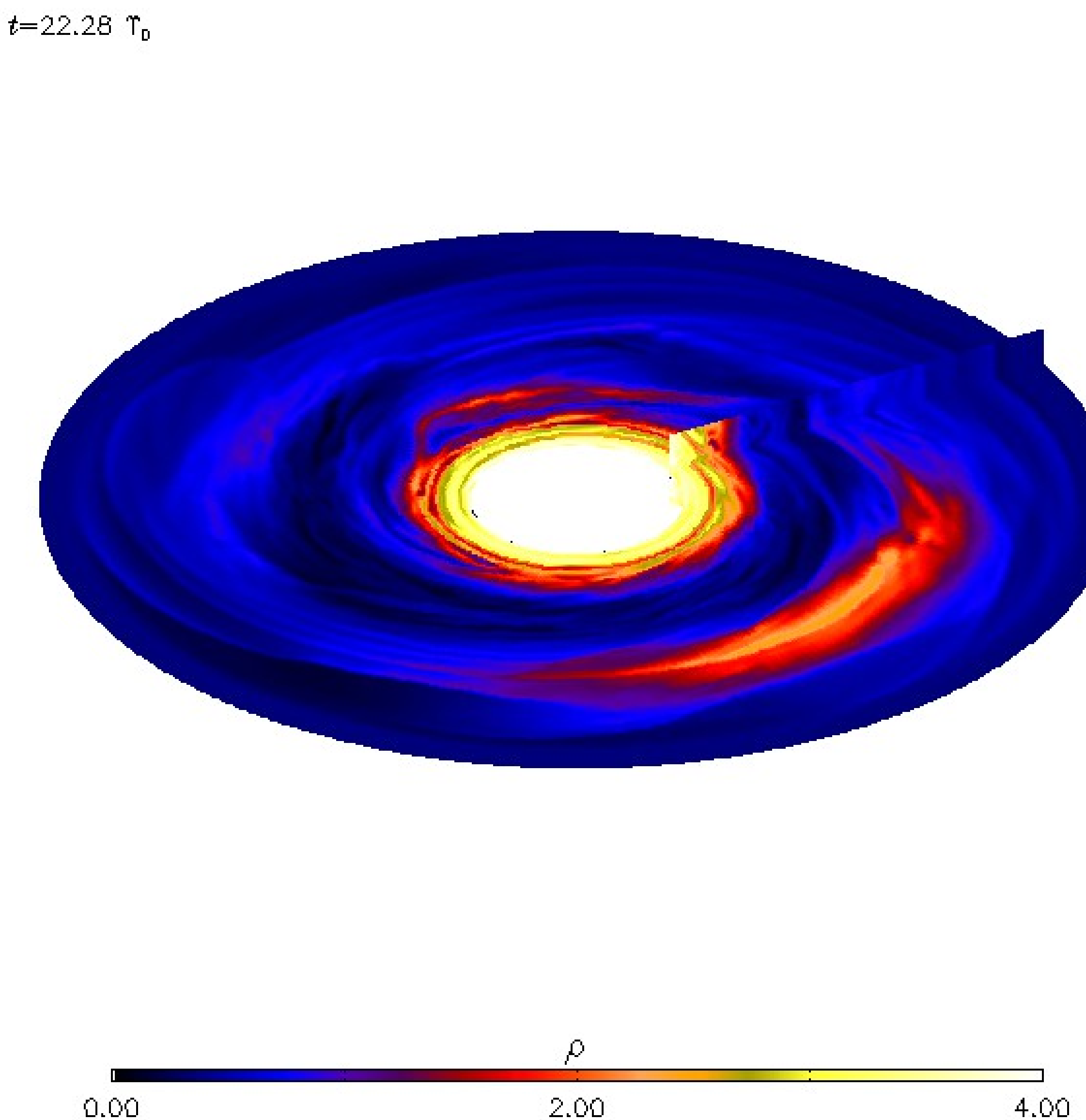
Baroclinic vortices survive
the hydro elliptic instability

