Vortices, Poincare Waves, and Critical Layers in Protoplanetary Disks

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I have only *15 minutes* to convince you that the Dead Zone is ***Not*** Dead, but fills spontaneously with large-amplitude waves and vortices

You already know why filling the zone with hydrodynamic features that can radially transport angular momentum and energy is important to complete star formation and make planets

Equations of an annular section of a PPD

are similar to those of

rotating, stratified, shearing Couette flow

PPD (Cartesian)

Tidal stretching term

Continuity Equation, and

$$\frac{d\boldsymbol{v}}{dt} = -2\Omega_0\hat{\boldsymbol{z}} \times \boldsymbol{v} + 3\Omega_0^2 x\hat{\boldsymbol{x}} - \frac{1}{\bar{\rho}}\nabla\tilde{p} - \frac{\tilde{\rho}}{\bar{\rho}}\Omega_0^2 z\hat{\boldsymbol{z}},$$

$$\frac{dT}{dt} = -(\gamma - 1)T(\nabla \cdot \boldsymbol{v})$$

PPD (Cartesian)

Advective derivative

Continuity Equation, and

$$\frac{d\boldsymbol{v}}{dt} = -2\Omega_0 \hat{\boldsymbol{z}} \times \boldsymbol{v} + 3\Omega_0^2 x \hat{\boldsymbol{x}} - \frac{1}{\bar{\rho}} \nabla \tilde{p} - \frac{\tilde{\rho}}{\bar{\rho}} \Omega_0^2 z \hat{\boldsymbol{z}},$$

$$\frac{dT}{dt} = -(\gamma - 1) T (\nabla \cdot \boldsymbol{v})$$

Coriolis term

Gravity proportional to *z* (vertically isothermal. Gaussian density)

Equations of Lab Boussinesq Couette Flow

Continuity Equation, and

$$\partial \mathbf{v}/\partial t \;=\; -(\mathbf{v}\cdot\nabla)\mathbf{v} - \frac{\nabla P}{\rho_0} + f\,\mathbf{v}\times\hat{\mathbf{z}} - \frac{(\rho-\rho_0)g}{\rho_0}\hat{\mathbf{z}}$$

$$\partial \rho/\partial t \;=\; -(\mathbf{v}\cdot\nabla)\rho$$

Pressure can absorb
the tidal stretching term

Evidence of Death (PPDs are stable)

Linear Stability: Rayleigh (angular momentum increases with radius), but this only applies to a constant density fluid. A stratified, rotating fluid could be linearly unstable to a strato-rotational instability (SRI) (Le Dizes, Le Bars & Le Gal *PRL* 2007), or to a mode-coupling instability (Tevzadze et al. *A&A* 2008), or to a rotational instability (Le Dizes, Meunier, Riedinger)

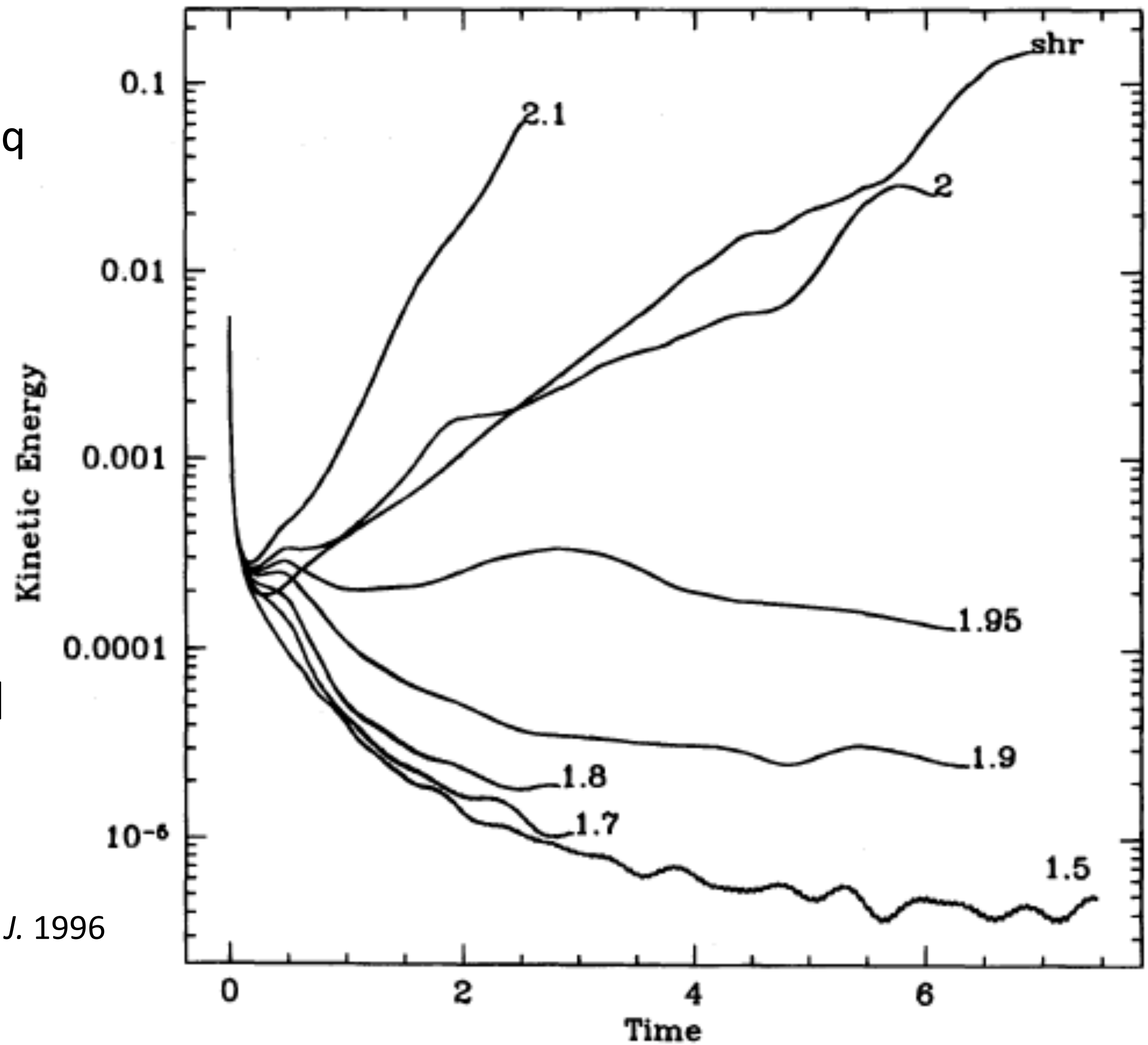
Evidence of Death (PPDs are stable)

Finite Amplitude Stability: A guessing game. Best (most cited) tests were for with a compressible code but initialized with a constant density fluid. A wide-spread belief that if a constant density shear flow is stable than the same shear flow with a stably stratified density is more stable.

$\Omega(r) \sim r^{-q}$

Keplerian is q=1.5

Rayleigh neutral to centrifugal instability Is q=2



Balbus et al. *Ap. J.* 1996

Evidence of Death (PPDs are stable)

Finite Amplitude Stability: Many other simulations of PPDs with ideal gases that are initially stratified and have initial perturbations do not show finite amplitude instability.

However, their computer codes may be highly dissipative at small scales.

Computational Method

- 3D Spectral Method
  - Horizontal basis functions: Fourier-Fourier or Chebyshev-Fourier basis
  - Vertical basis functions: Chebyshev polynomials or mapped Ferziger polynomials, or Fourier, and in finite infinite domain
- Equivalent to 7168 X 7168 X 7168 2nd -order F.D.
- Only report results when shearing sheet code agree with those of the "periodicized" radial code with sponge layers at the radial boundaries.
- Domain at least 16 H_o^3
- Nearly dissipationless at the small scales of a critical layer, unlike traditional grid codes

Evidence of Life (PPDs are unstable)

Finite Amplitude Instability:

(1)Spontaneous formation of off-midplane vortices starting with one midplane vortex

