THE HEAVY-CORE INSTABILITY IN DUST GATHERING PROTOPLANETARY VORTICES

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OUTLINE

- Introduction
- Equilibrium vortices with density gradients
- The Vortical Rayleigh-Taylor Instability
- Solution Methods
- The Heavy Core Instability and Planet Formation
- Issues, Problems, and Future Directions

INTRODUCTION

- By now, this is well known here:
  - Need km sized planetesimals
  - Start with um sized grains, with canonical 0.01 dust/gas ratio
  - ??
- Need gravitational instability in dust layer (?)
  - Requires overdensity of order ~ 20 -100x (when including K-H turbulence)

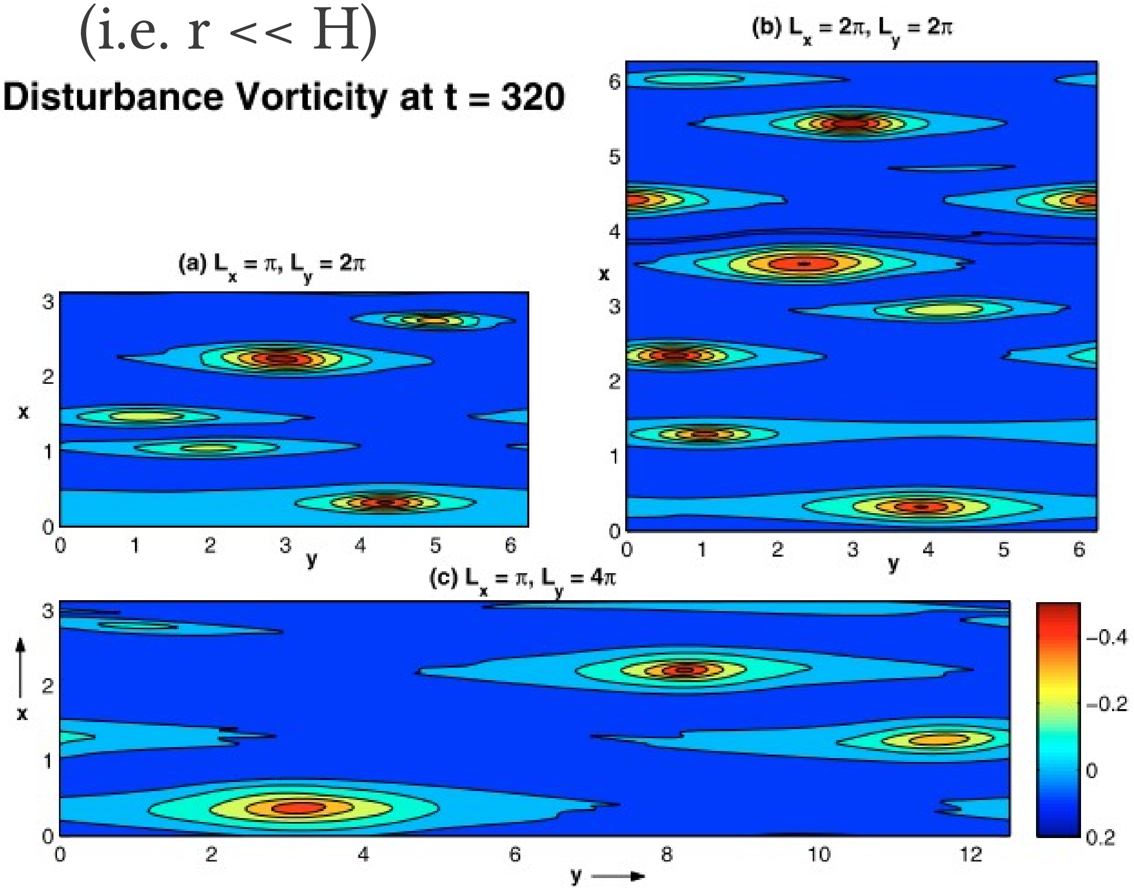
Brief Review

DISK VORTICES

- Are embedded in a shearing background
- Are typically anticyclonic, with high pressure cores
  - These trap dust