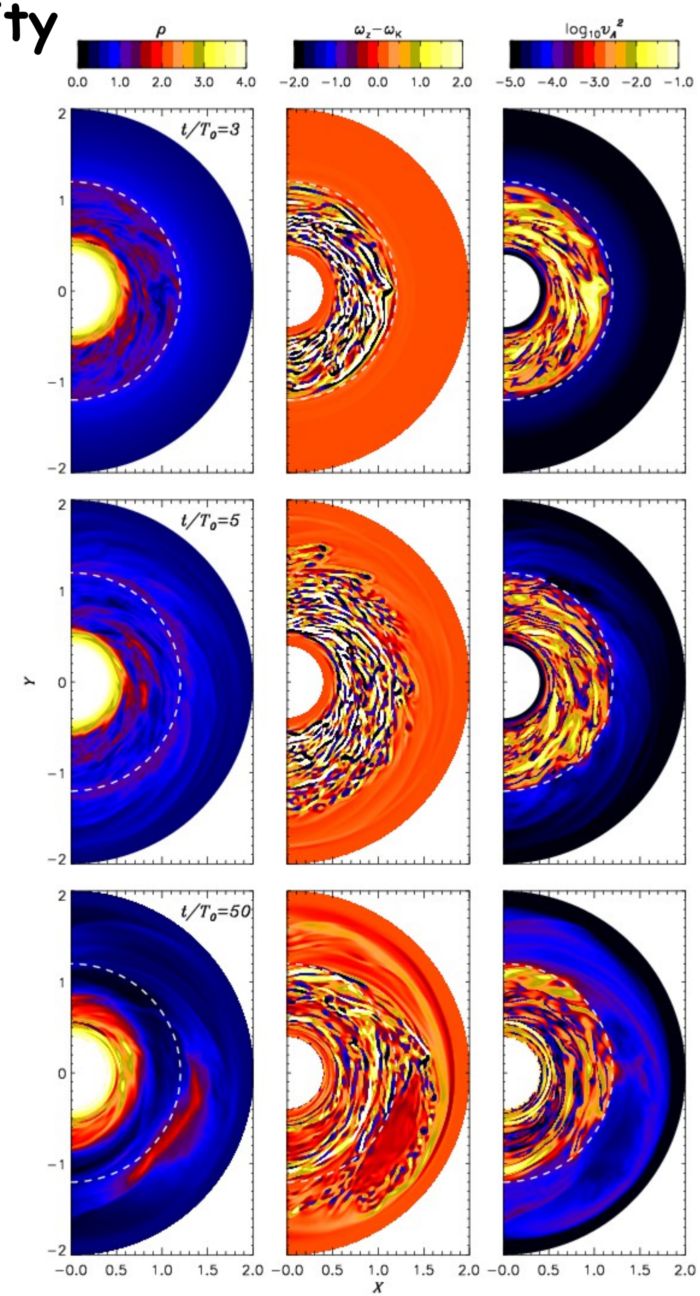
Do not survive the more violent
magneto-elliptic instability

Vortex gone!

Lyra & Klahr (2011)