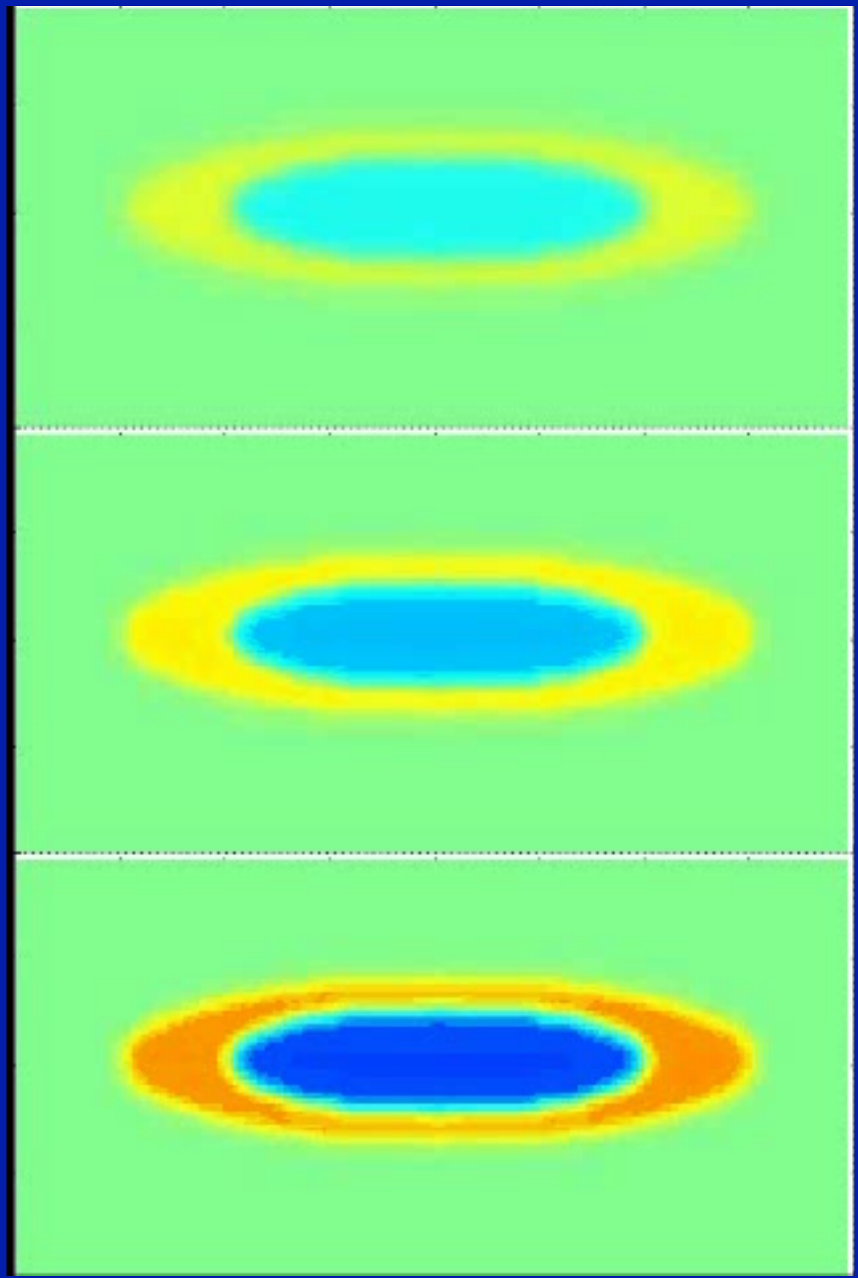
(2) Spontaneous formation of off-midplane vortices starting with one off-midplane vortex

(3) Spontaneous formation of vortex sheets rolling up into vortices starting with one vortex

$z = 1.5\ H_0$

$z = 1.0\ H_0$

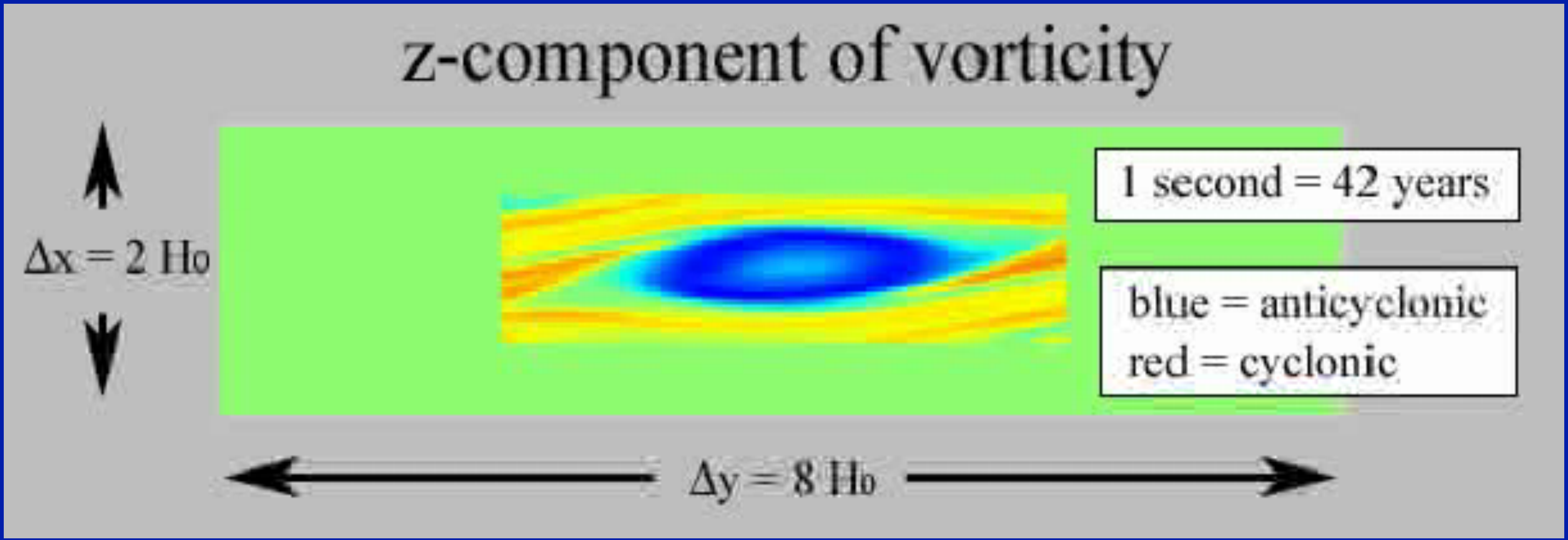
$z = 0\ H_0$



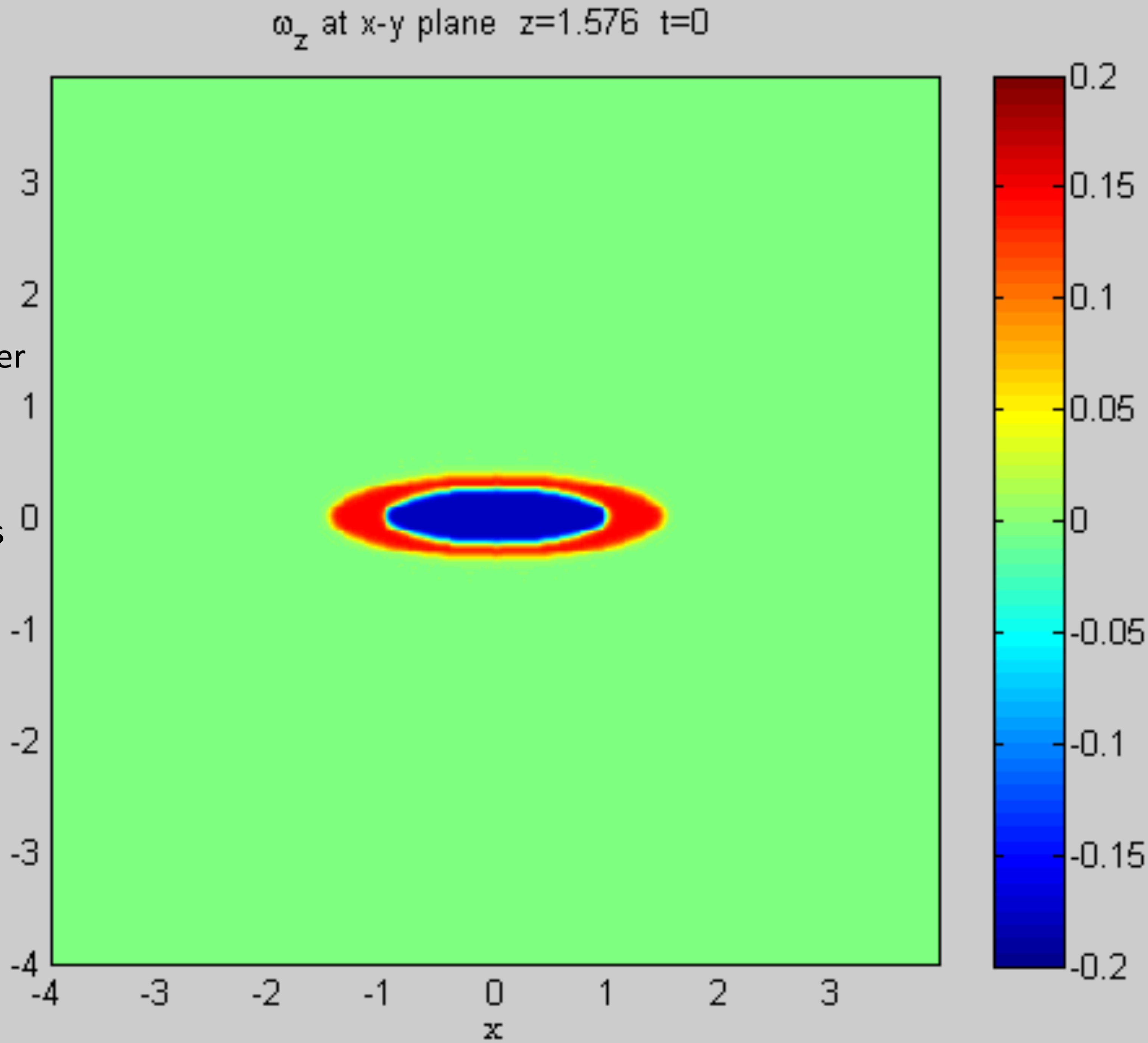
Barranco & Marcus *Ap. J.* 2005

Barranco & Marcus *JCP* 2006

Spontaneous Formation of Off-Midplane Vortices



Marcus
et al.
will be
submitted
To *PRL* after
this talk
if YOU
have no
objections



Traditional Critical Layers

In unidirectional, dissipationless shear flows with $V_y = \overline{V}(x)$, at location x_c where a neutrally stable eigenmode's phase velocity = $\overline{V}(x_c)$, there is a traditional *critical layer.*

The eigenmode's velocity in the *y* direction is singular, but this eigenmode is not important because it is difficult to excite. (Kelvin's cats-eyes?)