IN 2009, WE WERE THINKING OF SMALL SCALE VORTICES

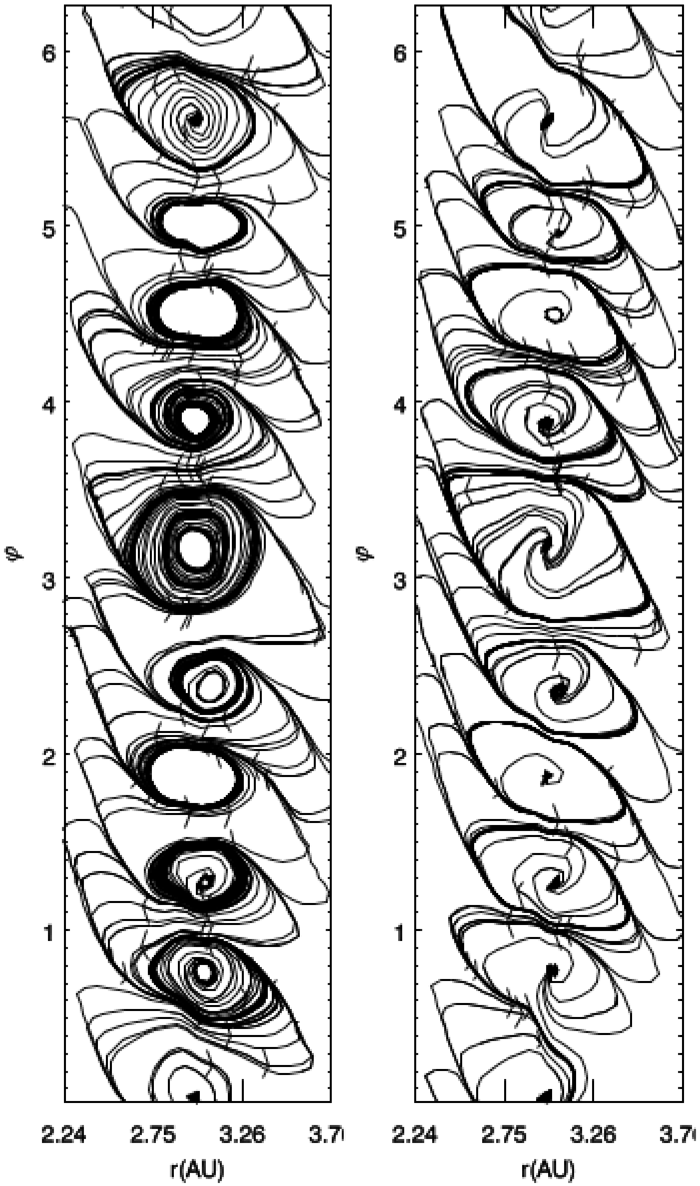
(i.e. $r << H$)



Umurhan & Regev (2005)

