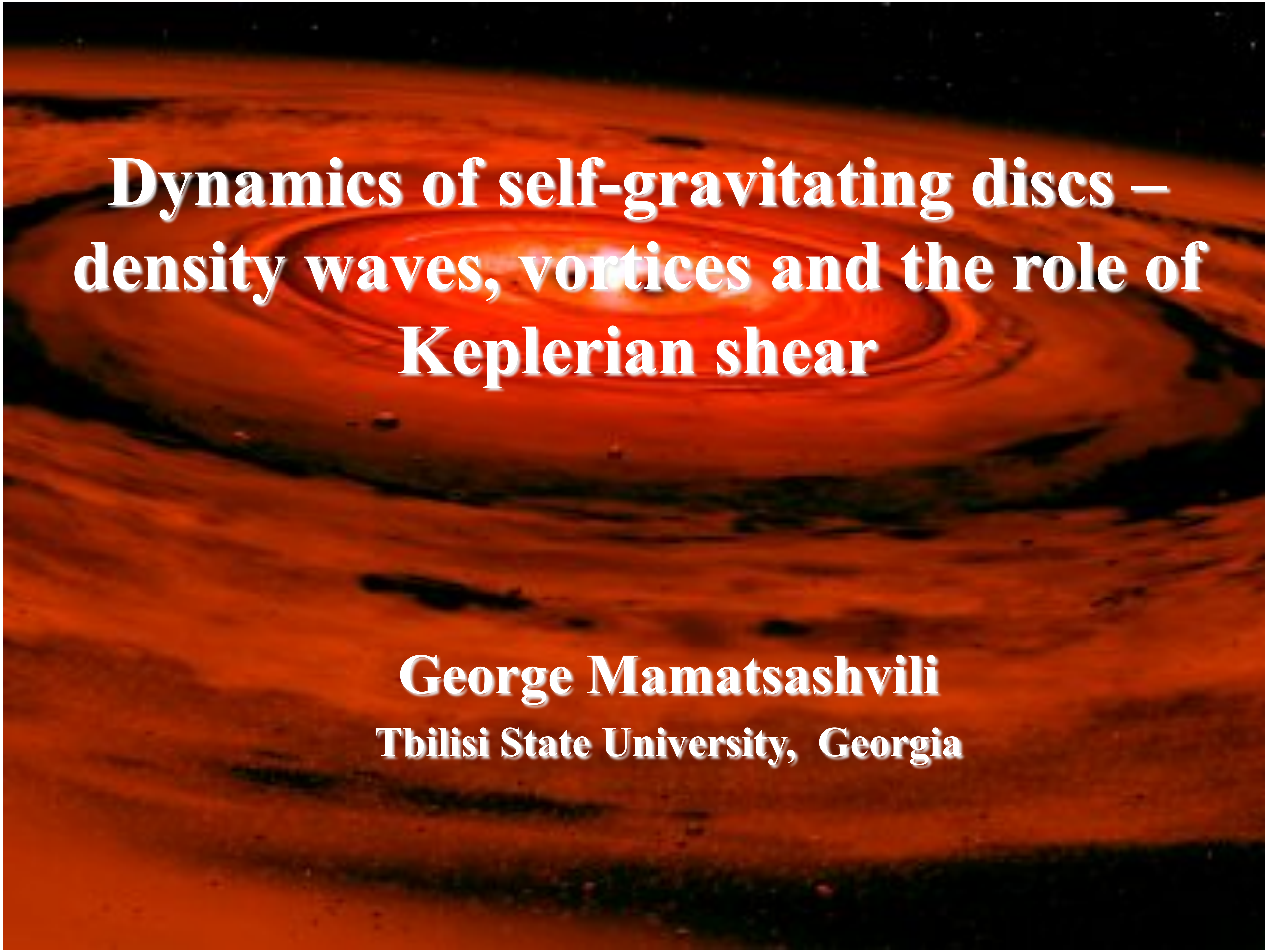

Dynamics of self-gravitating discs – density waves, vortices and the role of Keplerian shear

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Collaboration

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Outline

- Introduction:
  1. Overview of the basic dynamics of self-gravitating discs
  2. Recent issues in numerical modelling

- Revised classification of modes – density waves, vortices and their coupling

- Dynamics of vortices under the influence of self-gravity

- Conclusions and future directions

Basic dynamics of self-gravitating discs – a brief overview

- Self-gravity is important in the dynamics of protoplanetary discs at the early stages of evolution, before or sometimes in the T tauri phase, when discs are still massive enough

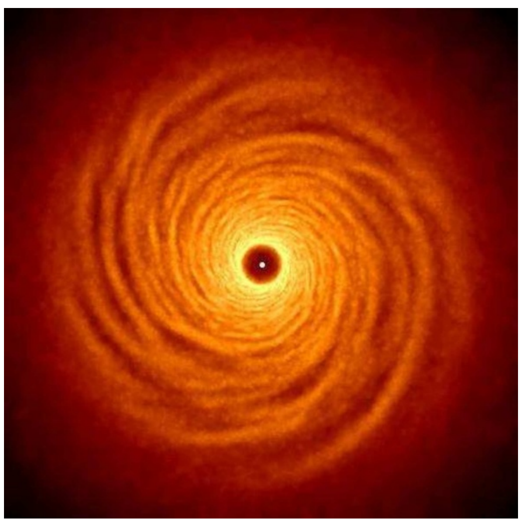
- Self-gravity renders a disc gravitationally unstable when Toomre's parameter

$$Q = \frac{c_s \Omega}{\pi G \Sigma} < 1$$

c_s - sound speed
Ω - angular velocity of disc rotation
Σ - surface density

- In fact gravitational instability develops at slightly larger values $Q = 1.5 - 2$ in the form of non-axisymmetric structures – spiral density waves -- on the disc surface (Durisen et al. 2007)

- Density waves exert torques on the disc via hydro and gravitational stresses transporting angular momentum outwards and matter inwards



Lodato & Rice (2004)

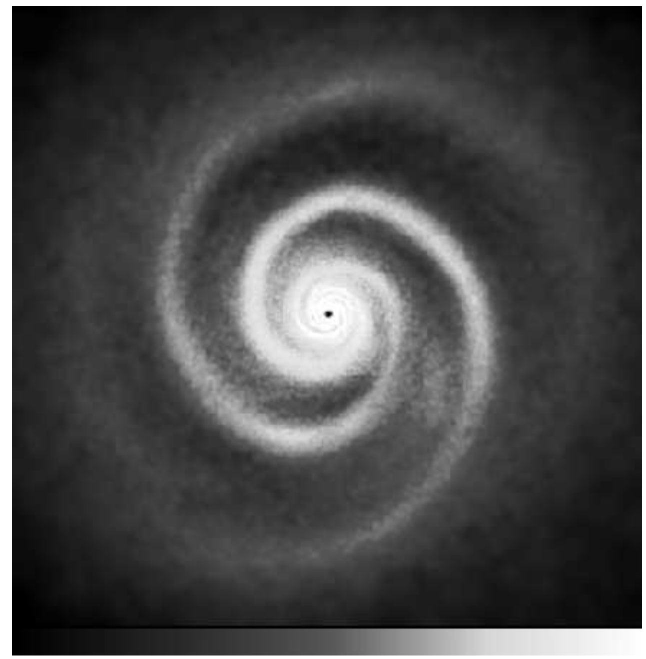
- Non-linear density waves dissipate energy via forming shocks. This shock heating balances the energy losses by radiative cooling. As a result, disc settles into a state of quasi-steady ***gravitoturbulence*** with $Q \approx 1$ (Gammie 2001, Rice et al. 2003, Lodato & Rice 2004),

- In gravitoturbulence, ***local*** balance between shock heating and cooling sets the total stress (α parameter) via effective cooling time $t_c = \beta \Omega^{-1}$ as

$$\alpha = \frac{1}{\gamma(\gamma - 1)\beta} \left| \frac{d \ln \Omega}{d \ln r} \right|^{-2}$$

In massive discs, $M_{disk} / M_{*} > 0.5$ angular momentum transport is dominated by large scale "grand design" density wave modes (e.g., Lodato & Rice 2005)

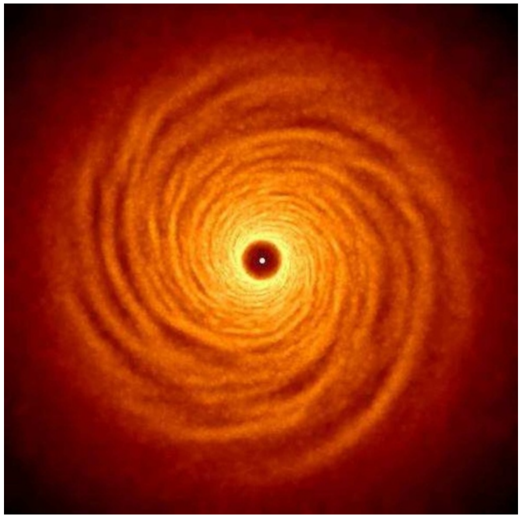
- The accretion process is episodic
- Heating-cooling balance is not local
- Quasi-steady gravitoturbulence is not reached



Lodato & Rice (2005)

In lighter discs, $M_{disk} / M_* < 0.5$ angular momentum transport is dominated by small scale ($\leq H$) modes (e.g., Gammie 2001, Lodato & Rice 2004, Boley et al 2006)

- Accretion process is quasi-steady
- Heating-cooling balance is local
- Disc stays in a quasi-steady gravitoturbulent state for many revolutions

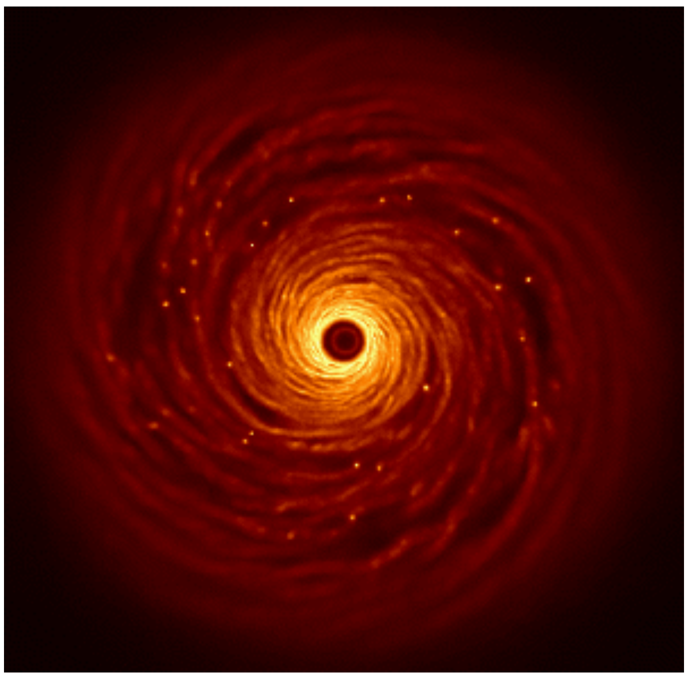


Lodato & Rice (2004)

- Gravitoturbulent state is thought to involve only spiral density wave mode

If cooling is too strong, typically $\beta < 3$ or $\alpha > \alpha_{max} = 0.06$
shock heating can not balance cooling, or
self-gravity cannot provide enough stress
consequently disc fragments into bound clumps

(Gammie 2001, Rice et al 2005, Paardekooper 2012)