New Critical Layers

In a stably vertically stratified, unidirectional, dissipationless shear flows with $V_y = \overline{\mathrm{V}}(x)$, & with a stable vertically stratified density with Brunt- Vaisala frequency *N*, there is a new critical layer where the neutrally stable eigenmode's velocity in the vertical *z* direction is singular, and this eigenmode is important because it is very easy to excite.

New Critical Layers

$$N(z) \equiv \sqrt{-g(1/\bar{\rho})(d\bar{\rho}/dz)}$$

$$e^{i(k_y y - s\, t)}$$

$$\bar{V}(x^*) = (s \pm N)/k_y$$

$$\bar{V}(x) \equiv \sigma x$$

New Critical Layers

$$k_y \equiv 2\pi m/L$$

$$x^* = (s \pm N)/(\sigma k_y) = L(s \pm N)/(2\pi\sigma m)$$

unit of length = $(LN)/(2\pi\sigma)$ unit of time = $1/N$

$$x^* = -(s \pm 1)/m$$

When N is linear in z, the critical layers are diagonal lines

New Critical Layers Create Large Vortex Layers

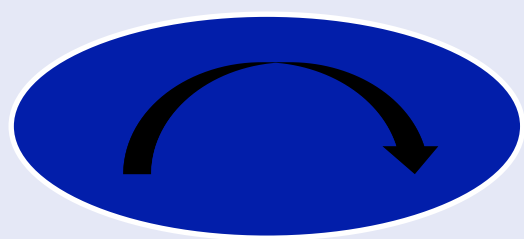
$$\partial \mathbf{v}/\partial t \; = \; -(\mathbf{v}\cdot\nabla)\mathbf{v} - \frac{\nabla P}{\rho_0} + f\mathbf{v}\times\hat{\mathbf{z}} - \frac{(\rho-\rho_0)g}{\rho_0}\hat{\mathbf{z}}$$

$$\boldsymbol{\omega} \equiv \nabla \times \left(\mathbf{v} - \bar{V}(x)\,\hat{\mathbf{y}}\right)$$

$$\begin{aligned}\partial \omega_z/\partial t \; &= \; -(\mathbf{v}\cdot\nabla)[\omega_z + (\partial\bar{V}/\partial x)] \\ &+ \; (\boldsymbol{\omega}\cdot\nabla)v_z + [f + (\partial\bar{V}/\partial x)](\partial v_z/\partial z),\end{aligned}$$

Very Stable

2D Vortices
in unidirectional flow
with shear σ (PSM, *Nature* 1988, *JFM* 1989, *Annual Rev Fluids* 1993)



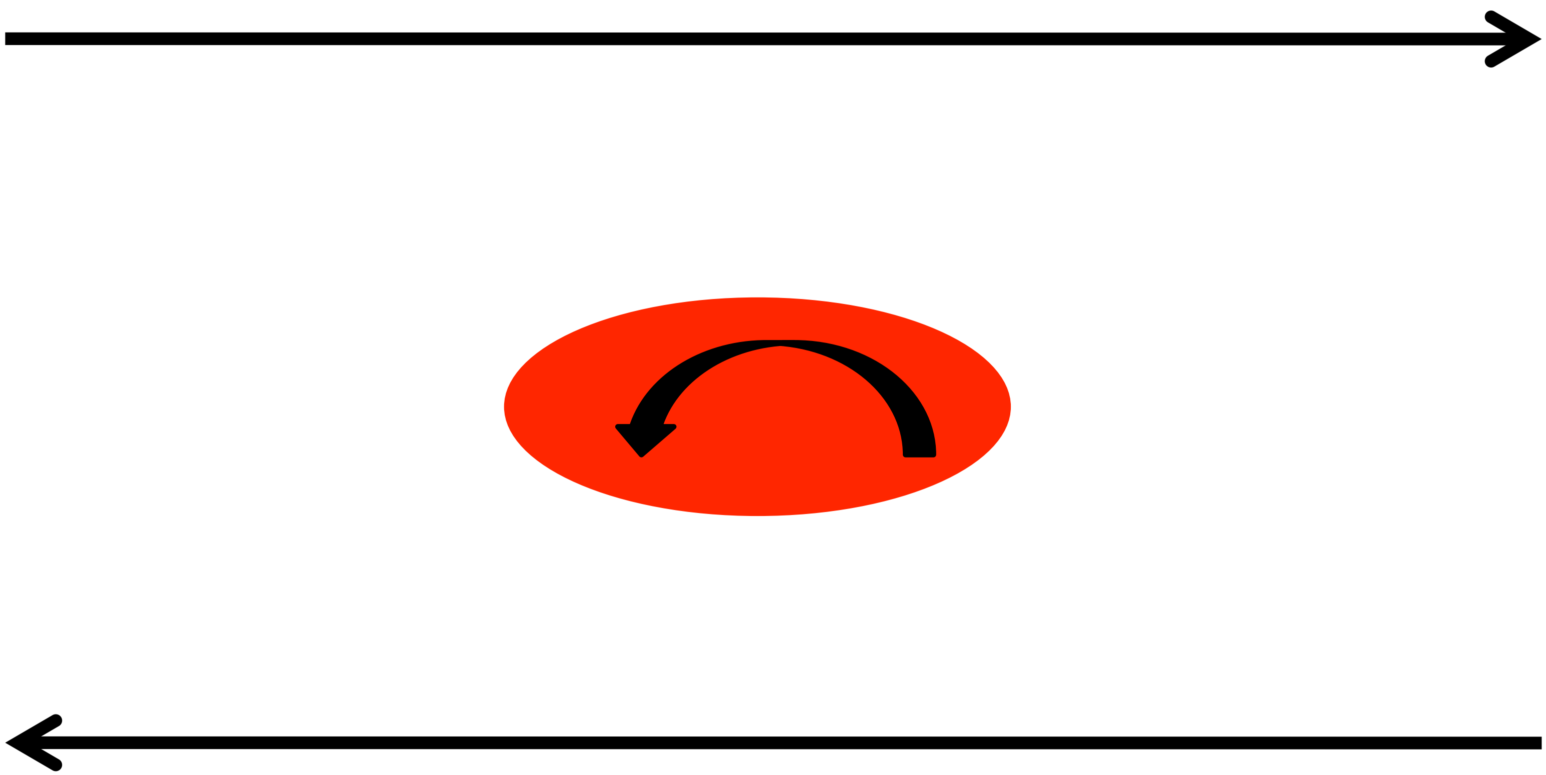
Shape is function
of σ/ω

(Moore & Saffman,
Kida)