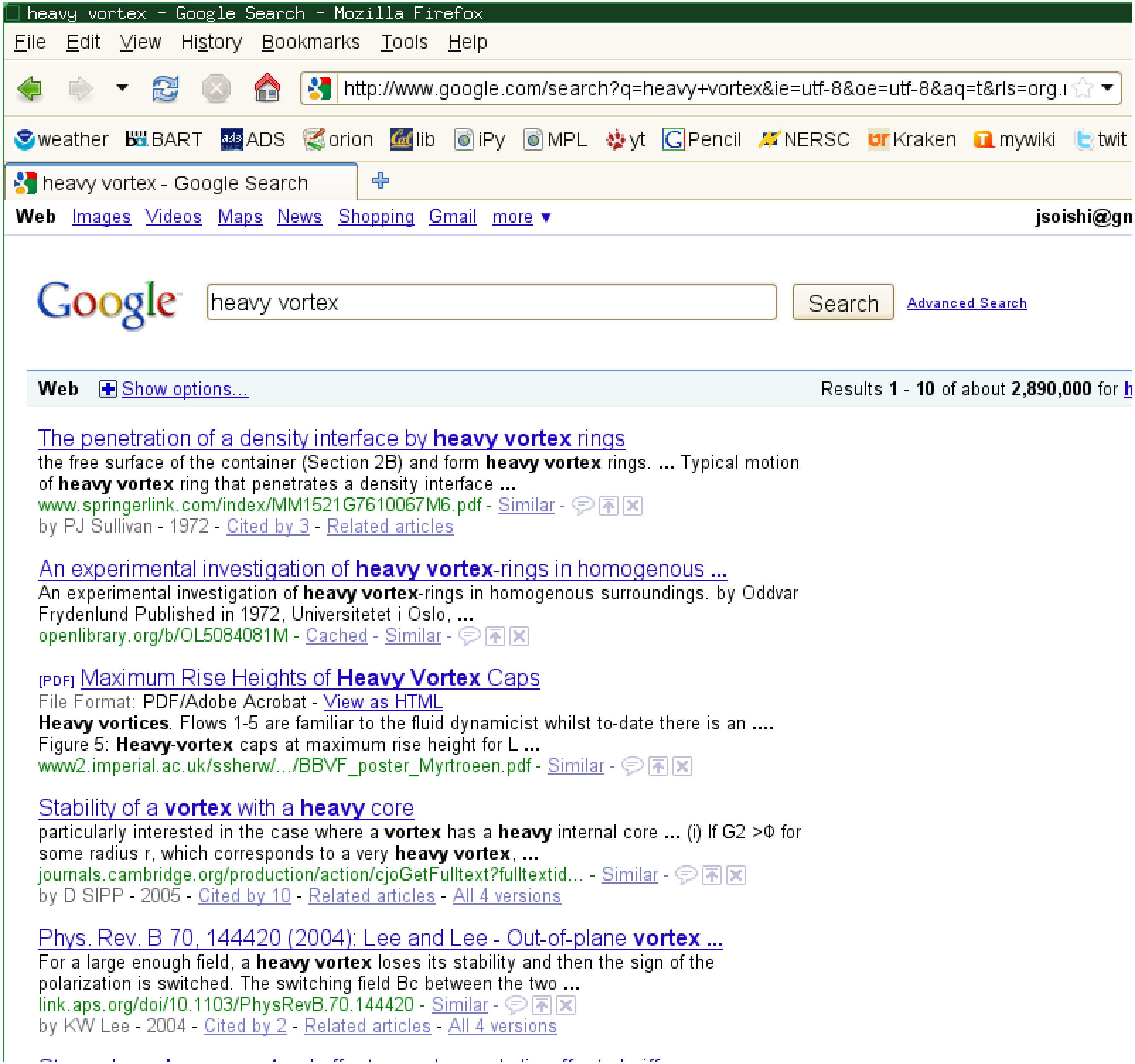
Of course, now we have RWI, SBI, etc too…

SIMULATIONS AGREE: VORTICES TRAP DUST



Meheut et al (2012)

But are dusty vortices stable?