Rossby Wave Instability

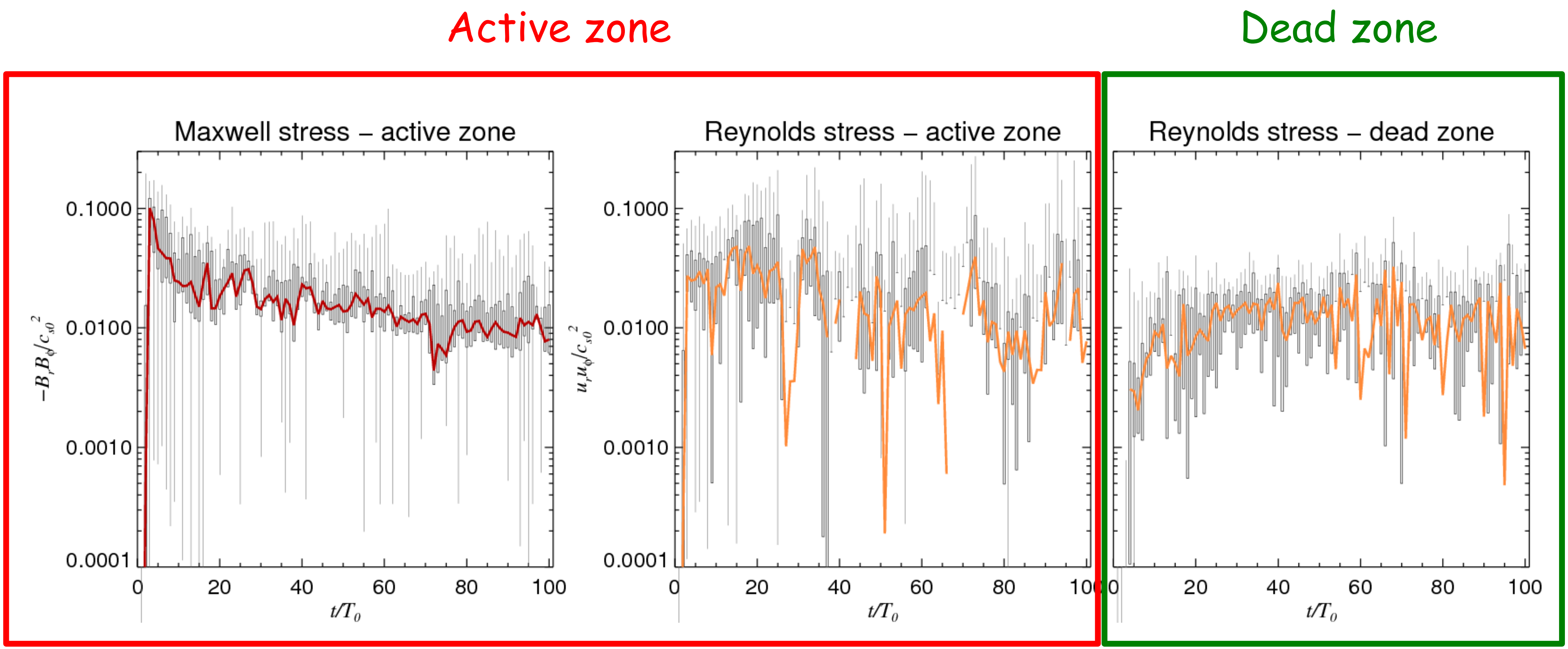




Magnetized inner disk + resistive outer disk

Lyra & Mac Low (2012)