$$\overline{V}_y = \sigma x$$

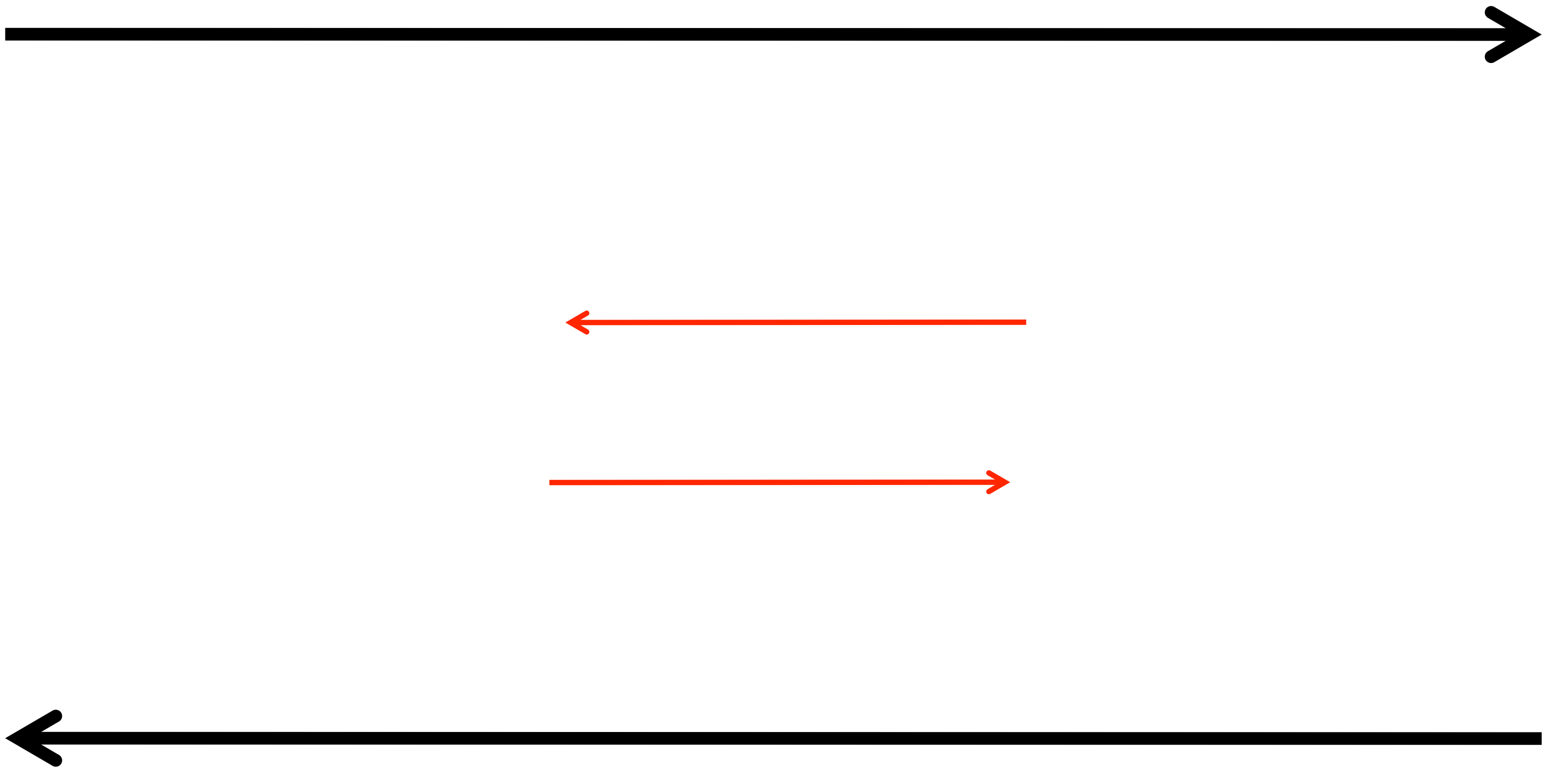
Generally, No Equilibrium

Moore & Saffman

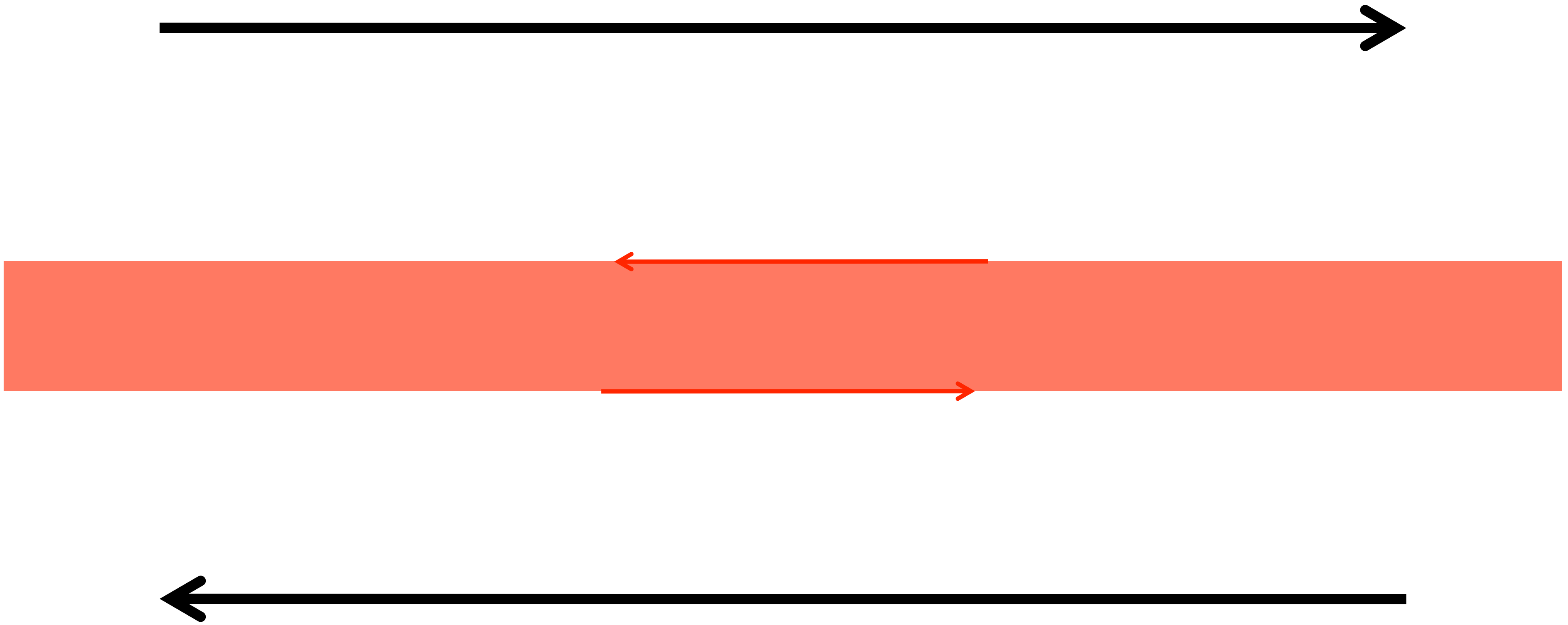


$$\overline{V}_y = \sigma x$$

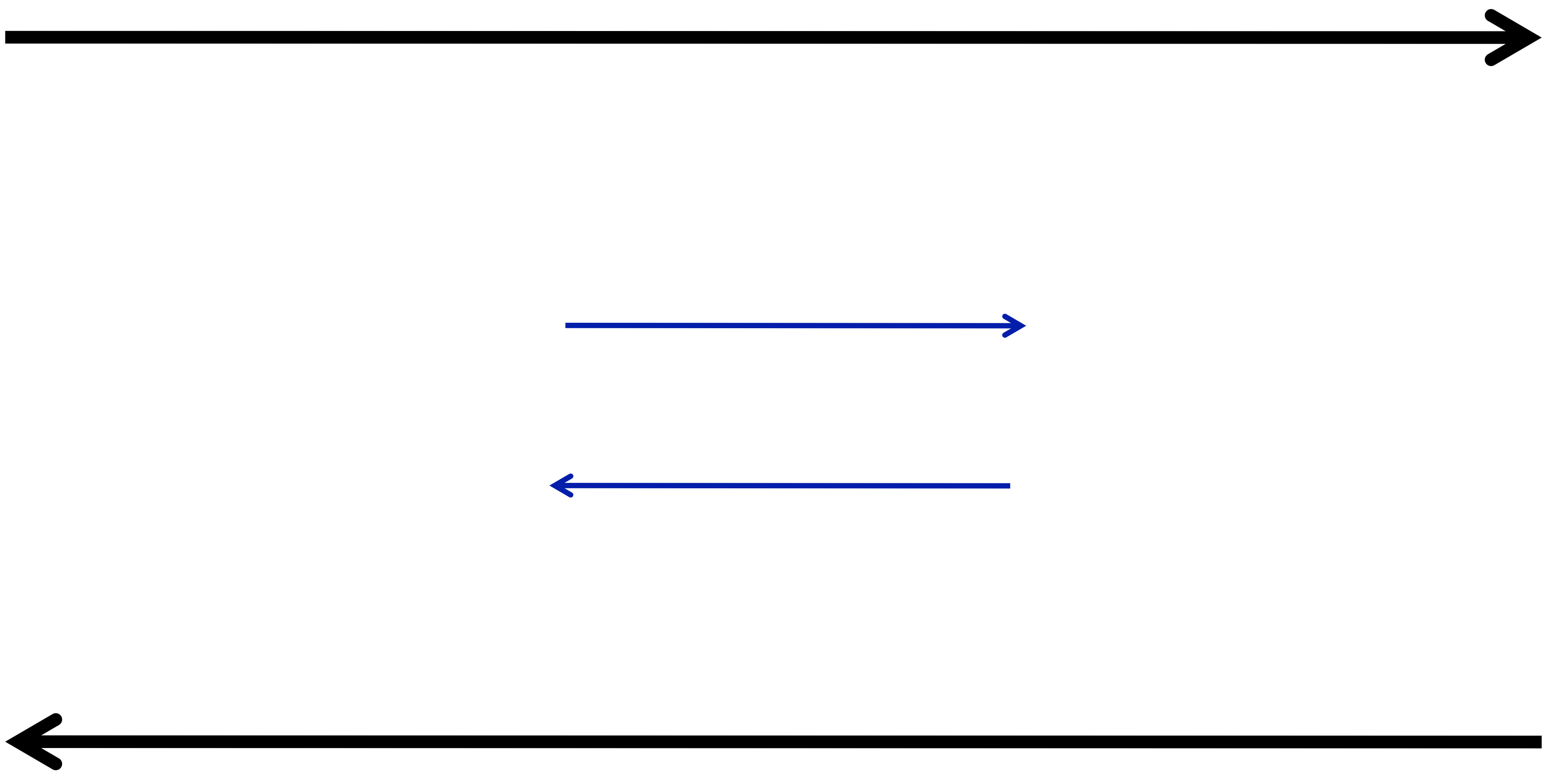
2D QuasiGeostrophic Linear Stability of Shear Layers