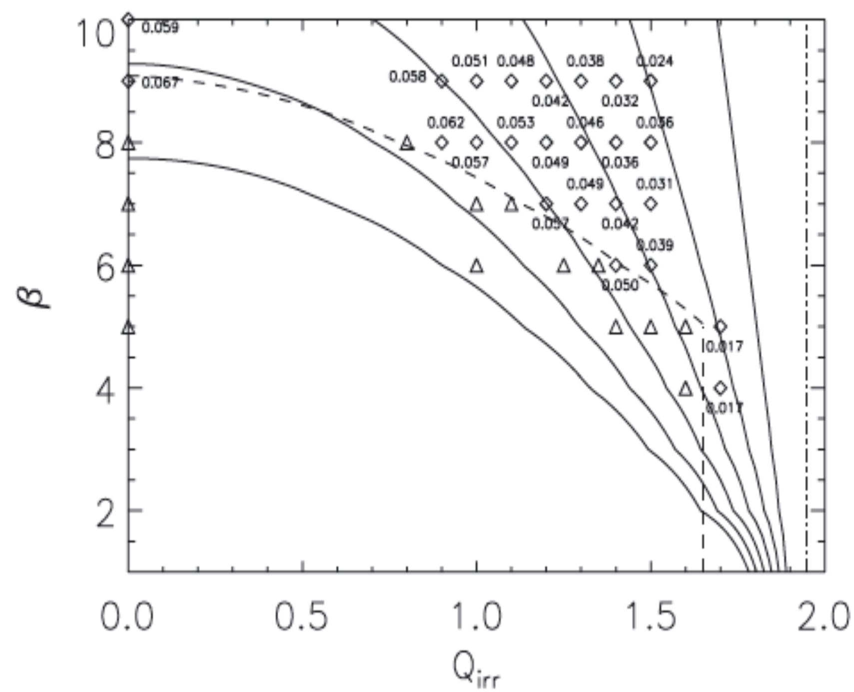
Rice et al (2003)

Role of irradiation – how it changes fragmentation boundary

We (Rice et al 2011) used local shearing sheet simulations to investigate fragmentation as a function of irradiation strength ($Q_{irr} = c_{s,irr}\Omega / \pi G \Sigma$) and local cooling time t_c

- Fragmentation occurs for $t_c < \beta_{crit}\Omega^{-1}$
- Irradiation reduces saturated α

$$\alpha \approx \frac{4}{9\gamma(\gamma-1)\Omega\tau_c}\left(1 - \frac{Q_{\rm irr}^2}{Q_{\rm sat}^2}\right),$$



dashed line shows fragmentation boundary β_{crit} vs. irradiation Q_{irr}

Latest numerical issues – non-convergence of fragmentation boundary

Recent high-resolution simulations indicate that fragmentation boundary β_{crit} can increase with resolution (Meru & Bate 2011, Paardekooper et al 2011)

likely reasons:

1. Simple cooling law prescription in SPH (Rice et al 2011)
2. Numerical viscosity in SPH (Lodato & Clarke 2011, Meru & Bate 2012)
3. Initial conditions, boundary and global effects (Paardekooper et al 2011)

Revisiting mode classification in self-gravitating discs

In self-gravitating discs, angular momentum transport is commonly attributed to spiral density waves

(e.g., Lynden-Bell & Kalnajs 1972, Durisen et al 2007)

- Linear (WKB) dispersion relation: $\omega^2 = c_s^2 k^2 + \kappa^2 - 2\pi G \Sigma_0 k$

  These are actually sound waves modified by rotation and self-gravity, become unstable for

  $$Q = \frac{c_s \kappa}{\pi G \Sigma} < 1$$

- Density waves are thought to be the only mode affected by self-gravity and determining gravitational instability of the disc
- ***Density waves aren't thought to carry potential vorticity !***

Perturbation modes in *non-self-gravitating* discs:

Sound (density) waves due to compressibility

- Linear dispersion relation: $\omega^2 = c_s^2 k^2 + \kappa^2$

- Carry no potential vorticity, i.e., are mostly divergent

- Play an important role in:

1. Enhancing angular momentum transport (e.g., in dead zone)

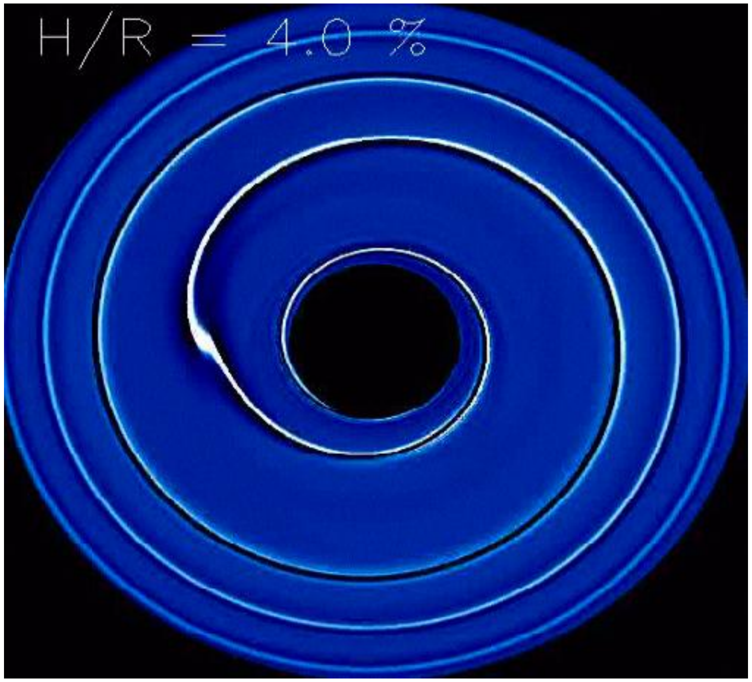
(Johnson & Gammie 2005, Oishi & Mac Low 2009)

2. Slowing down planet migration (Nelson 2005, Uribe et al 2011)

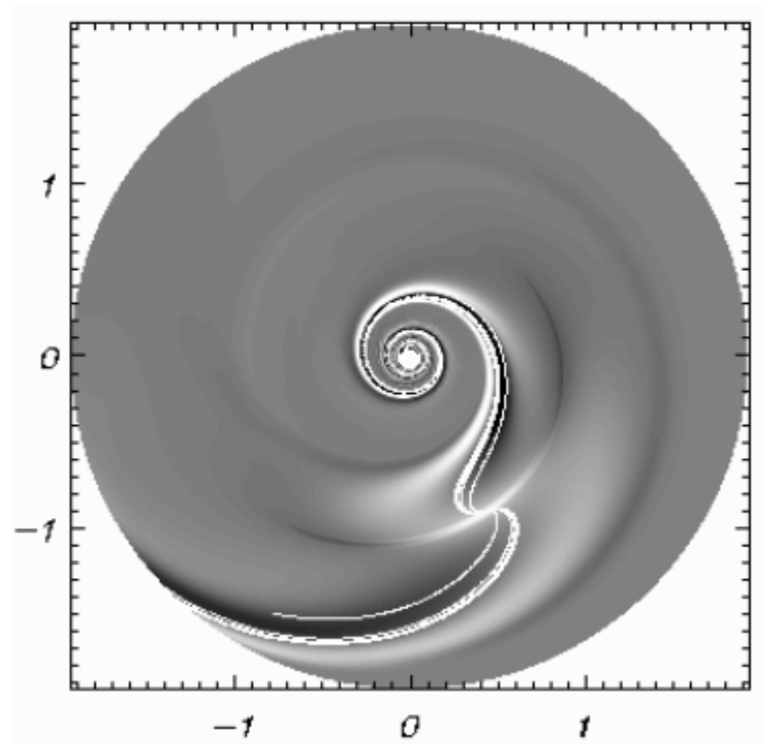
3. Disc-planet interaction is mediated through density waves

(Goldreich & Tremaine 1980, Lin & Papaloizou 1986)

Visualization of spiral density waves in non-self-gravitating discs



(http://www.maths.qmul.ac.uk/~masset/)



Bodo et al (2005)

Spiral density waves generated by a low-mass planet in orbital motion

Spiral density waves generated by a localized vortex

Vortices / Rossby waves due to global and local baroclinic effects

- Necessary criterion for the instability: extrema in $\frac{\Sigma\Omega S^{2/\gamma}}{\kappa^2}$ and $\kappa^2 + N^2 < 0$

  (Lovelace et al 1999, Li et al 2000, Klahr & Bodenheimer 2003)

- Linear dispersion relation: $\omega^2 = -c_s^4 k_y^2 / L_p L_s (c_s^2 k^2 + \kappa^2)$

  L_p, L_s - characteristic length scales of pressure and entropy radial structure

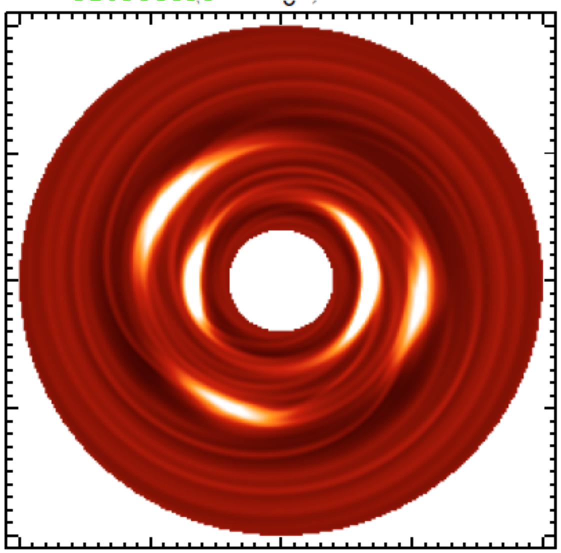
  (In the local shearing sheet model, vortices correspond to $\omega = 0$)

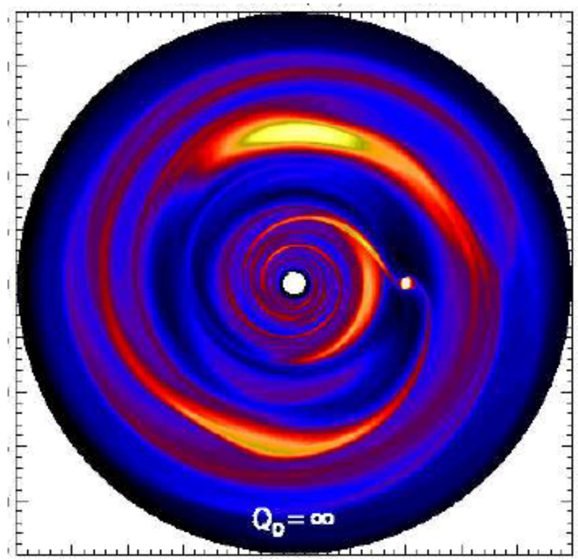
- Carry nonzero potential vorticity

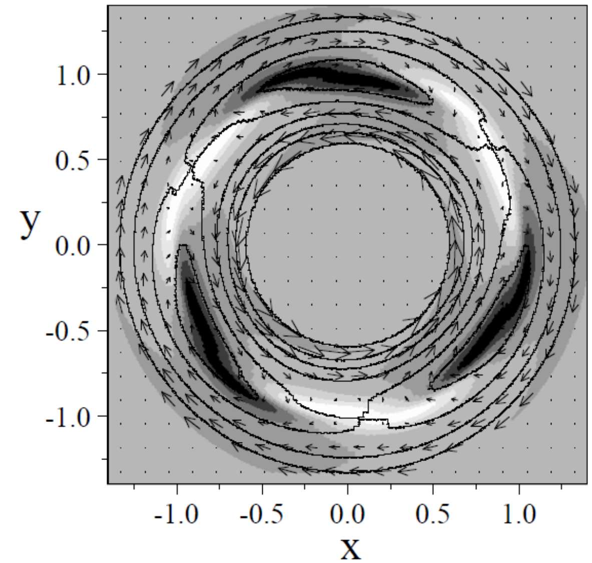
- Play a role in:

1. Angular momentum transport (Li et al 2001, Klahr & Bodenheimer 2003)
2. Planetesimal formation (Barge & Sommeria 1995, Johansen et al 2004)
3. Dead zone dynamics (Lyra et al 2009)
4. Planet migration - planetary gaps are unstable to vortex formation (Lin 2012)

Visualization of vortices/Rossby waves in non-self-gravitating discs







Lyra et al (2009)