Significant angular momentum transport

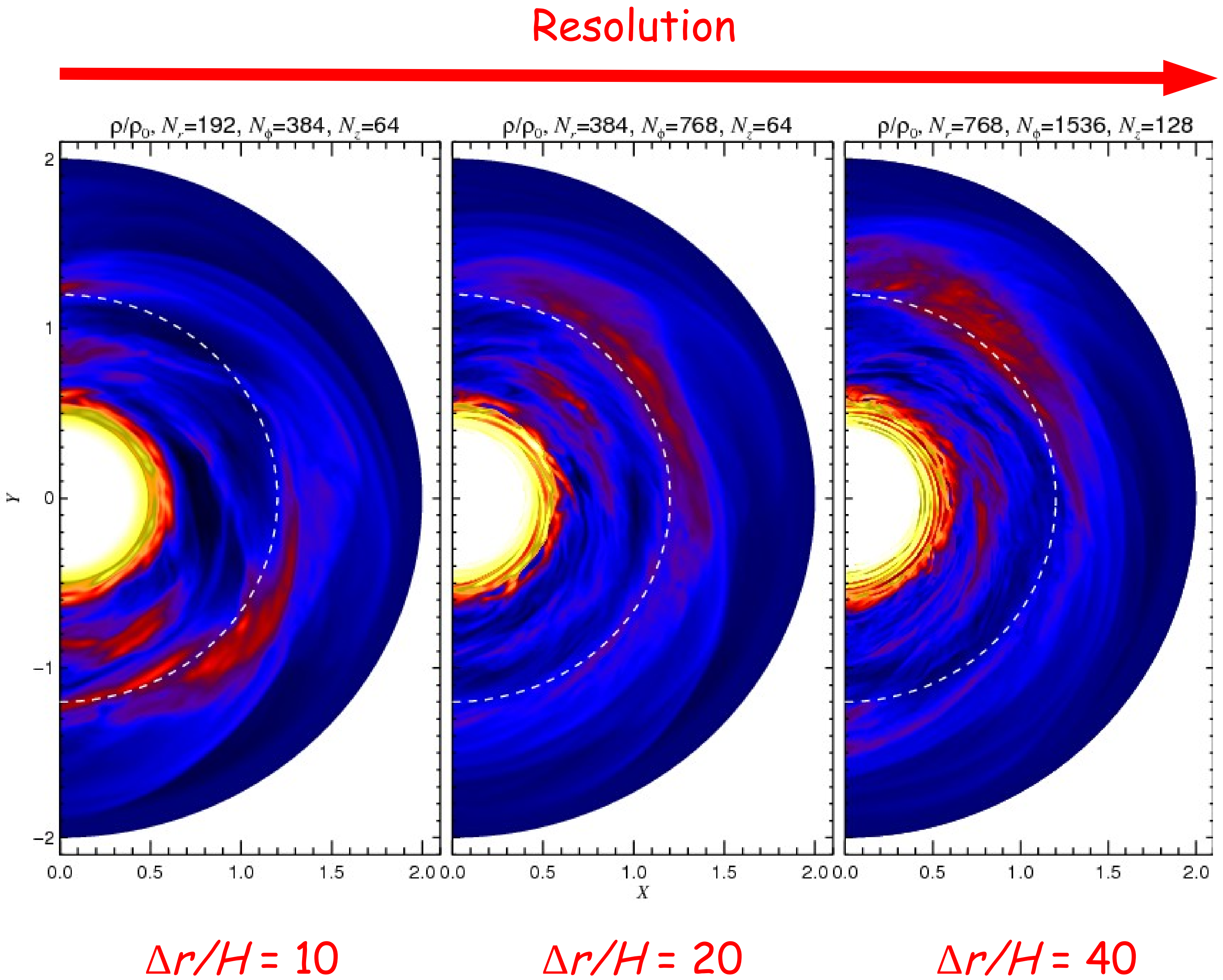


Large mass accretion rates in the dead zone,

comparable to the MRI in the active zone!

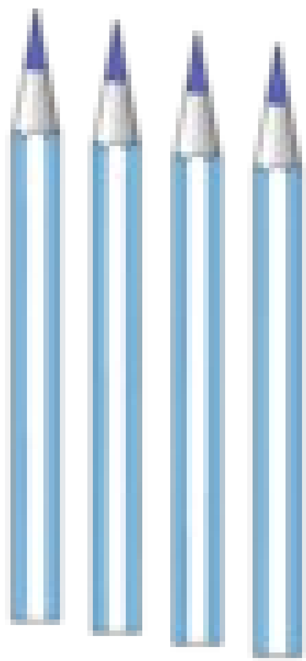
Lyra & Mac Low (2012)

Fishy vortex in the active zone...