Results **1** - **10** of about **2,890,000** for

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the free surface of the container (Section 2B) and form **heavy vortex** rings. **...** Typical motion of **heavy vortex** ring that penetrates a density interface **...**
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[PDF] Maximum Rise Heights of **Heavy Vortex** Caps
File Format: PDF/Adobe Acrobat - View as HTML
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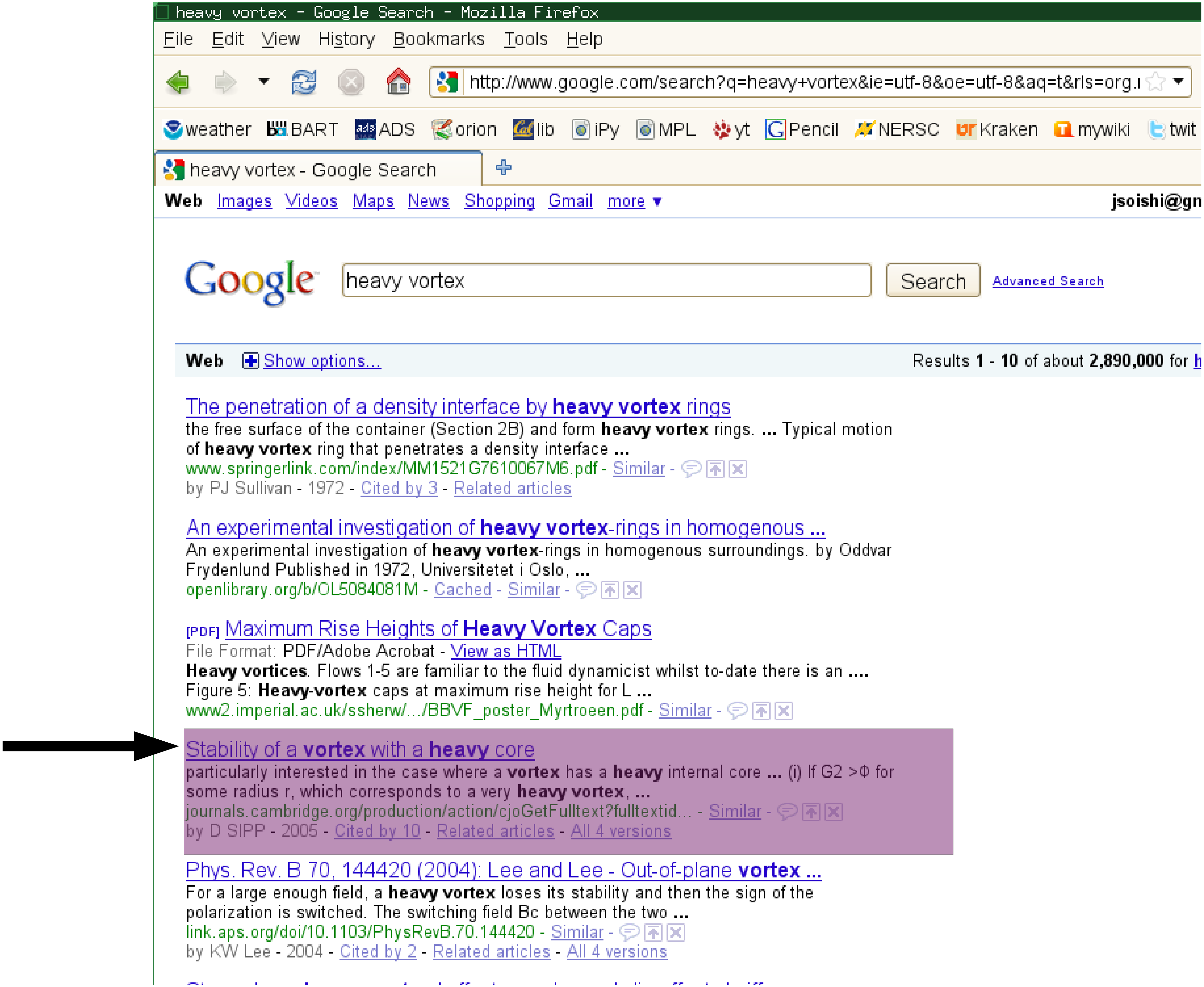