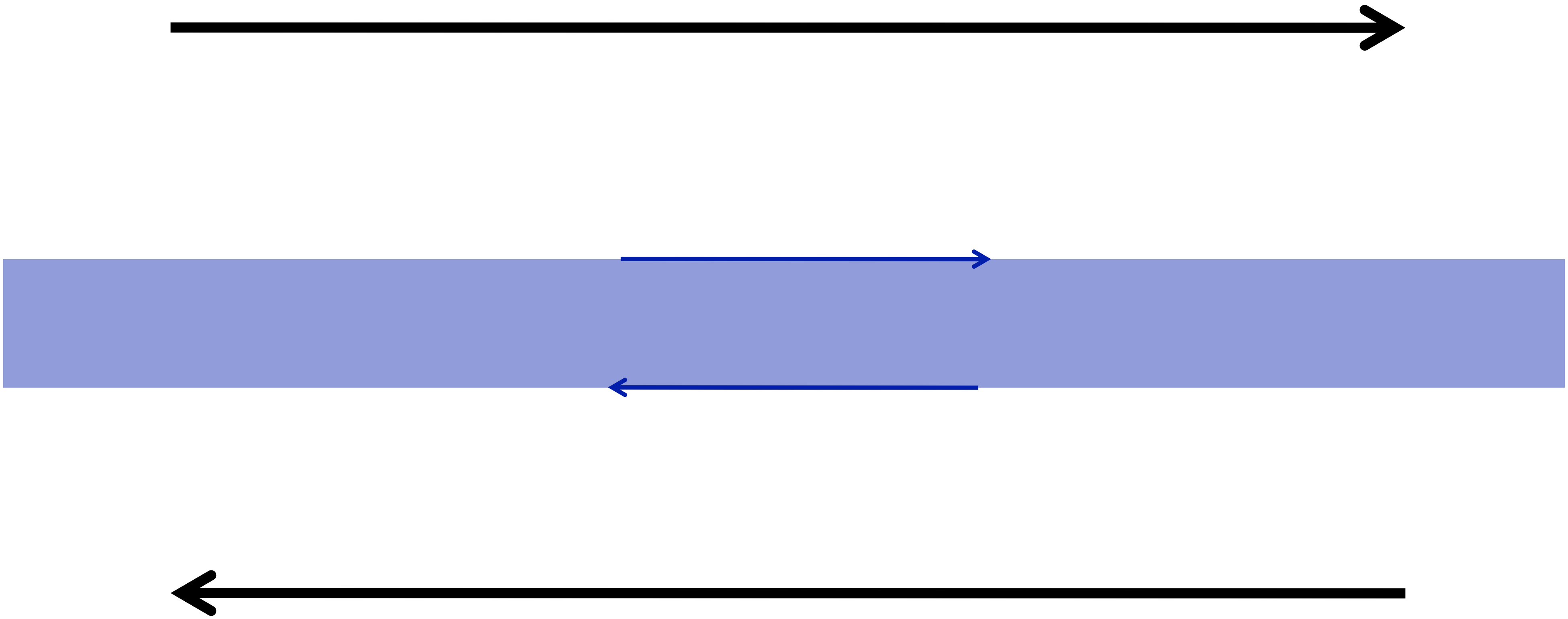
Stable



2D QuasiGeostrophic Linear Stability of Shear Layers



UNSTABLE: Waves on the
boundaries, breaks into separate lumps of vorticity, roll up
into one large vortex