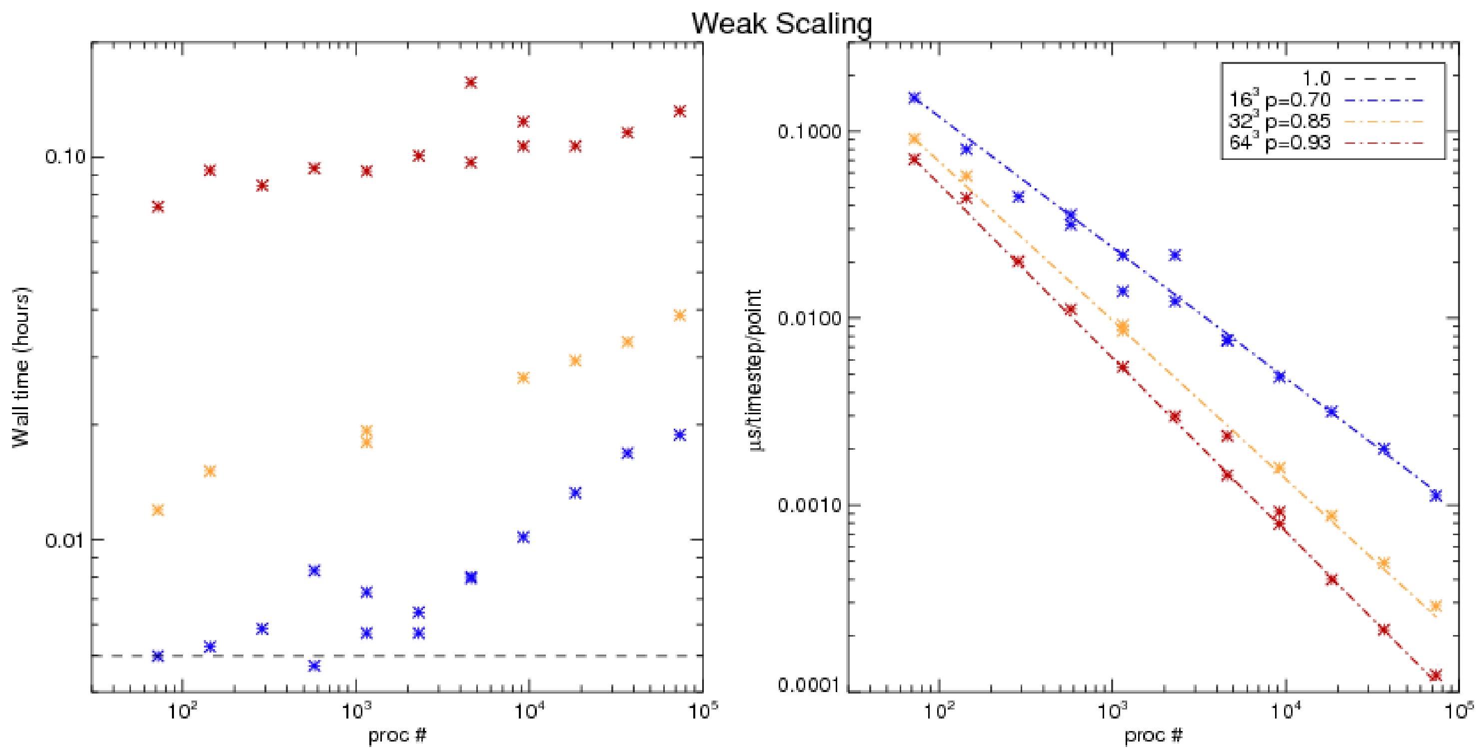
Lyra & Mac Low (2012)

High end computing



The Pencil Code

Brandenburg & Dobler (2002)