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WE CONSTRUCT A SIMPLE MODEL

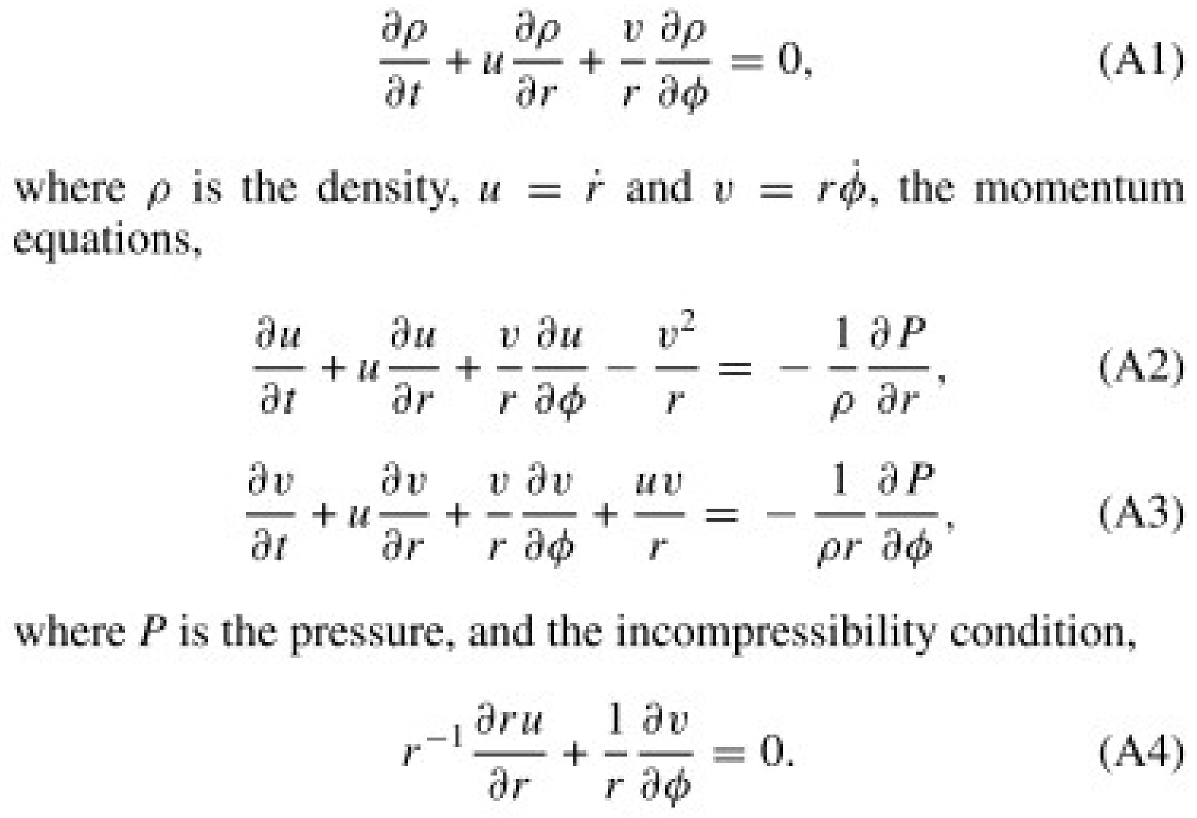
- Incompressible
  - Though with density stratification *within* the vortex
- One-fluid
- Linear
- Based on *simple* equilibrium solutions
  - Kida vortex
  - Goldreich, Narayan, & Goodman (1987) "planet" solution

…AND GET THE HEAVY CORE INSTABILITY

- A 2D linear, parametric instability in elliptic, protoplanetary disk vortices
  - Typically anticyclonic, high pressure cores

…So how does this thing work?