Vortices around dead zone edges

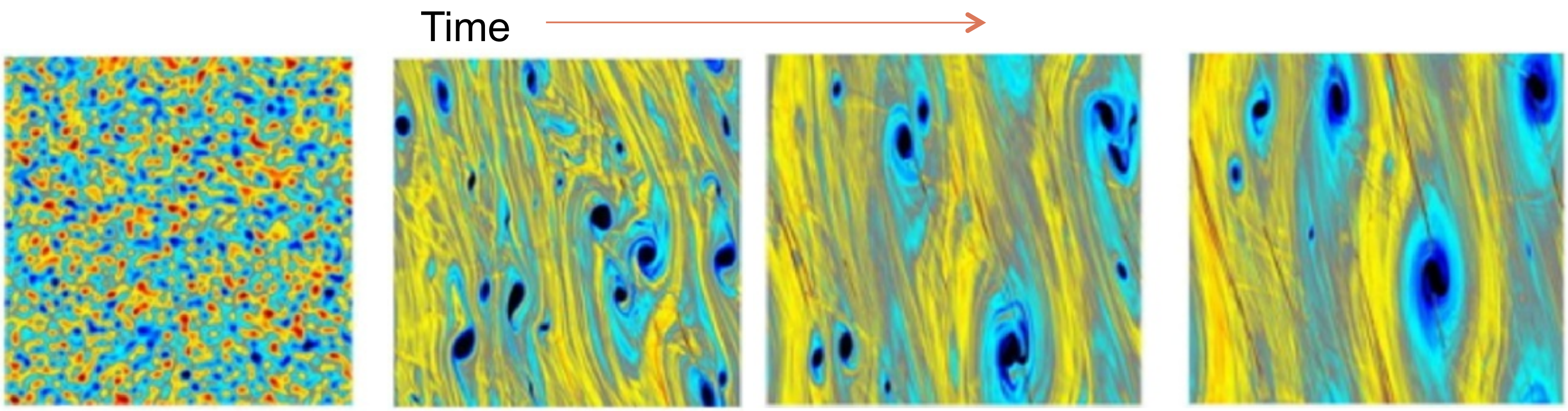
Lin (2012)

Vortices around a gap opened by planet

Li et al (2000)

linear vortex mode due to a density bump

In non-self-gravitating discs, vortices once formed merge into each other forming larger and larger vortices



Johnson & Gammie (2005)

Evolution of potential vorticity (PV) starting with random noise. Anti-cyclonic Vortices survive and grow in size via merging until their size reaches local scale-height

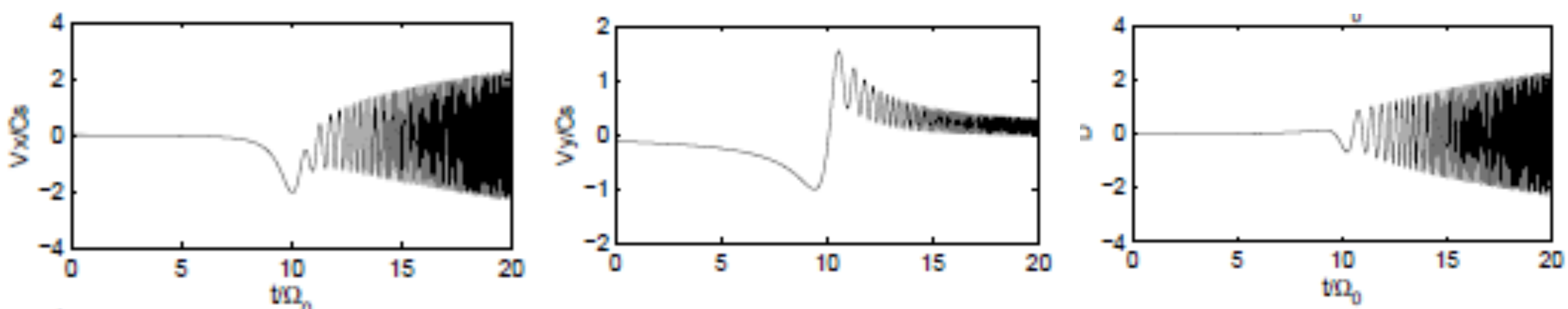
Vortices are also important participant in energy exchange processes in discs –

- **Undergo transient amplification** (Lominadze etal‘88,Chagelishvili etal‘03)
- **Couple with density waves due to Keplerian differential rotation -- vortices generate density waves**

(Chagelishvili et al ‘97, Vanneste & Yavneh ‘04, Bodo et al. ‘05, ‘07, Johnson & Gammie ‘05

Mamatsashvili & Chagelishvili 2007, Heinemann & Papaloizou 2009, Paardekooper et al 2010)

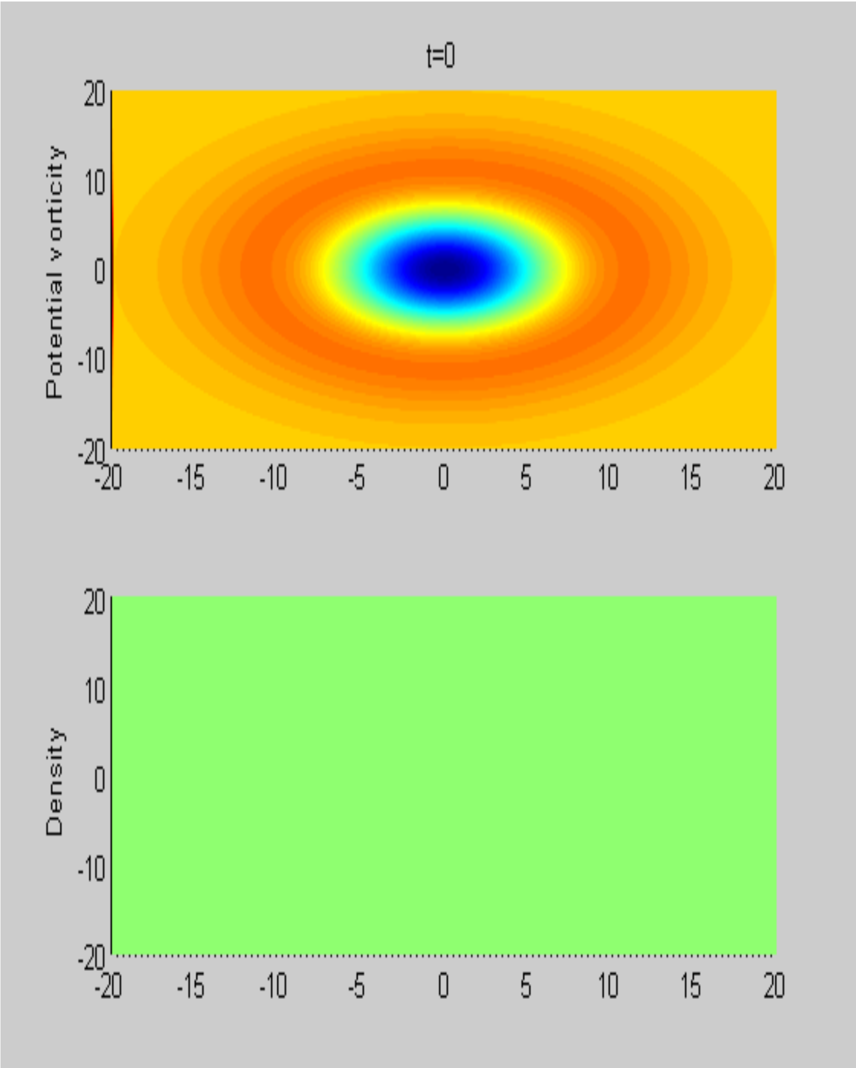
Linear excitation of density waves by non-oscillatory vortex (in terms of shearing waves).



(Calculations are performed in the 2D shearing sheet by Bodo et al 2005)

Wave generation is most efficient at $k_y H \sim 1$

Spatial aspect of vortex-wave coupling:
nonlinear simulations in the shearing sheet



Initial vortex configuration:

$$u_x = -aye^{-\frac{x^2+y^2}{l^2}}, \quad u_y = axe^{-\frac{x^2+y^2}{l^2}},$$

$$\Sigma = 1, \quad a = 0.6$$

Potential vorticity

$$I = \frac{1}{\Sigma}\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y} + (2-q)\Omega\right)$$

After a few orbital times the vortex adjusts and emits shocks of density waves

Up to recently, vortices/Rossby waves, or generally potential vorticity evolution has not been considered in the context of self-gravitating discs

Recent developments have shown that in fact vortices also play an equally important role as density waves in the dynamics of self-gravitating discs (e.g. Mamatsashvili & Chagelishvili 2007, Mamatsashvili & Rice 2009, Lyra et al 2009, Lin & Papaloizou 2011, Lin 2012):

- Can be induced by similar baroclinic effects (e.g., RWI)
- Couple with spiral density waves
- Subject to the effects of self-gravity and exhibit gravitational instability

Linear dynamics of vortex mode in self-gravitating discs

(Mamatsashvili & Chagelishvili 2007)

Linearized 2D shearing sheet equations with Poisson's equation for self-gravity

Perturbations are decomposed into shearing waves

$$F(\mathbf{r},t) = F(t)\exp[\mathrm{i}K_x(t)x + \mathrm{i}K_y y], \quad K_x(t) = K_x(0) + qK_y t,$$

where $F \equiv (u_x, u_y, \Sigma)$

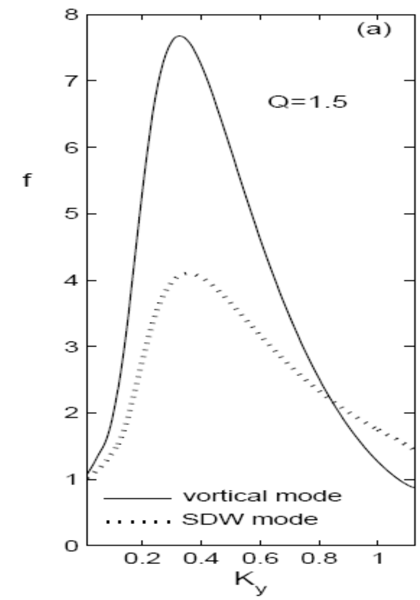
differential equation for gravitational potential

$$\frac{d^2\phi}{dt^2} + \omega^2(K_x(t), K_y)\phi = -\frac{4}{QK(t)}\left(1 - \frac{qK_y^2}{K^2(t)}\right)\mathcal{I},$$

I - linearized potential vorticity

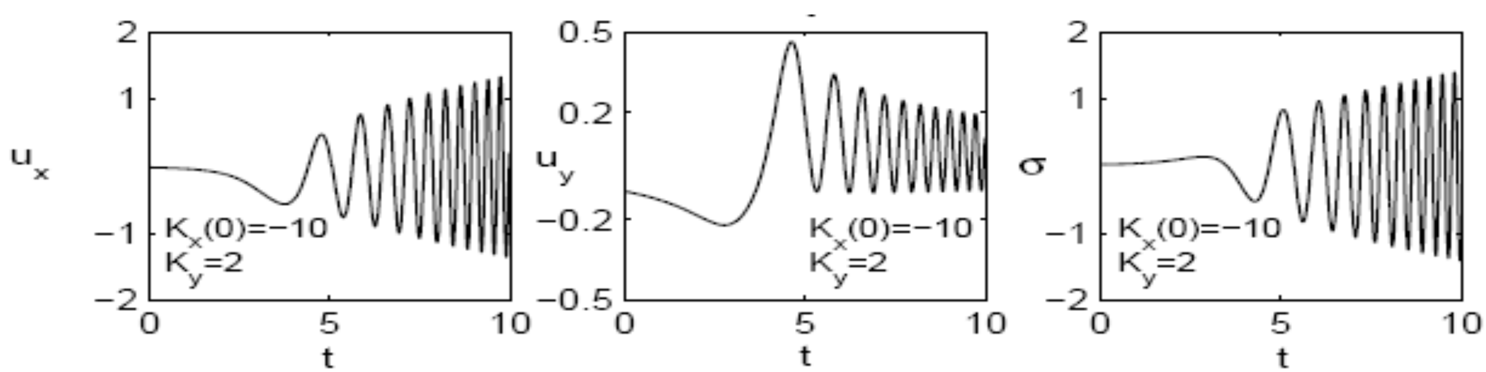
Frequency of density waves $\omega^2(K_x(t), K_y) = 1 + K^2(t) - \frac{2}{Q}K(t) - \frac{4qK_y^2}{K^2(t)} + \frac{3q^2K_y^4}{K^4(t)}$