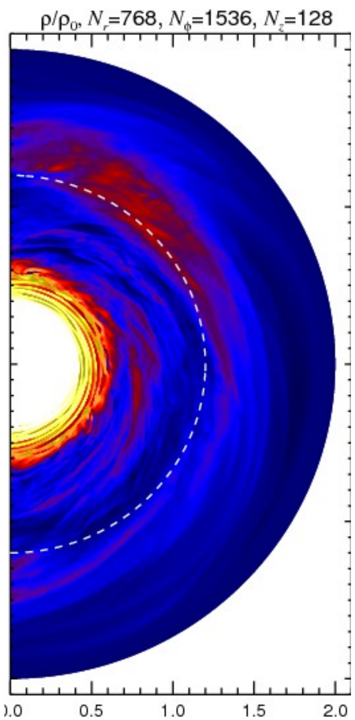


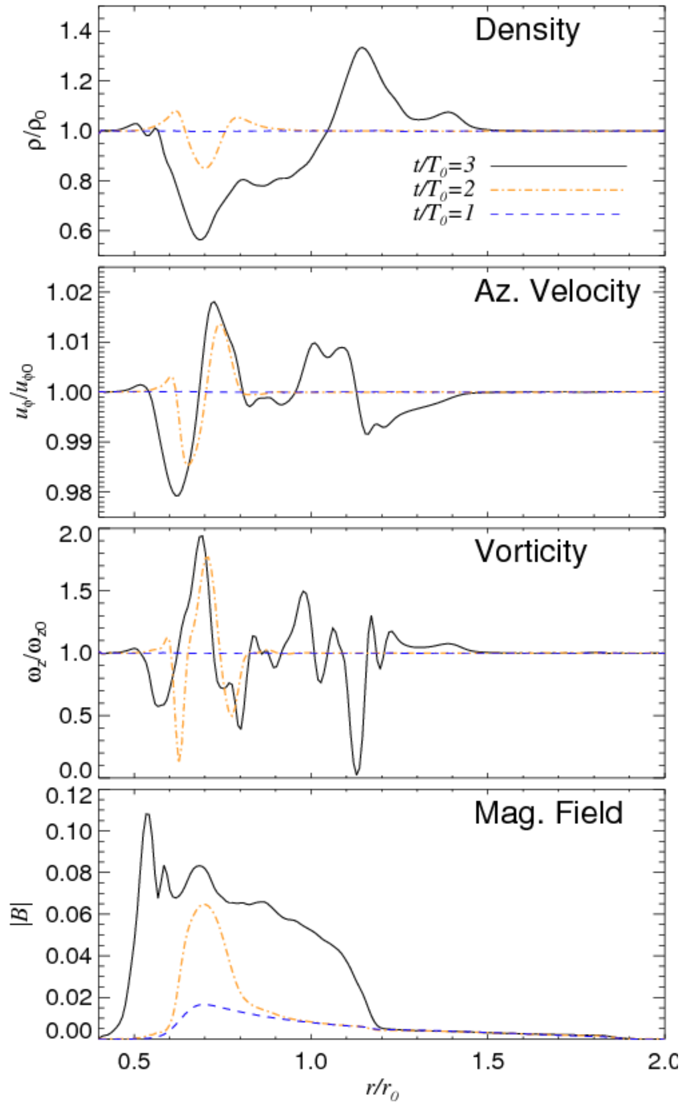
Good scaling up to > **70,000 processors !**

(At NICS - Kraken)

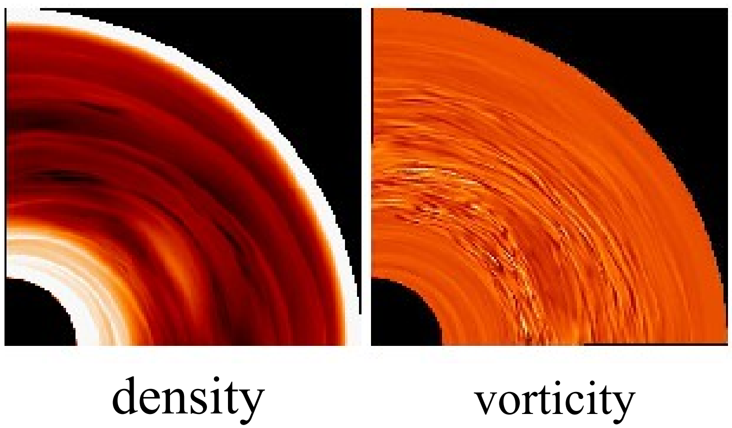
A zonal flow?

(see also R. Lovelace's talk)

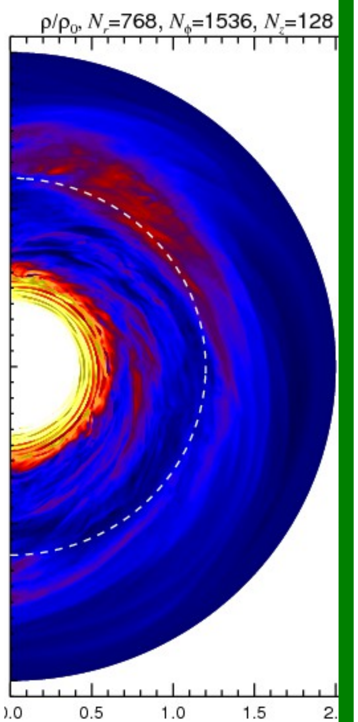




Lyra & Mac Low (2012)



Fromang & Nelson (2005)



L

Facts

Rossby vortices survive
the magneto-elliptic instability,
whereas baroclinic vortices do not.