Simple Case: No shear, Gaussian Vortex



$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \frac{v}{r}\frac{\partial \rho}{\partial \phi} = 0, \qquad \text{(A1)}$$

where ρ is the density, $u = \dot{r}$ and $v = r\dot{\phi}$, the momentum equations,

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial \phi} - \frac{v^2}{r} = -\frac{1}{\rho}\frac{\partial P}{\partial r}, \qquad \text{(A2)}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + \frac{v}{r}\frac{\partial v}{\partial \phi} + \frac{uv}{r} = -\frac{1}{\rho r}\frac{\partial P}{\partial \phi}, \qquad \text{(A3)}$$

where P is the pressure, and the incompressibility condition,

$$r^{-1}\frac{\partial r u}{\partial r} + \frac{1}{r}\frac{\partial v}{\partial \phi} = 0. \qquad \text{(A4)}$$

This goes way back to Howard (1973); Gans (1975); Eckhoff (1984), &c...and is nicely summarized in Sipp et al (2005)

Dispersion Relation

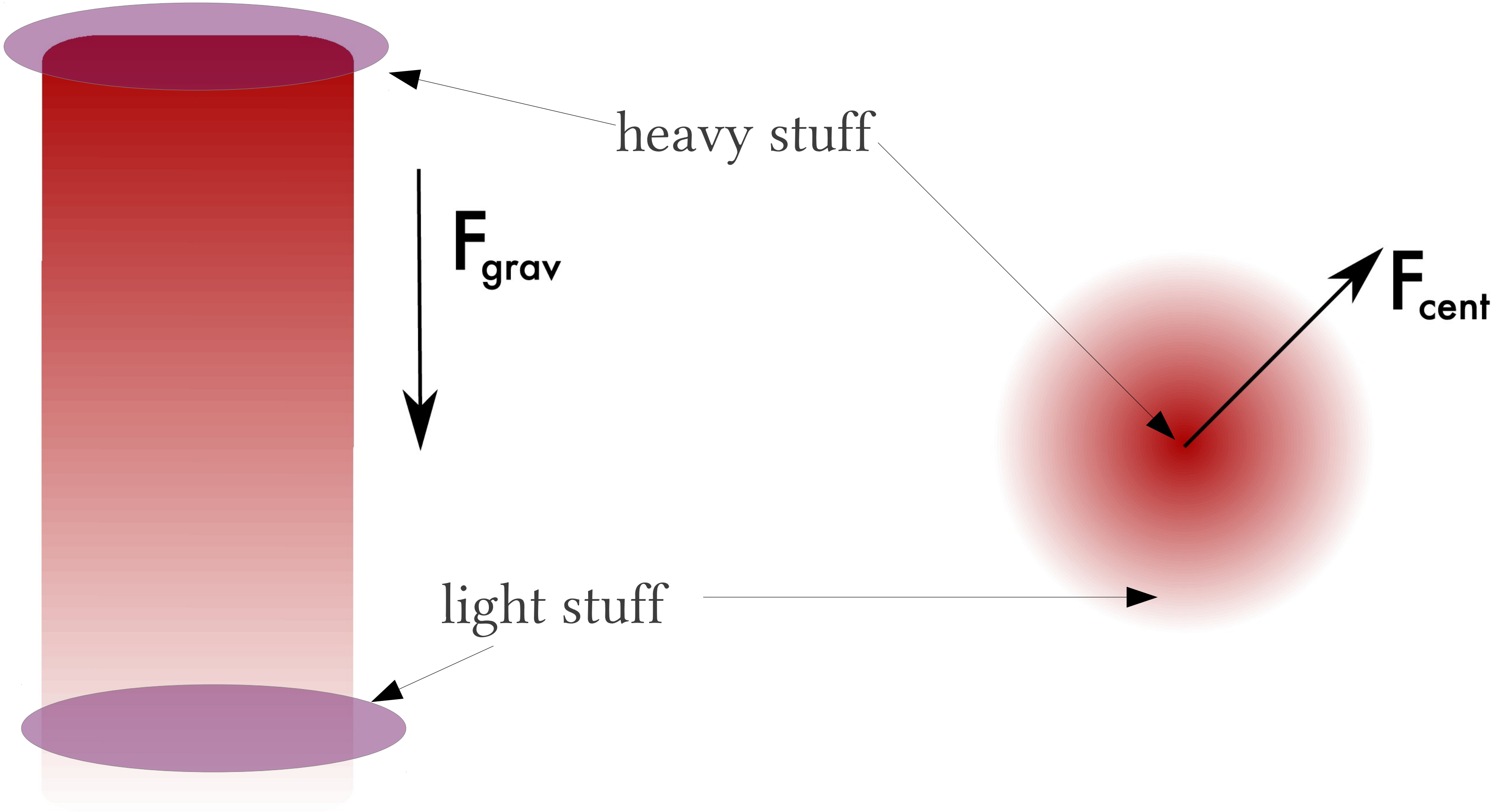
$$\bar{\sigma} \;=\; (\omega - mv/r).$$

$$\bar{\sigma}^2\left(\frac{m^2}{r^2}+k^2\right)+i\bar{\sigma}k\frac{m}{r}\left(\frac{\partial v}{\partial r}-\frac{v}{r}\right)-\frac{m^2}{r^2}\frac{v^2}{r}\frac{\partial \ln\rho}{\partial r}=0. \quad \text{(A10)}$$

$$\bar{\sigma}^2=\frac{m^2/r^2}{m^2/r^2+k^2}\frac{v^2}{r}\frac{\partial \ln\rho}{\partial r}, \qquad \text{(A11)}$$