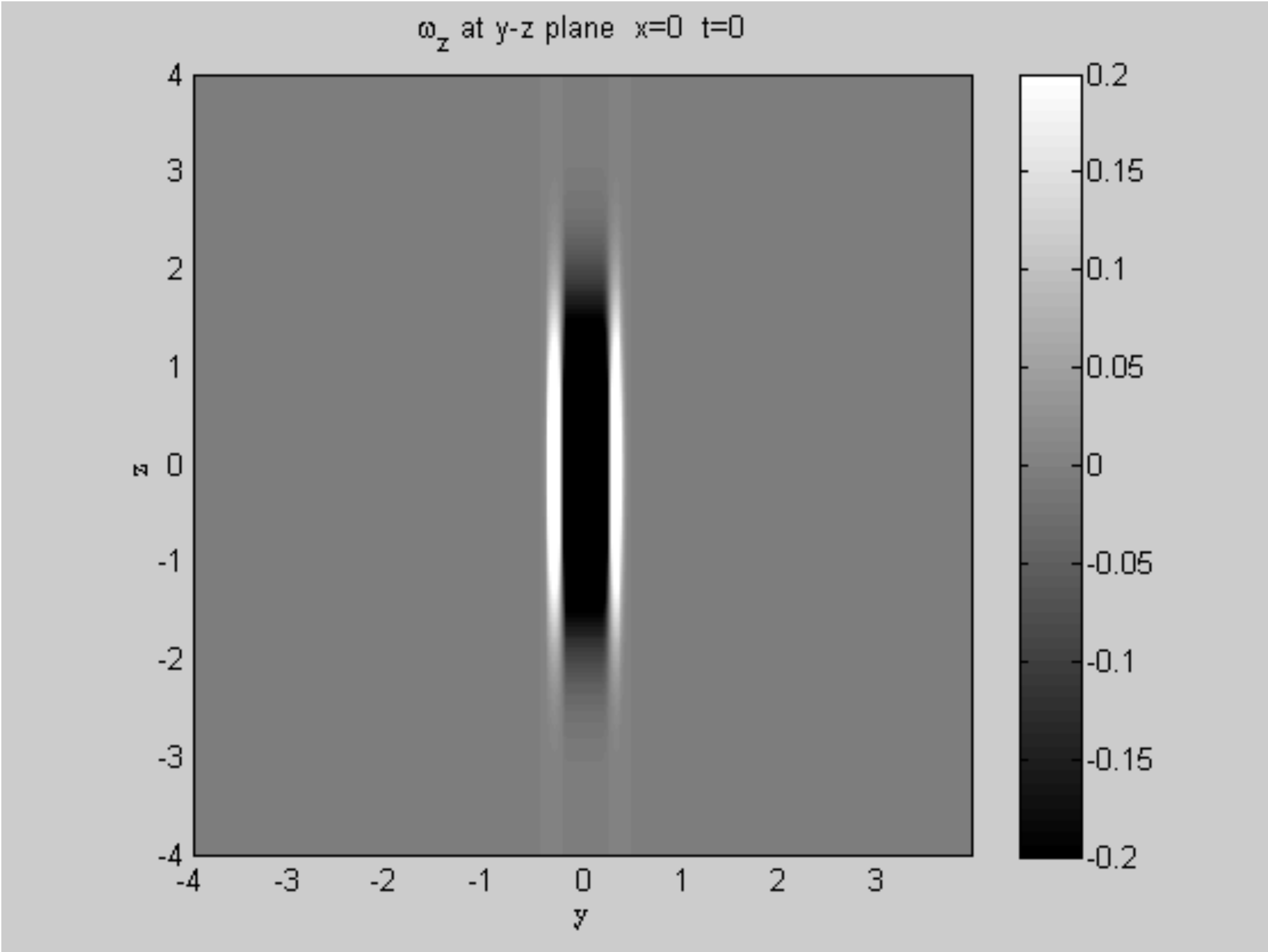
New Critical Layers

$$k_y \equiv 2\pi m/L$$

$$x^* = (s \pm N)/(\sigma k_y) = L(s \pm N)/(2\pi\sigma m)$$

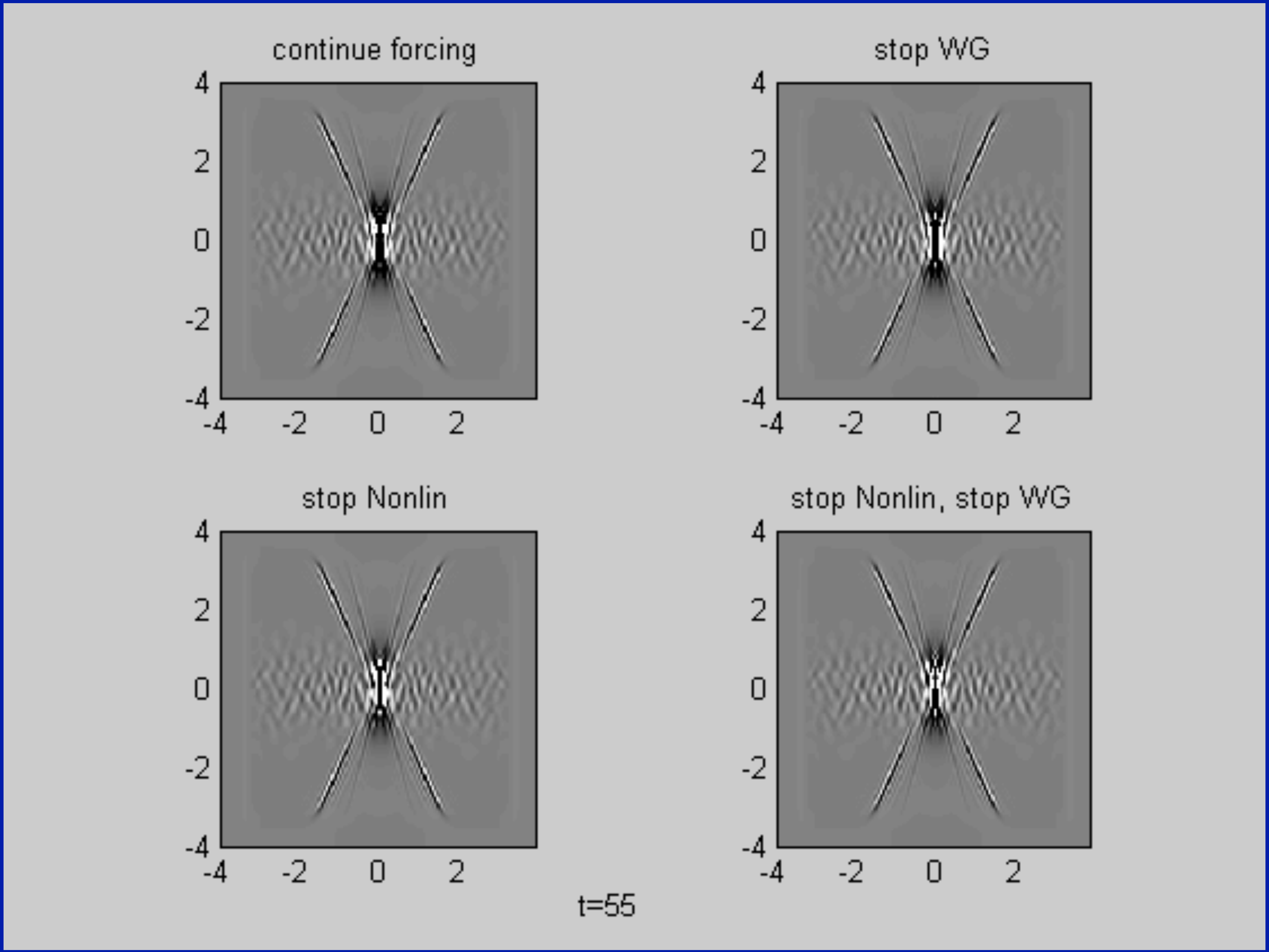
When N is linear in z,
the critical layers are diagonal lines
with slope in the x-z plane
proportional to $1/m$

Generally, $m = 1$ or 2



ωz at y-z plane x=0 t=0
z
y
4
3
2
1
0
-1
-2
-3
-4
-4
-3
-2
-1
0
1
2
3
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2

Critical Layers Draw Energy from Keplerian Rotation



New Critical Layers

$$k_y \equiv 2\pi m/L$$

$$x^* = (s \pm N)/(\sigma k_y) = L(s \pm N)/(2\pi\sigma m)$$

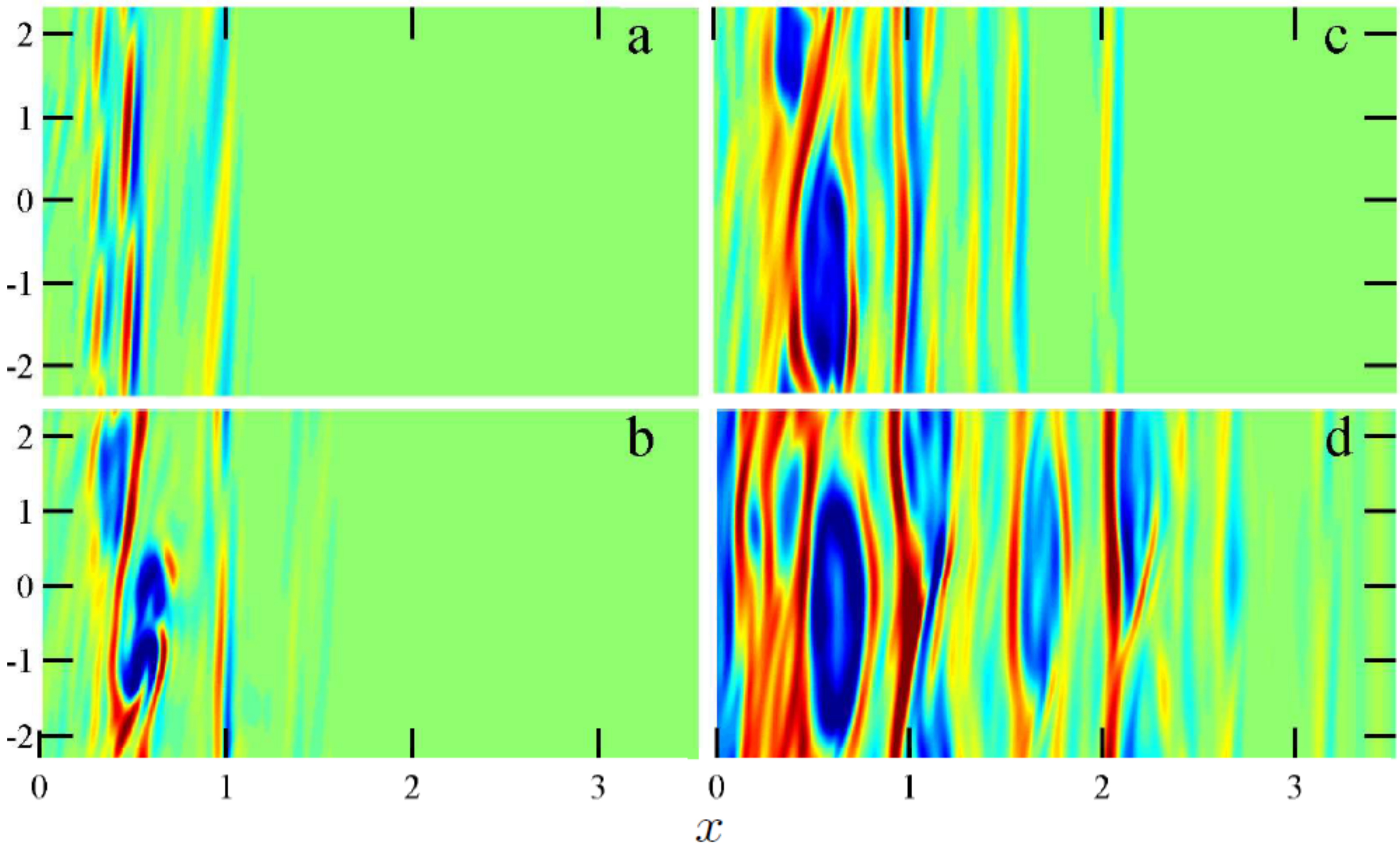
unit of length = $(LN)/(2\pi\sigma)$ unit of time = $1/N$

$$x^* = -(s \pm 1)/m$$

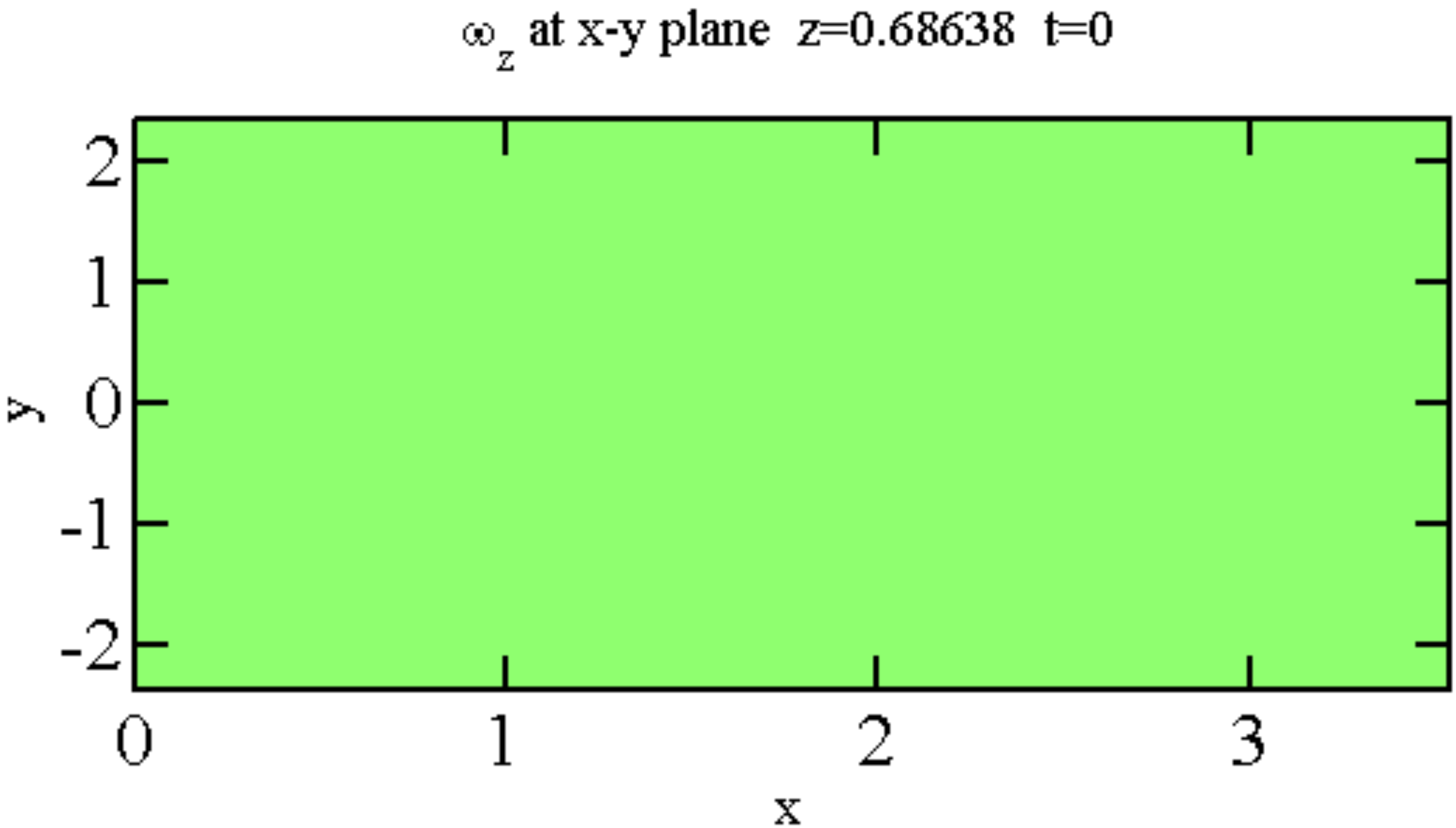
For a perturbation from a steady vortex,