Vortex mode is subject to the effects of self-gravity and exhibits gravitational instability



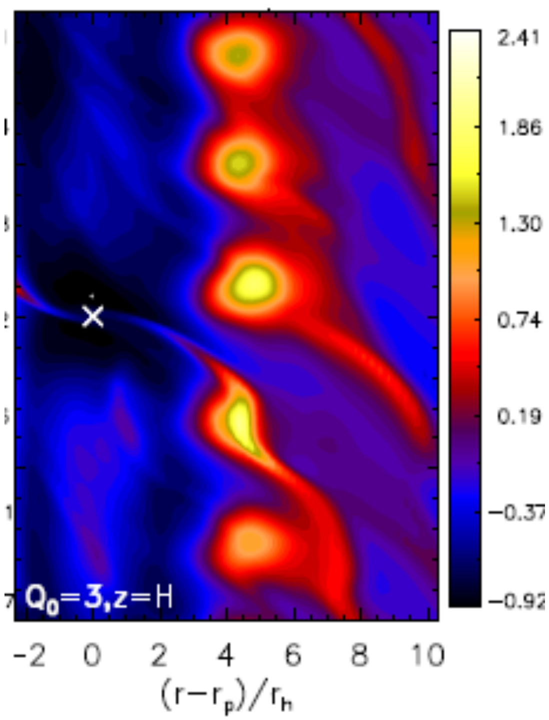
Vortical mode grows about 3 times larger due to self-gravity than density waves when the azimuthal scale is comparable to the scale height *H* (K_y=0.3)

Coupling with density waves

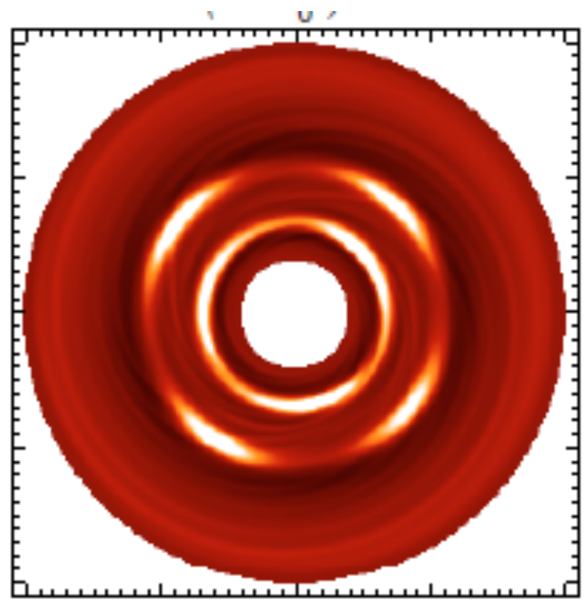


Vortical mode undergoes transient growth due to self-gravity and generates density waves at $K_x(t)$=0 ; Wave generation is most efficient when K_y ~ 1, i.e., λ_y ~ *H*

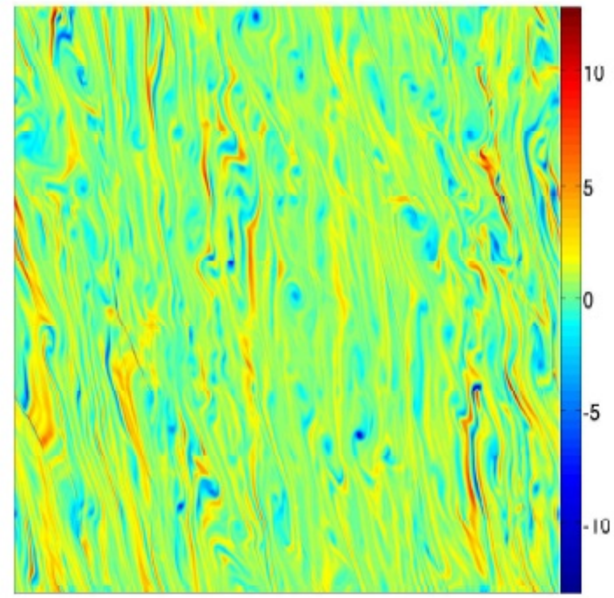
Vortices/Rossby waves in self-gravitating discs



Vortices at the outer edge of a planetary gaps (Lin 2012)



Vortices at the boundaries of a dead zone (Lyra et al 2009)



Vortices without baroclinic driving under self-gravity in the shearing sheet (Mamatsashvili & Rice 2009)

Effects of self-gravity on the vortex dynamics

Self-gravity opposes inverse cascade of energy and merging of vortices – smaller scale vortices are favoured more compared with the non-self-gravitating case

Rossby vortices (Lyra et al 2009, Lin & Papaloizou 2011, Lin 2012)

- occurs at minima of vortensity and Toomre Q
- shifts to higher azimuthal wavenumbers *m* as *Q* decreases
- vortex merging is resisted -- the merging time increases with self-gravity, so that multi-vortex configurations can stay longer at large self-gravity (small Q).

Vortices in self-gravitating discs (without baroclinic/Rossby wave driving)

We study vortex dynamics in a self-gravitating disc using the shearing sheet approximation

$$\frac{\partial\Sigma}{\partial t} + \nabla\cdot(\Sigma\mathbf{u}) - q\Omega x\frac{\partial\Sigma}{\partial y} = 0,$$

$$\frac{\partial u_x}{\partial t} + (\mathbf{u}\cdot\nabla)u_x - q\Omega x\frac{\partial u_x}{\partial y} = -\frac{1}{\Sigma}\frac{\partial P}{\partial x} + 2\Omega u_y - \frac{\partial\psi}{\partial x},$$

$$\frac{\partial u_y}{\partial t} + (\mathbf{u}\cdot\nabla)u_y - q\Omega x\frac{\partial u_y}{\partial y} = -\frac{1}{\Sigma}\frac{\partial P}{\partial y} + (q-2)\Omega u_x - \frac{\partial\psi}{\partial y}.$$

$$\frac{\partial U}{\partial t} + \nabla\cdot(U\mathbf{u}) - q\Omega x\frac{\partial U}{\partial y} = -P\nabla\cdot\mathbf{u} - \frac{U}{\tau_c},$$

Basic Keplerian shear flow
$\boldsymbol{u}_0=(0, -q\Omega x)$, $q=1.5$.

Shearing sheet rotates with Ω

X-radial coordinate
Y-azimuthal coordinate

Simple cooling law with constant $\tau_c=20\Omega^{-1}$

Equation of state $P = (\gamma - 1)U, \quad \gamma = 2$

Poisson'e quation $\Delta\psi = 4\pi G\Sigma\delta(z).$

$\boldsymbol{u}(u_x,u_y)$ – perturbed velocity relative to mean Keplerian shear flow $\boldsymbol{u}_0$,
Σ – surface density, P – pressure, U – internal energy, ψ– gravitational potential,

Potential vorticity (PV), or vortensity – a basic quantity characterizing vortex formation and evolution

$$I = \frac{1}{\Sigma}\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y} + (2-q)\Omega\right)$$

Evolution equation for PV

$$\left(\frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla - q\Omega x\frac{\partial}{\partial y}\right)I = \frac{1}{\Sigma^3}\left(\frac{\partial \Sigma}{\partial x}\frac{\partial P}{\partial y} - \frac{\partial \Sigma}{\partial y}\frac{\partial P}{\partial x}\right)$$

Below we follow evolution of ***PV, Σ, P*** in two cases:

- Initially imposed single vortex
- Vortices in the quasi-steady *gravitoturbulent* state

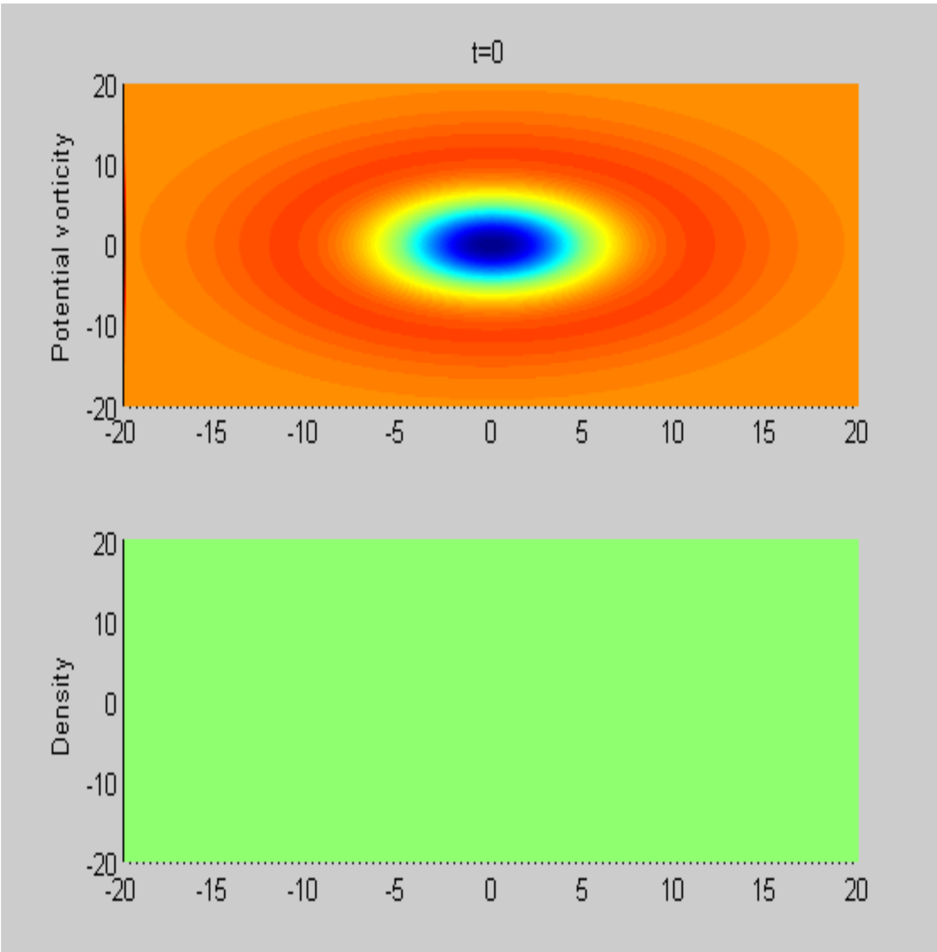
Numerical technique

- ZEUS code suited for the shearing sheet (Gammie 2001, Johnson & Gammie 2003, 2005)

  — with modified treatment of advection by large Keplerian velocity (FARGO scheme, Masset 2000)

- Shearing sheet boundary conditions (Hawley et al. 1995).
- Poisson equation for self-gravity is solved via FFT technique modified for shearing coordinates