Unstable if gradient is *negative:*
heavy core

PUT SIMPLY,



heavy stuff

F_{grav}

F_{cent}

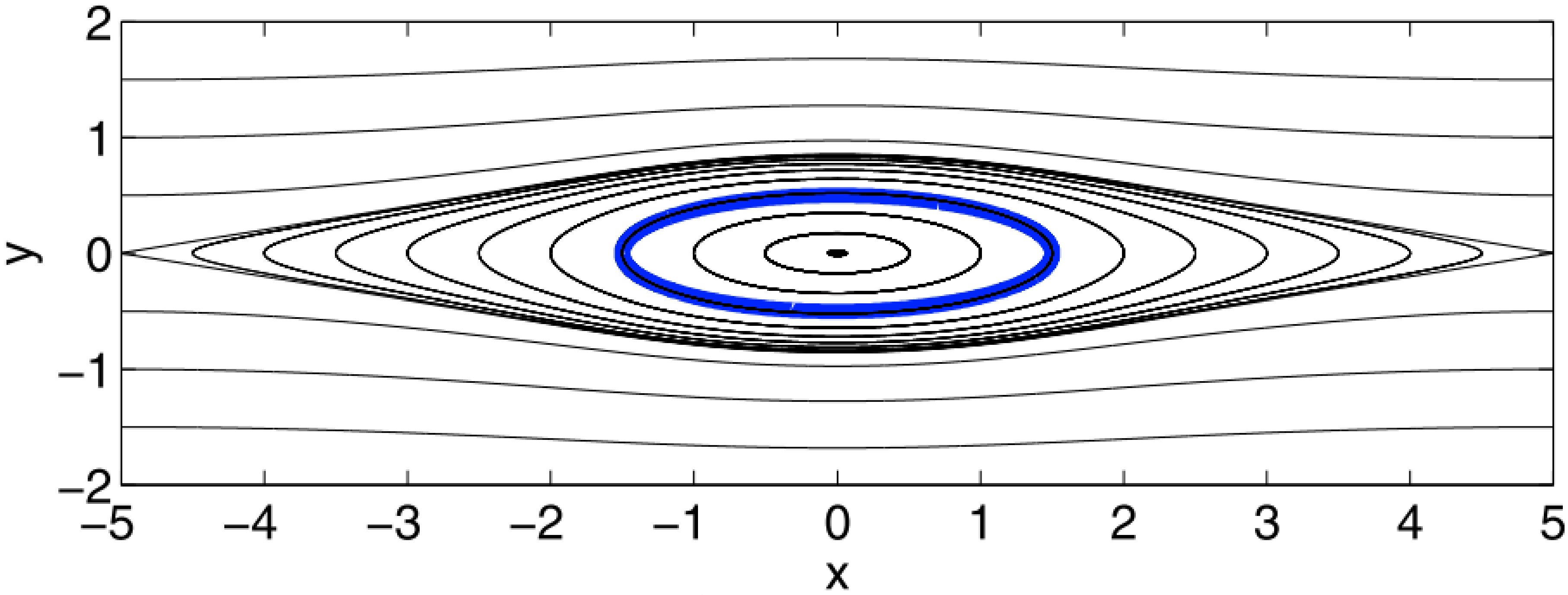
light stuff

Rayleigh-Taylor

Vortical Rayleigh-Taylor