$s = 0$

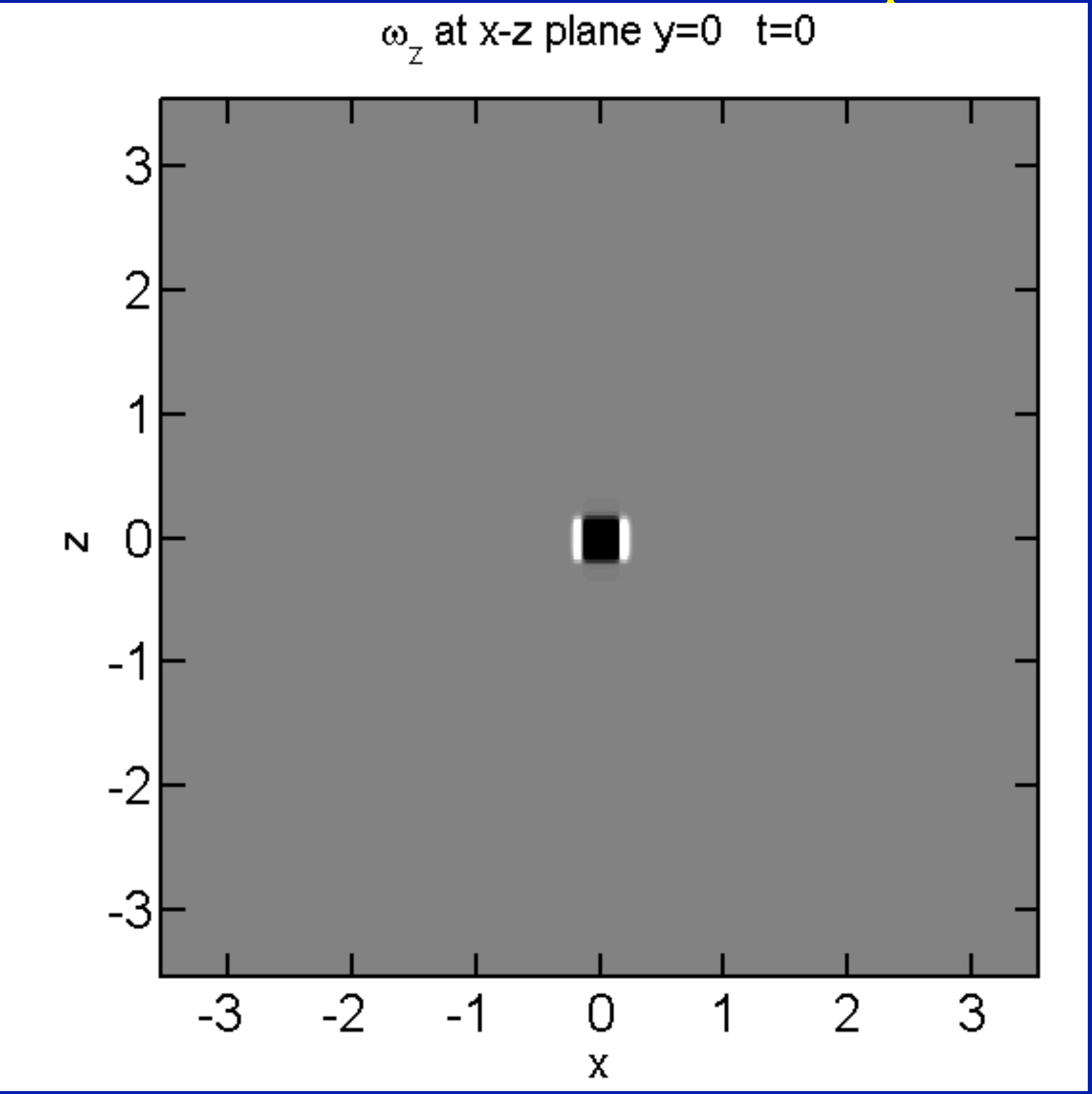
and the largest value of $|x^*|$ is unity