Evolution of a single vortex in a self-gravitating disc



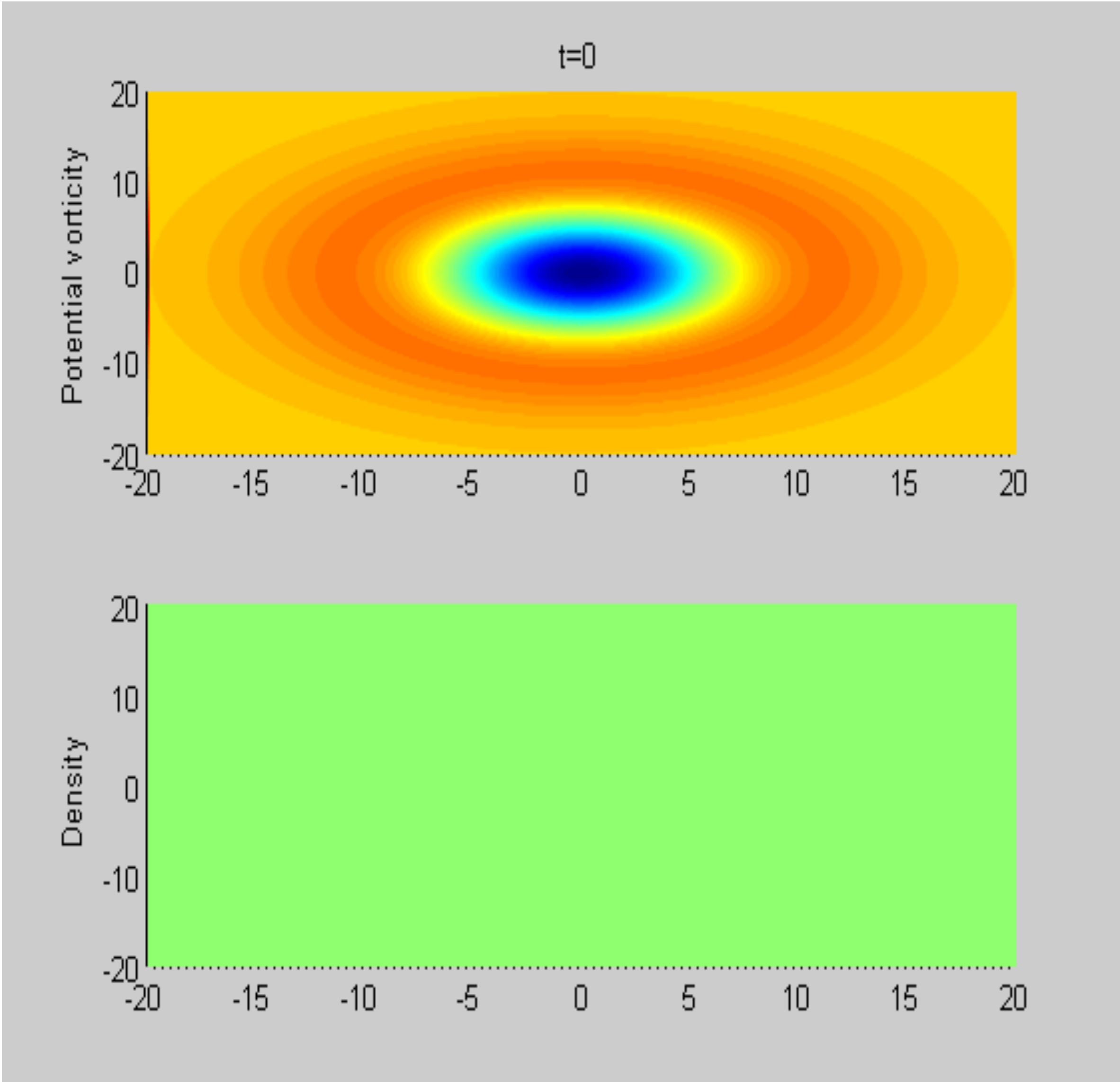
Initial vortex configuration:

$$u_x = -aye^{-\frac{x^2+y^2}{l^2}}, \quad u_y = axe^{-\frac{x^2+y^2}{l^2}},$$

$$\Sigma = 1 \quad Q = 1 \quad l = 1.3\lambda_J$$

Initial vortex becomes gravitationally unstable and the subsequent evolution shows the development of a gravitoturbulent state. The initial vortex has been washed out

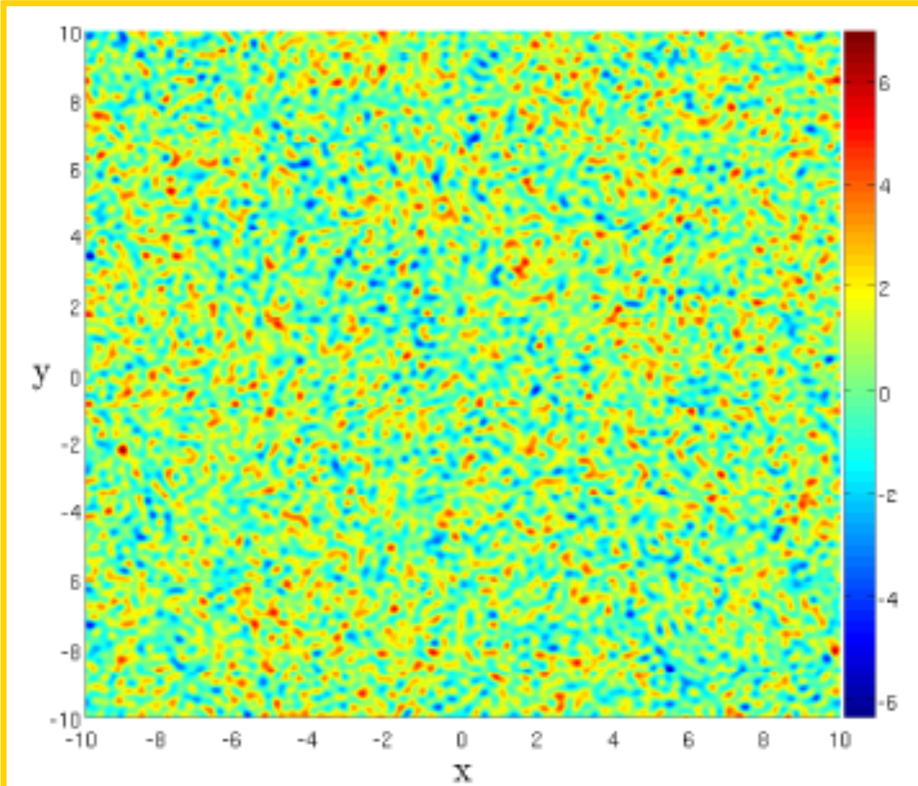
$$t_c = 20\Omega^{-1}$$



$$t_c = 500\Omega^{-1}$$

Vortices in a quasi-steady gravitoturbulent state

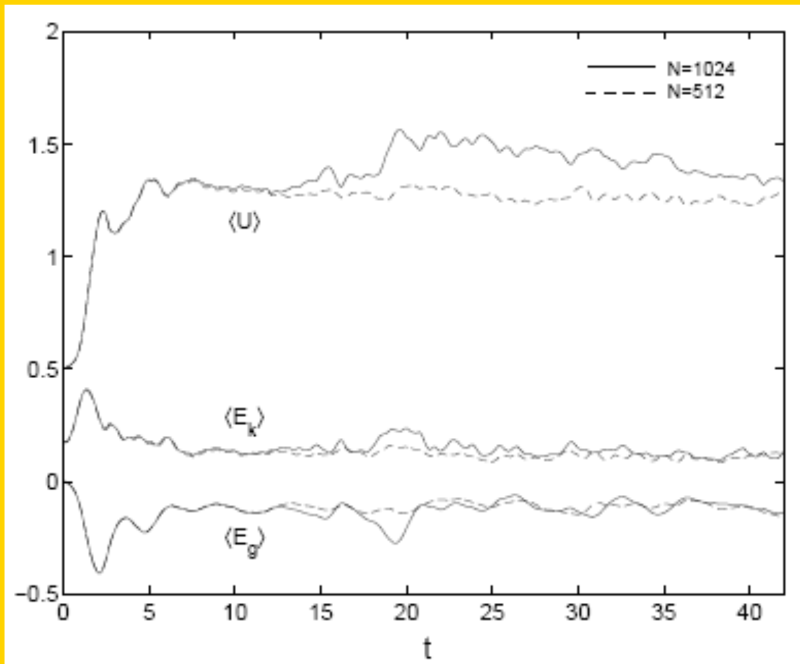
Initial conditions

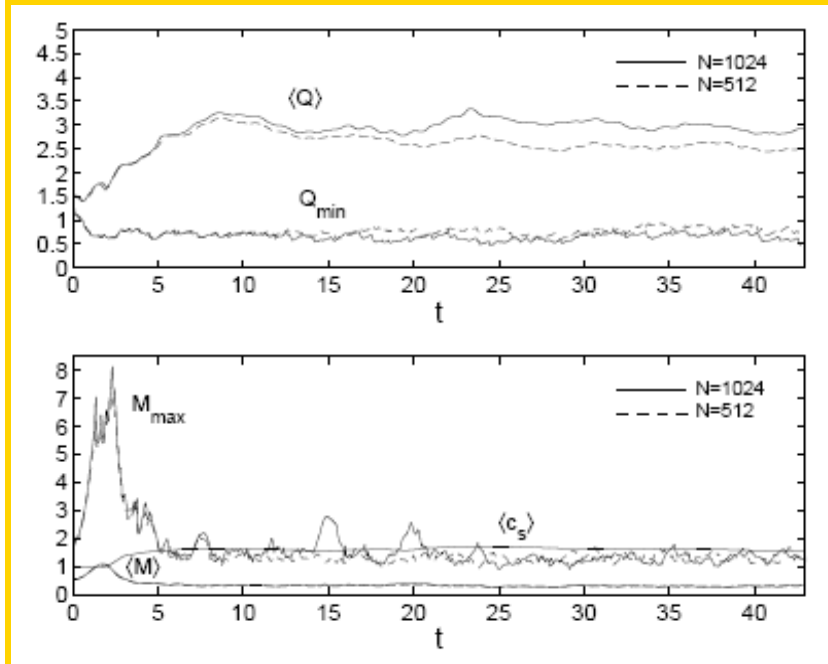


Initial random distribution of PV

Random/chaotic (Kolmogorov spectrum) velocity perturbations are imposed initially with nonzero potential vorticity (PV). Other variables are not perturbed initially.

Evolution of vortices in a gravito-turbulent state





- Average kinetic, internal and graviational energies as well as Toomre's *Q* and Mach number (=u/c_s) after an initial transient settle down to constant values – onset of quasi-steady gravitoturbulence