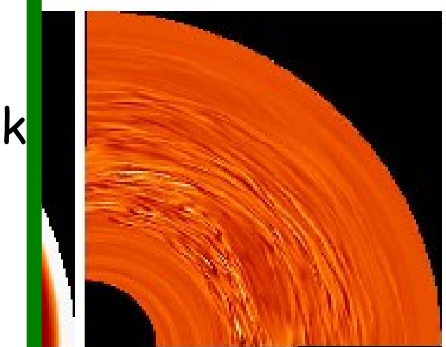
"Vortex survival is a balance
between production and destruction"
- John Papaloizou's talk

Conjecture

Active zone Rossby vortex survives because RWI
produces vorticity faster than the MEI destroys,
whereas BI does not.

Strenght of vorticity injection/destruction

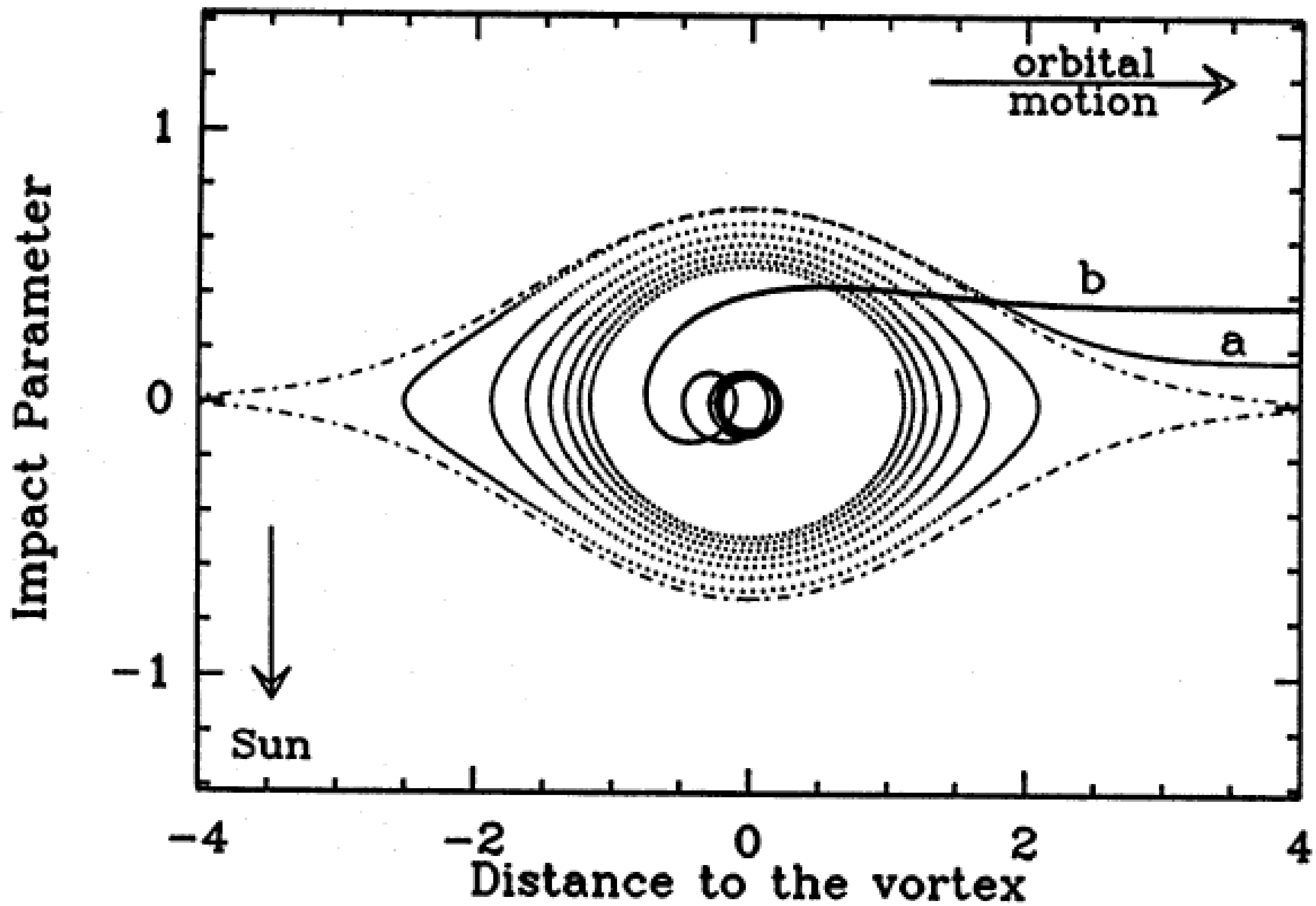
EI < BI < MEI < RWI



vorticity

Nelson (2005)