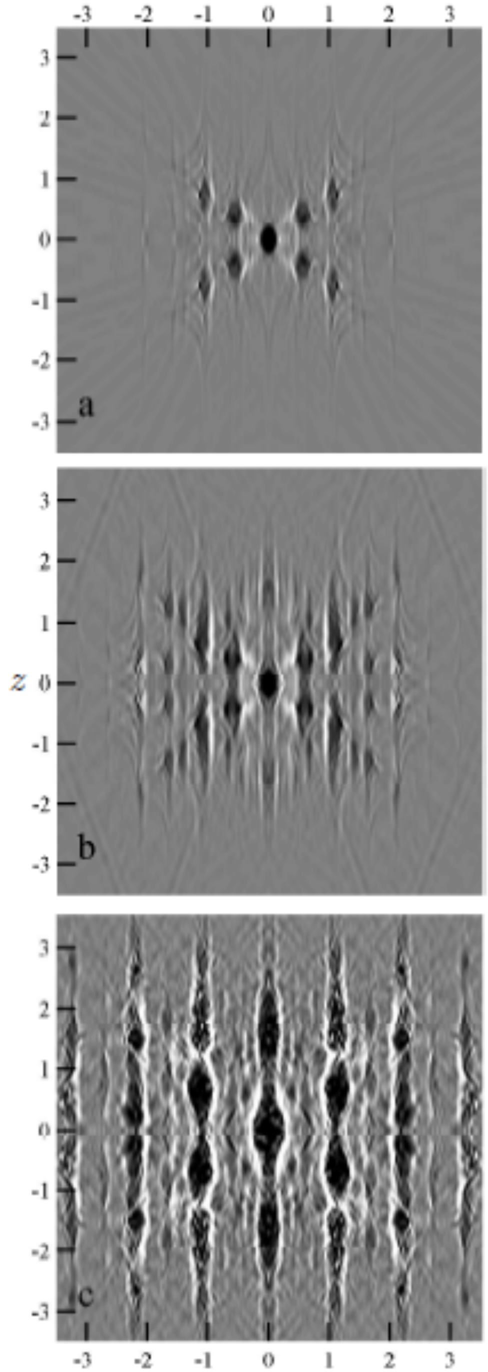

a
b
c
d
2
1
0
-1
-2
0
1
2
3
x

Critical Layers Draw Energy from Keplerian Rotation

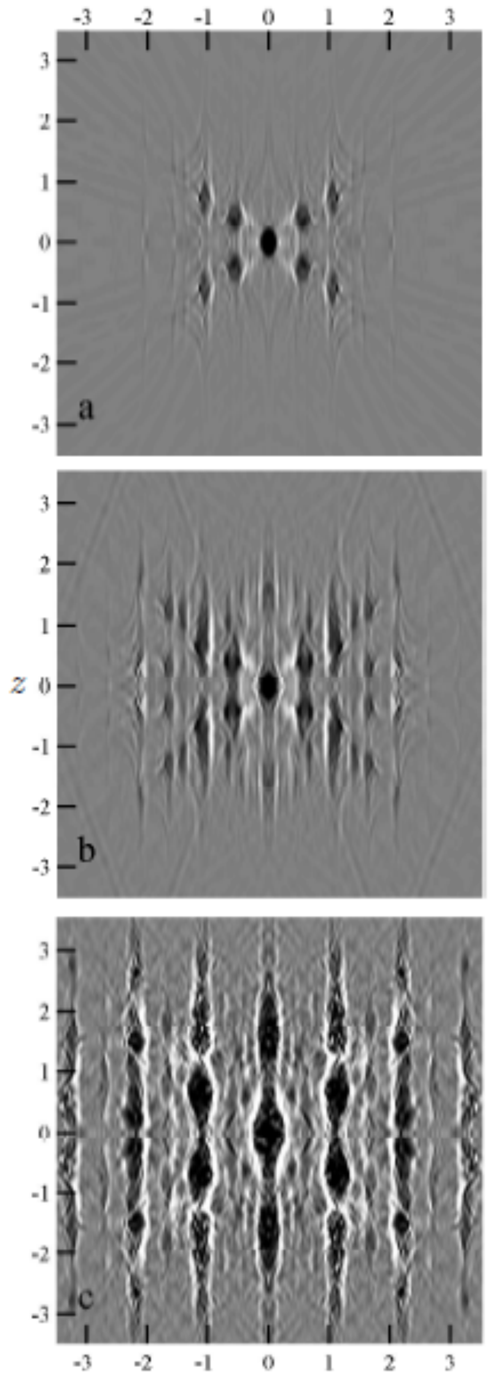


Perturbation of 1 vortex – Boussinesq *N* constant





-3 -2 -1 0 1 2 3
3 2 1 0 -1 -2 -3
a
b
c
z



The vortices in this lattice are stable, and their computed equilibrium solutions, as parameterized by their vertical to horizontal aspect ratio, were confirmed in the lab across the street (Aubert et al. *JFM*, last month) , also consider the GRS

Shielded vortices.

New Critical Layers

$$k_y \equiv 2\pi m/L$$

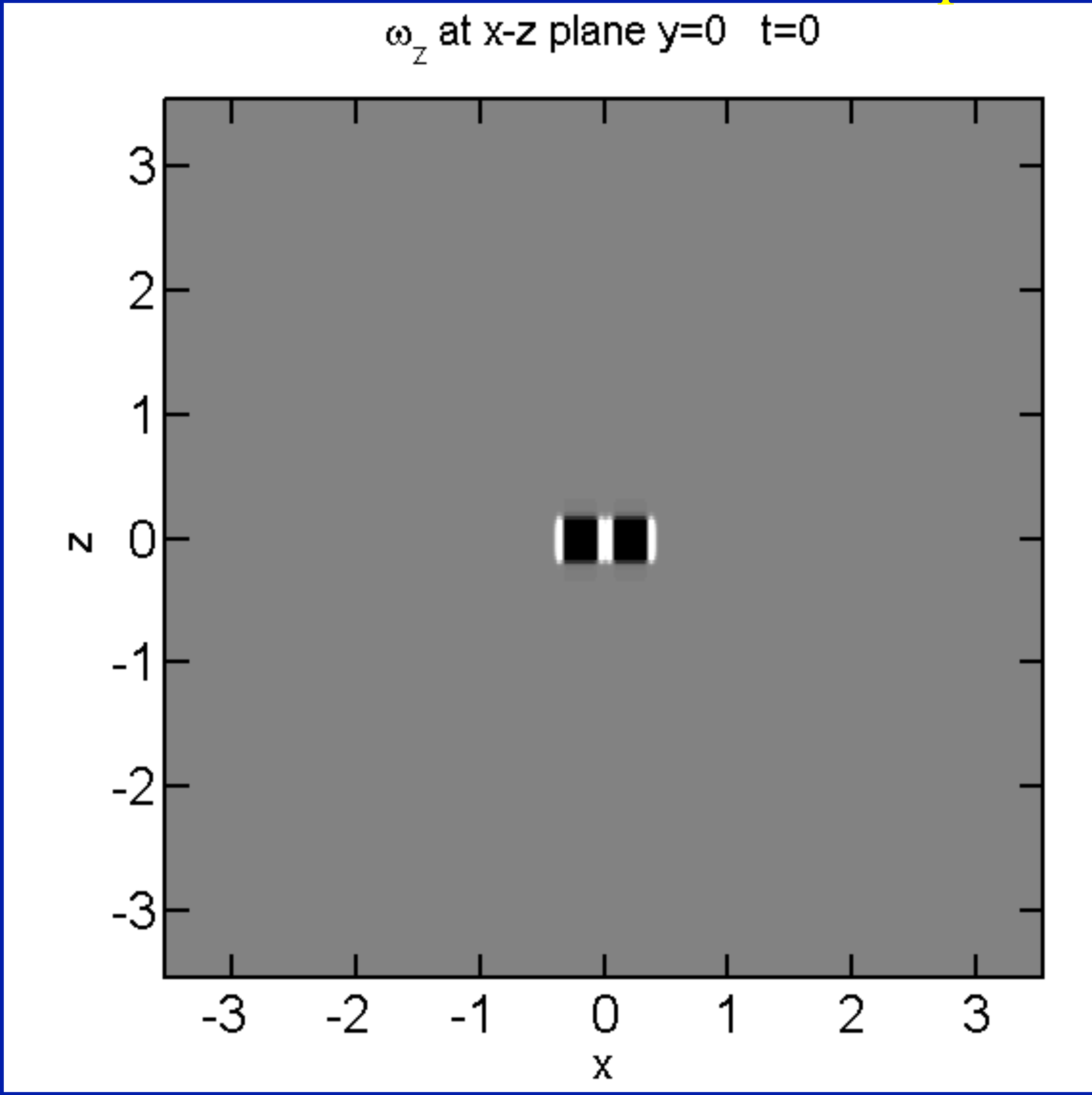
$$x^* = (s \pm N)/(\sigma k_y) = L(s \pm N)/(2\pi\sigma m)$$

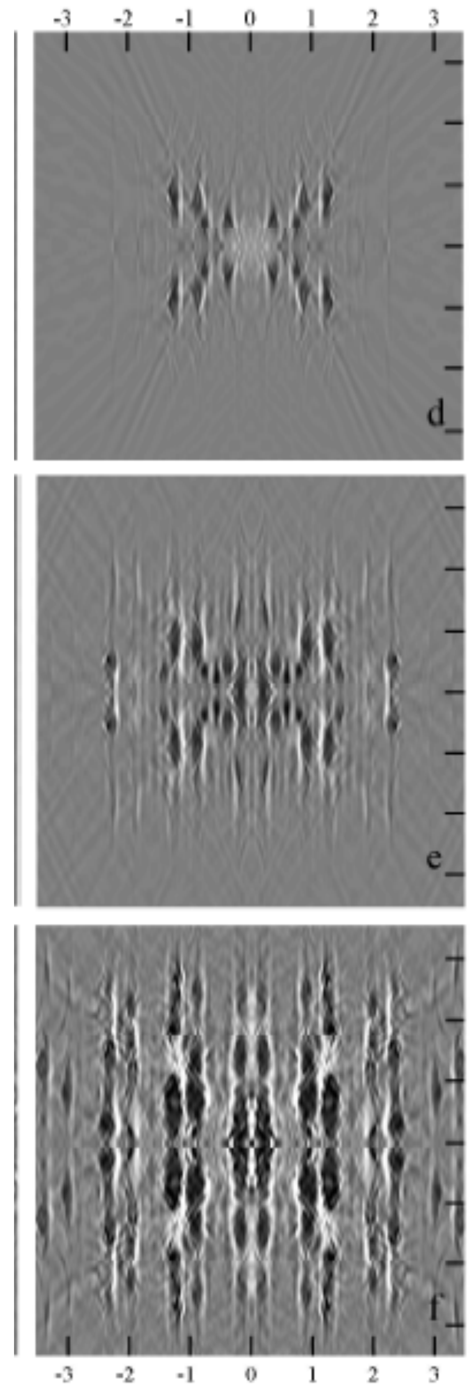
unit of length = $(LN)/(2\pi\sigma)$ unit of time = $1/N$

$$x^* = -(s \pm 1)/m$$

Perturbation from a wave creates two small vortices separated in x by Δ. This creates a perturbation with $s \neq 0$ and two new critical layers for each m that are separated in x by Δ

Perturbation of 2 vortices – Boussinesq *N* constant





-3 -2 -1 0 1 2 3
d
e
f
-3 -2 -1 0 1 2 3

Conclusions

Strong non-axisymmetric Critical Layers in PPDs created from a variety of possible small perturbations

Single vortices, vortex pairs, and create Poincare waves, can excite these new critical layers with large vertical velocities

Large-amplitude vortex layers form (via the Coriolis term), which roll-up into large-amplitude vortices (Rossby numbers of 0.3 – 0.4, so rounder than "bananas")

The process self-replicates, filling the dead zone at all radii with large vortices and waves.

Conclusions

Critical layers, and the vortices and waves they create draw their energies from the Keplerian shear, an example of finite-amplitude instability of the disk.

The same phenomena should occur in Laboratory Experiments stratified, shearing, rotating Couette flows.

Amplitudes of the vortices depend on the slopes of the critical layers and therefore on the stratification as a function of the vertical coordinate

Find the minimum strength disturbance to start the self-replication and find the maximum numerical dissipation allowable that allows a code to reproduce these results.