- Saturated angular momentum transport parameter α is given by

$$\alpha = \frac{1}{q\gamma(\gamma-1)\Omega\tau_c},$$

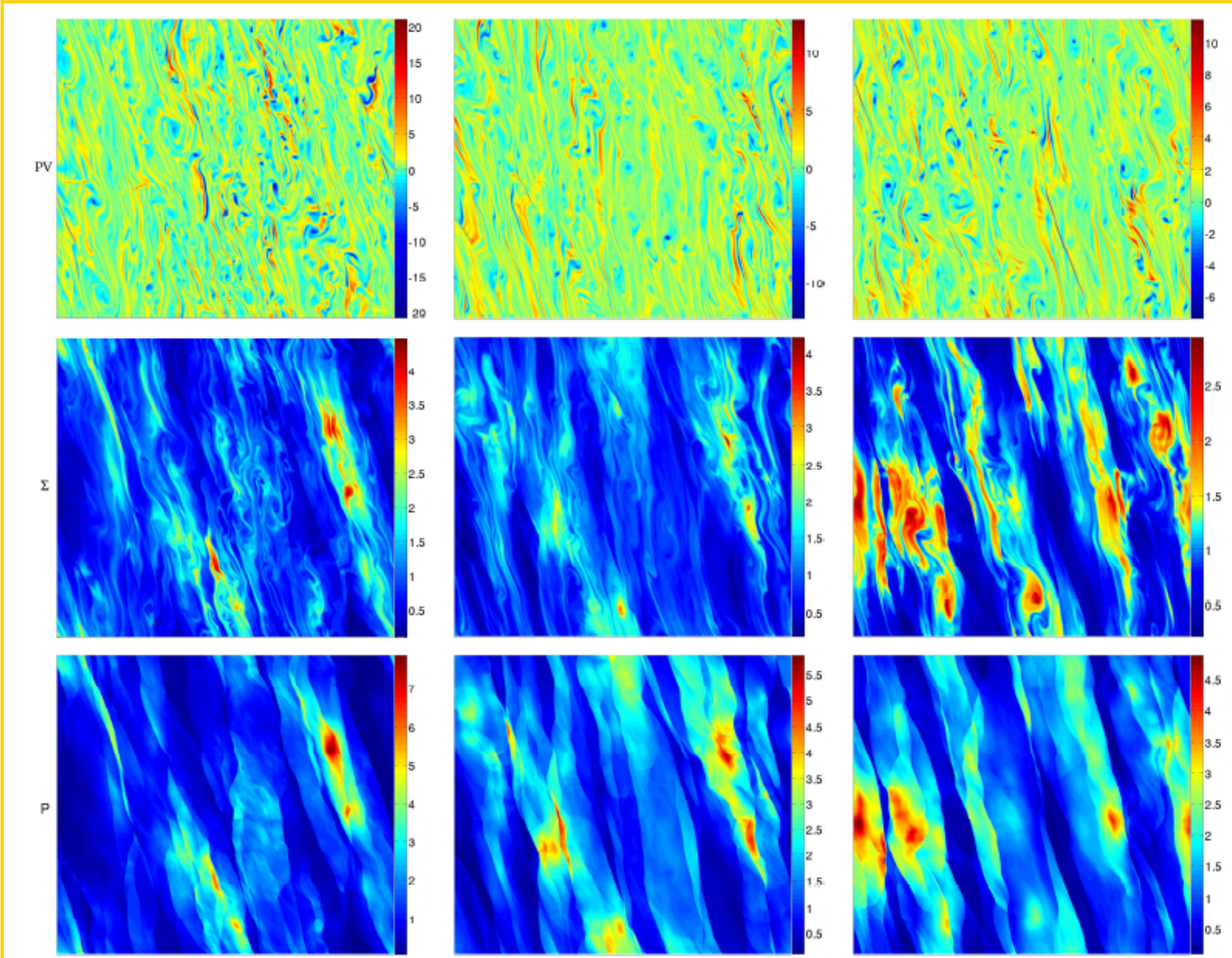
- **<u>Note</u>** minimum *Q* is small (0.6-0.7) and is associated with vortices

Evolution of vortices (potential vorticity) – snapshots at different time moments

PV

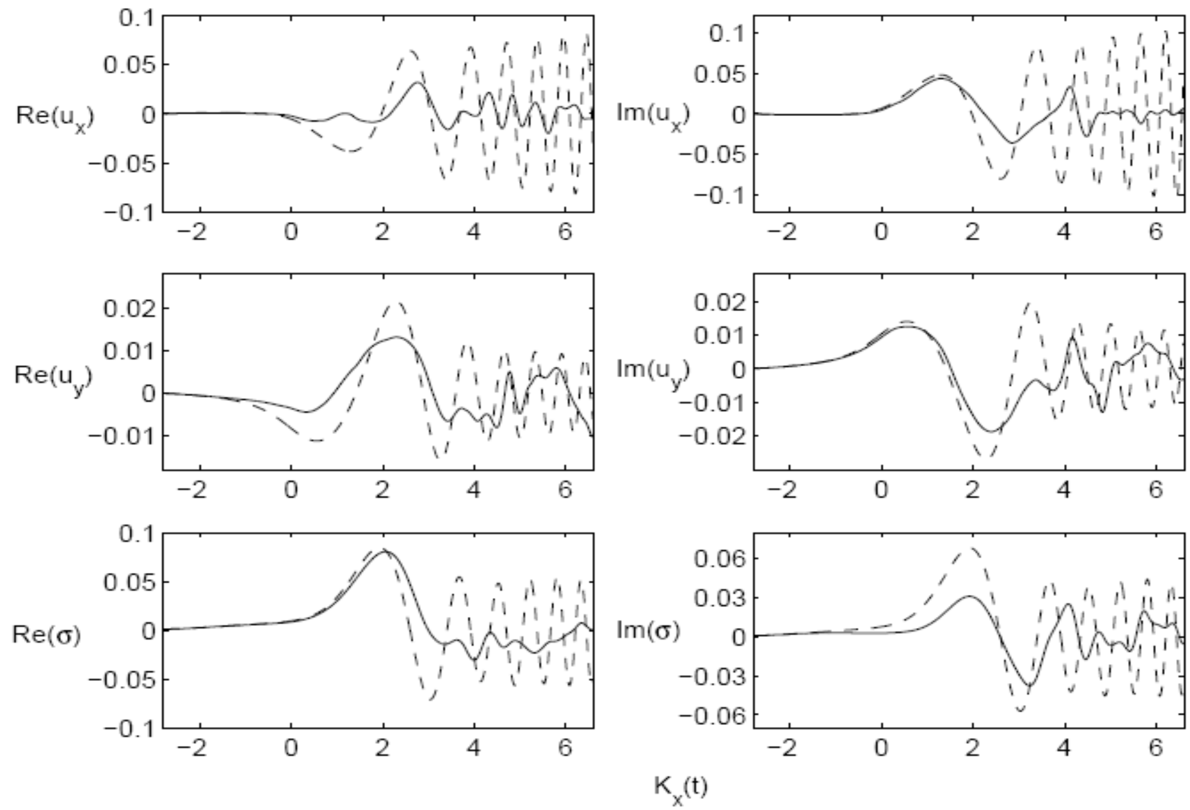
Σ

P



tΩ=14.7 tΩ=29.3 tΩ=44

Vortex-wave coupling in the non-linear evolution of a shearing wave



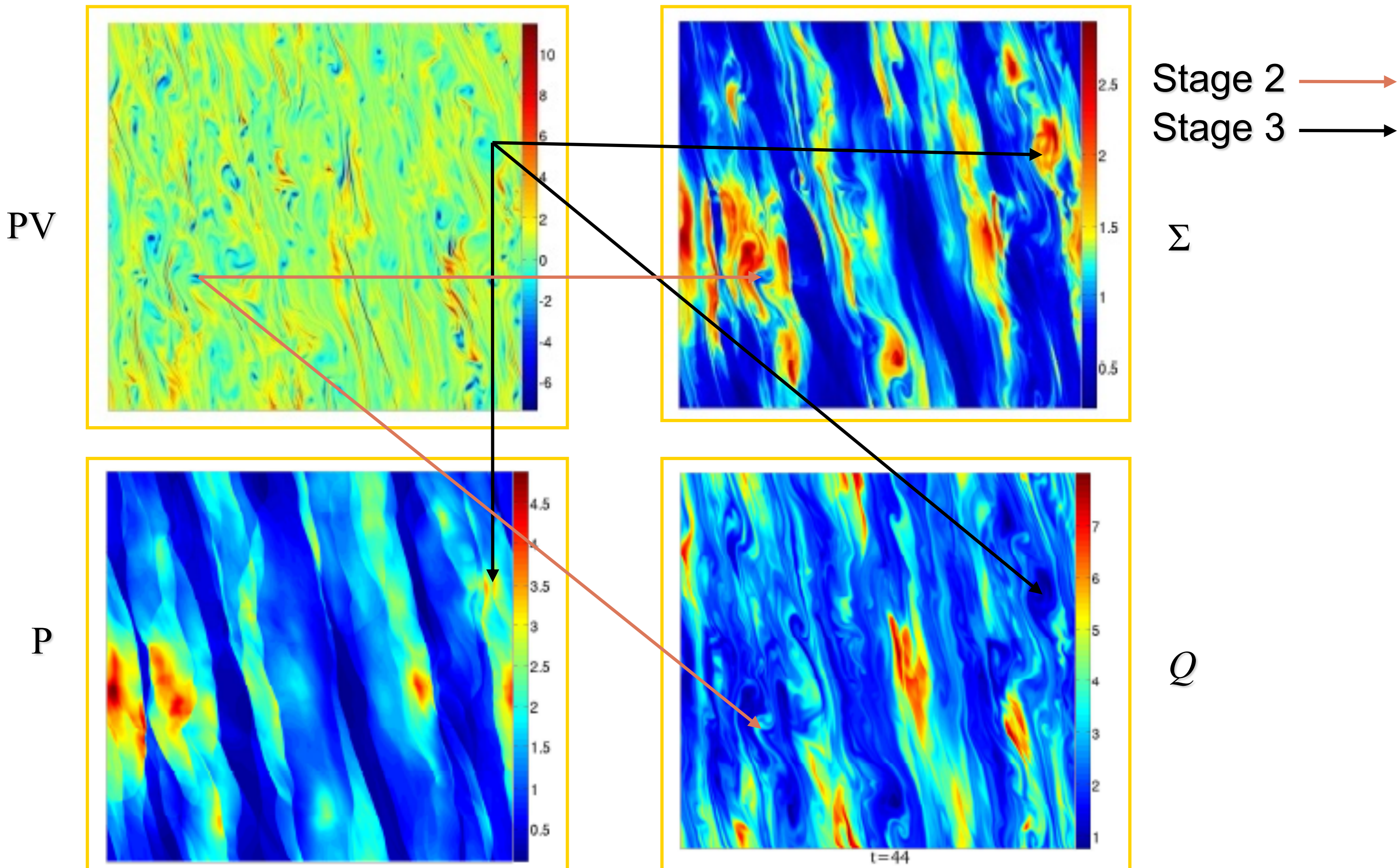
transient (swing) amplification of individual vortical shear wave in the gravitoturbulent state is accompanied by density wave generation similarly as in the linear regime above, but now it is damped in the trailing phase because of shock formation

(Heinemann & Papaloizou 2009)

Evolution of vortices – 4 key evolutionary stages

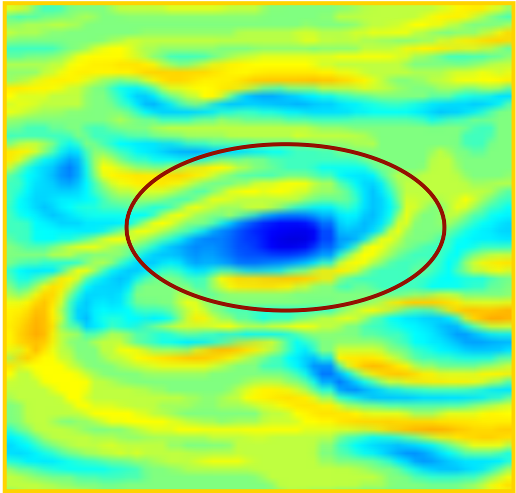
- Formation of small-scale anticyclonic vortices from vortex strips
- Gradual growth in size, underdense centre surrounded by overdense regions – sites of density wave emission
- Vortices with the local Jeans scale, self-gravity comes into play, vortices have a single overdense region. PV is smaller by absolute value. *Q* gradually drops
- *Q* is sufficiently small (0.6-0.7) and vortices are in the process of shearing by self-gravity/gravitational instability and Keplerian shear

Evolution of vortices – snapshot at a single instant ($t\Omega=44$).
Correlations/correspondence for structures in PV, Σ, P and Q fields

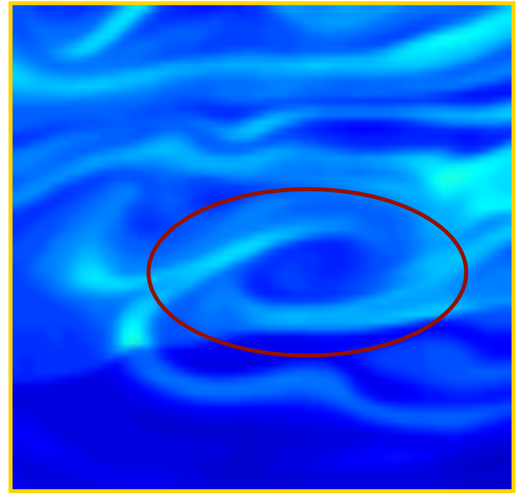


Evolution of vortices – analogy with other simulations (stage 2, underdense and overdense ring-like region – sites of coupling with waves)