Forming Planets



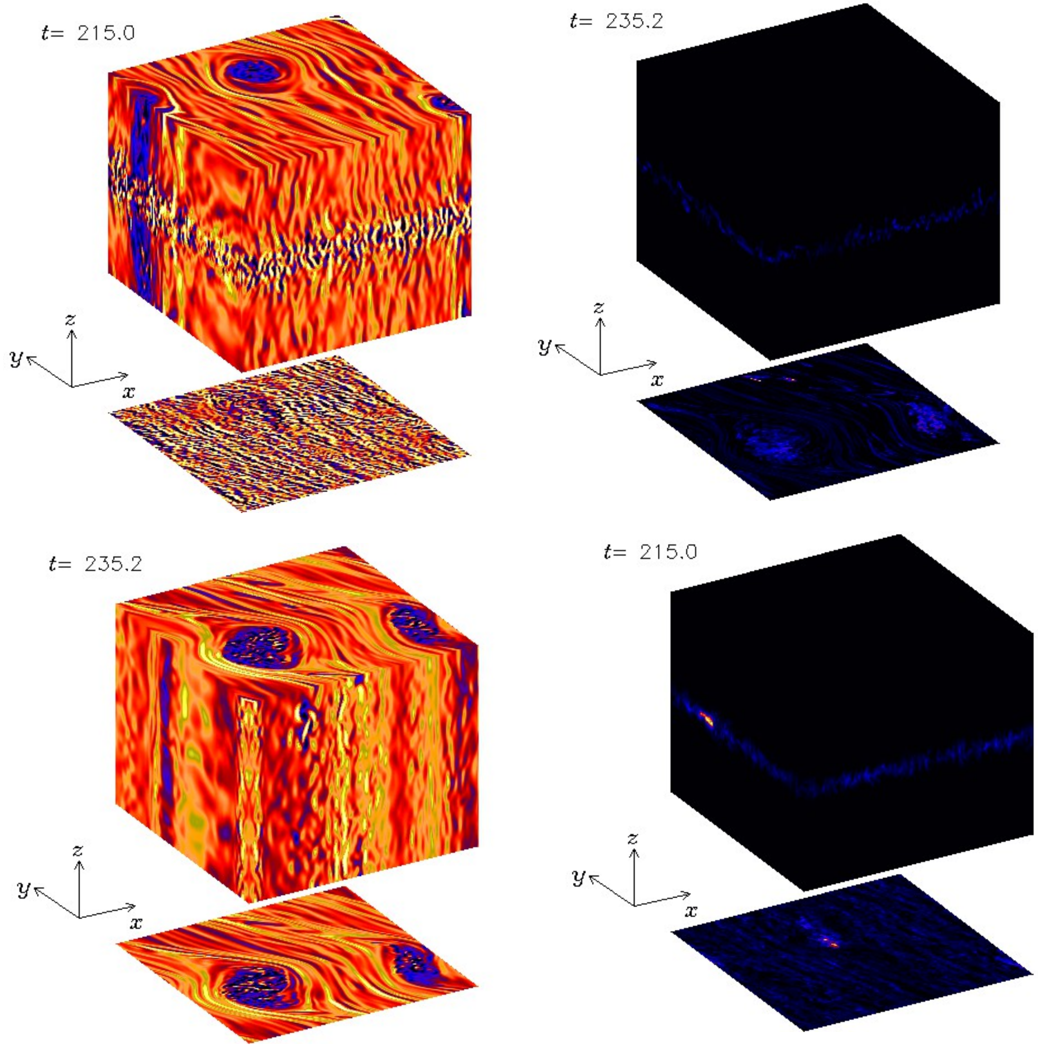
Barge & Sommeria (1995)

Forming Planets

Gas | Particles

Backreaction

No Backreaction



Vortices are
not destroyed
by heavy particle load.

Raettig, Lyra & Klahr (to be submitted)