PV

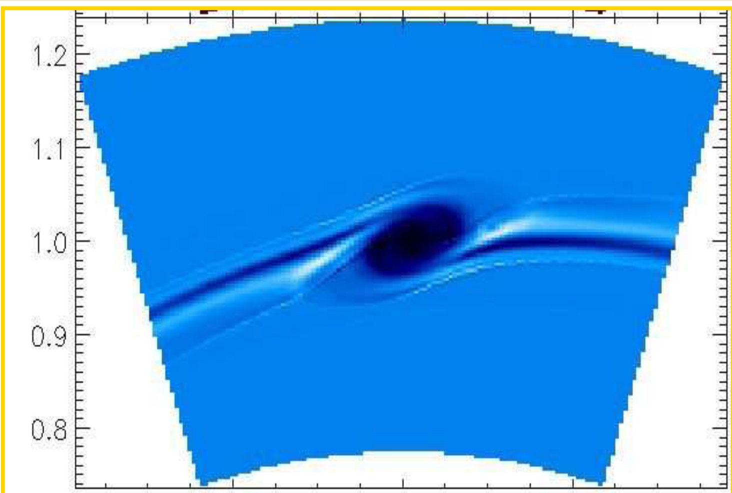


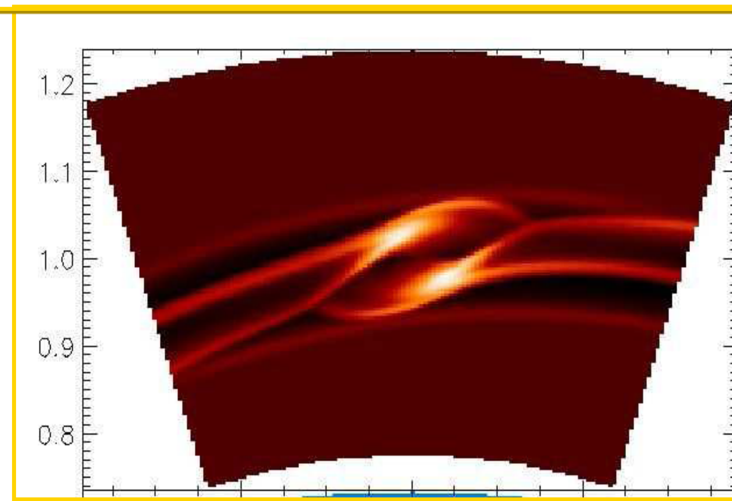
Our plots



Σ

PV



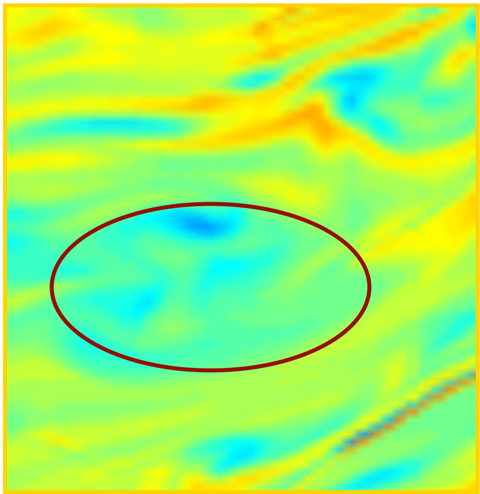


Σ

Simulations of the adjustment of a single vortex without self-gravity (Bodo et al. 2007)

Evolution of vortices – analogy with other simulations (final stages 3-4, only stronger overdense region with lower *Q* is left and is gradually getting sheared)

PV



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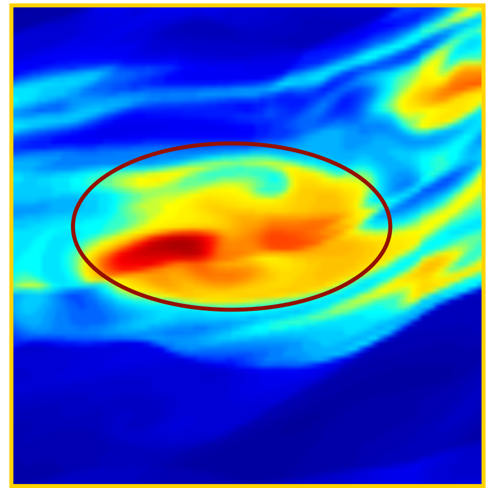
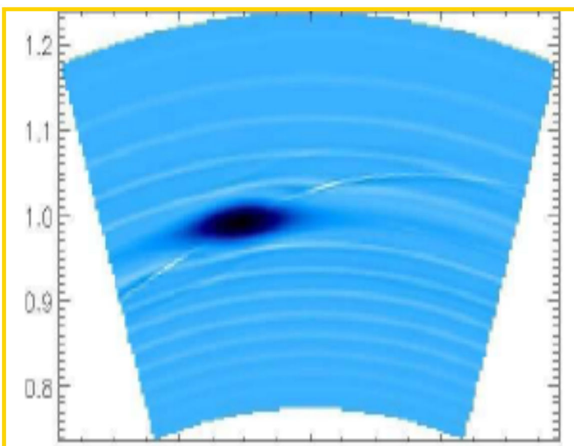
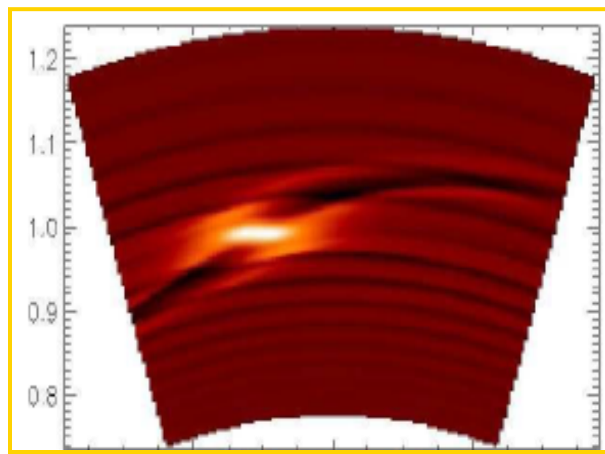


Σ

PV



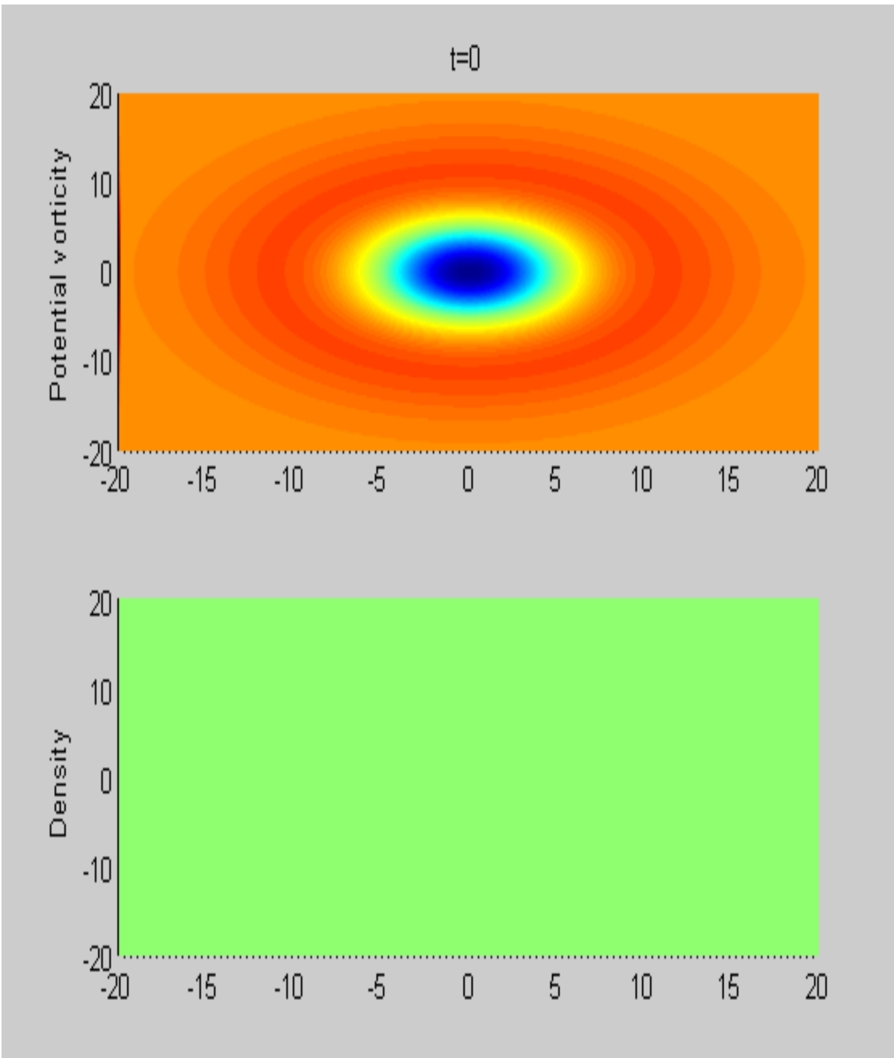


Σ

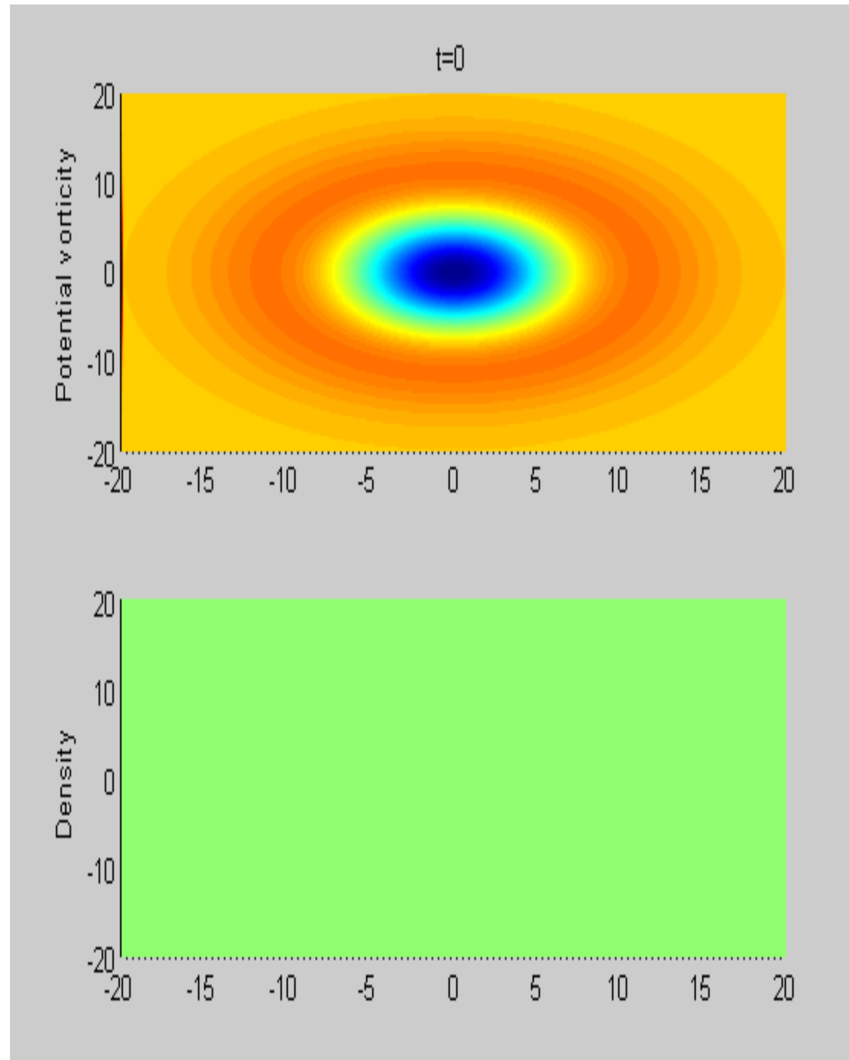
Simulations of the adjustment of a single vortex without self-gravity (Bodo et al. 2007)

Contrast with non-self-gravitating case – a single vortex

self-gravity



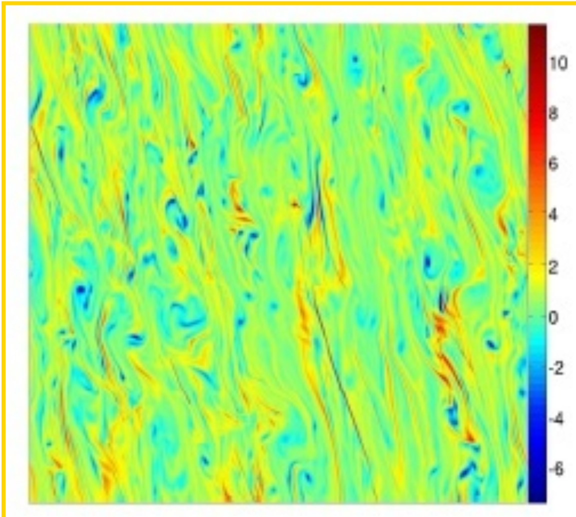
No self-gravity



Contrast with non-self-gravitating case -- gravitoturbulence

self-gravity

PV



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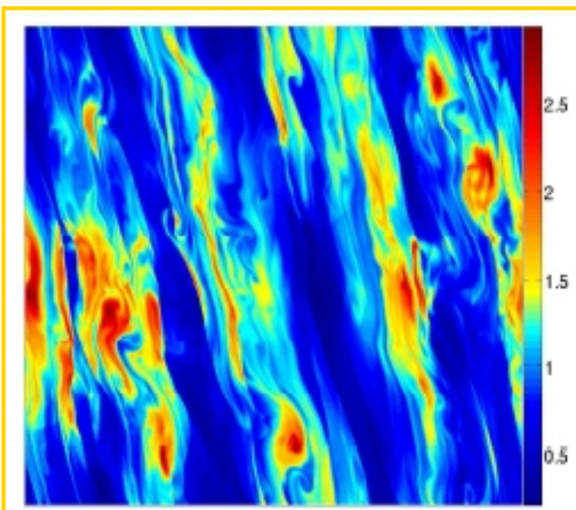
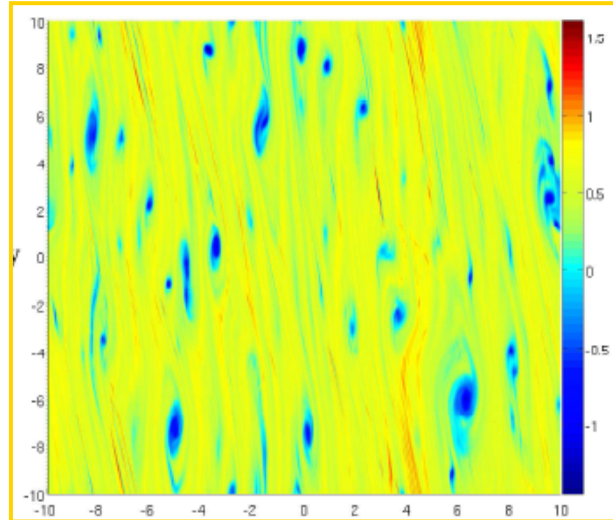
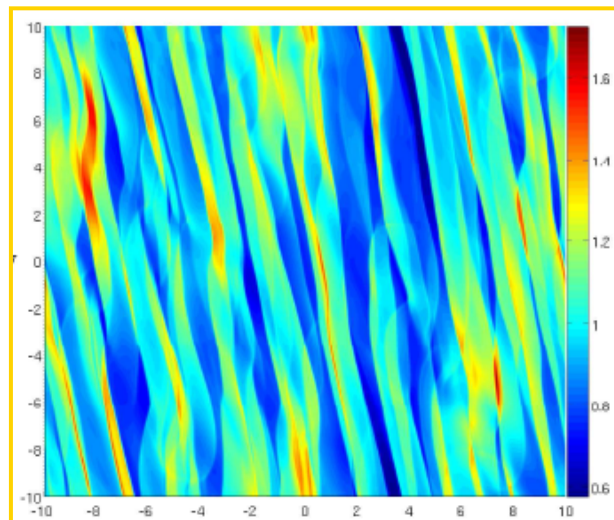


No self-gravity



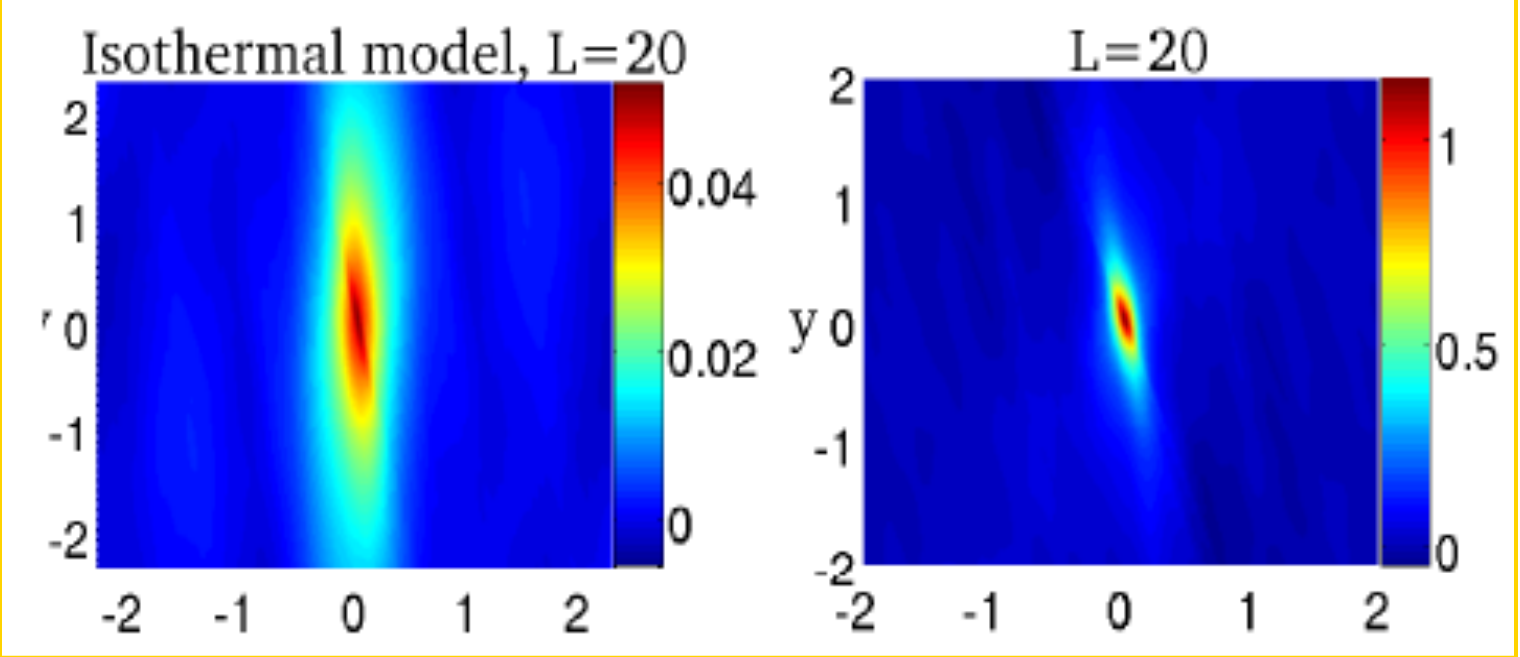
PV



Σ

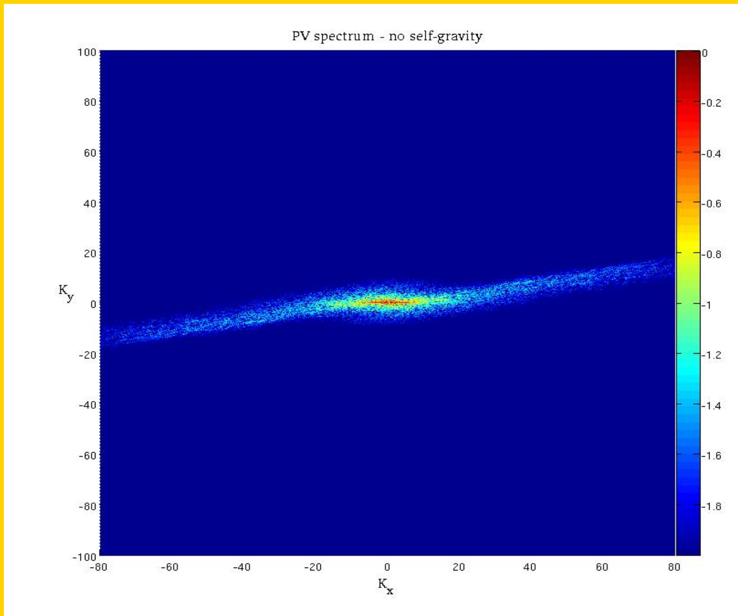
Contrast with non-self-gravitating case – autocorrelation functions

$$R_I(x,y) = \frac{\Sigma_0^2}{\Omega^2 L_x L_y} \int \delta I(x',y') \delta I(x+x',y+y') dx' dy',$$

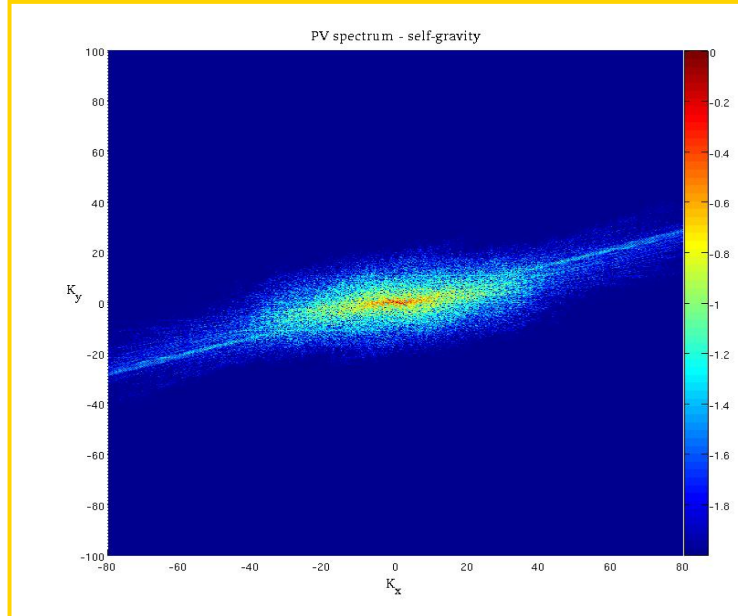


Non-self-gravitating **Self-gravitating**

Contrast with non-self-gravitating case – potential vorticity spectra



Non-self-gravitating



Self-gravitating

- PV (turbulent) spectra in both cases are strongly anisotropic due to the main Keplerian shear flow
- Spectrum in the self-gravitating case is broader than that in non-self-gravitating case – self-gravity opposes inverse cascade of power towards larger scales

Conclusions

- Vortices are dynamically as important as density waves in self-gravitating discs.
- Vortices couple with density waves due to Keplerian shear
- Self-gravity prevents the development of long-lived coherent vortices, they instead are short-lived and transient structures (if there is no baroclinic/RWI driving).
- Self-gravity opposes the inverse cascade energy to larger scales

Possible future developments

- Dynamics of vortices in 3D stratified self-gravitating discs
- Effects of self-gravity on the development of baroclinic and RWI
- Possibility of trapping particles by self-gravitating vortices

Thank you