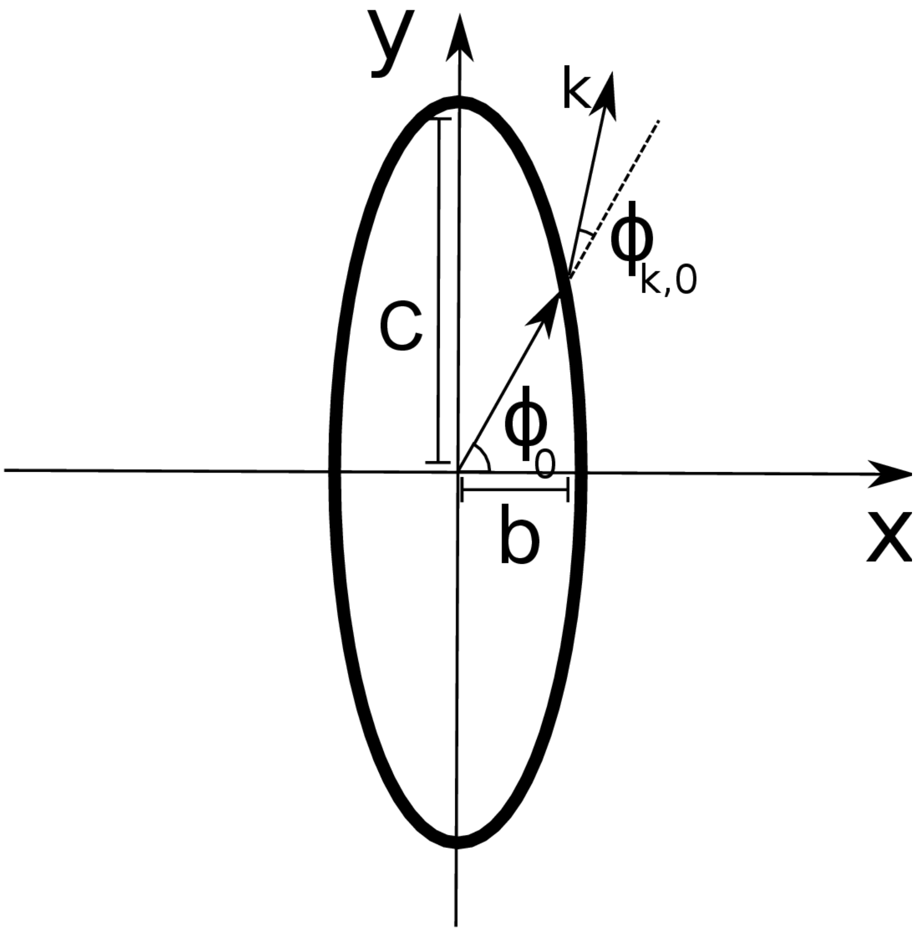
ADD SHEAR

Use either Kida or GNG "Planet" solution, both adjusted to have a *density gradient* specified (balanced by non-trivial pressure distribution)



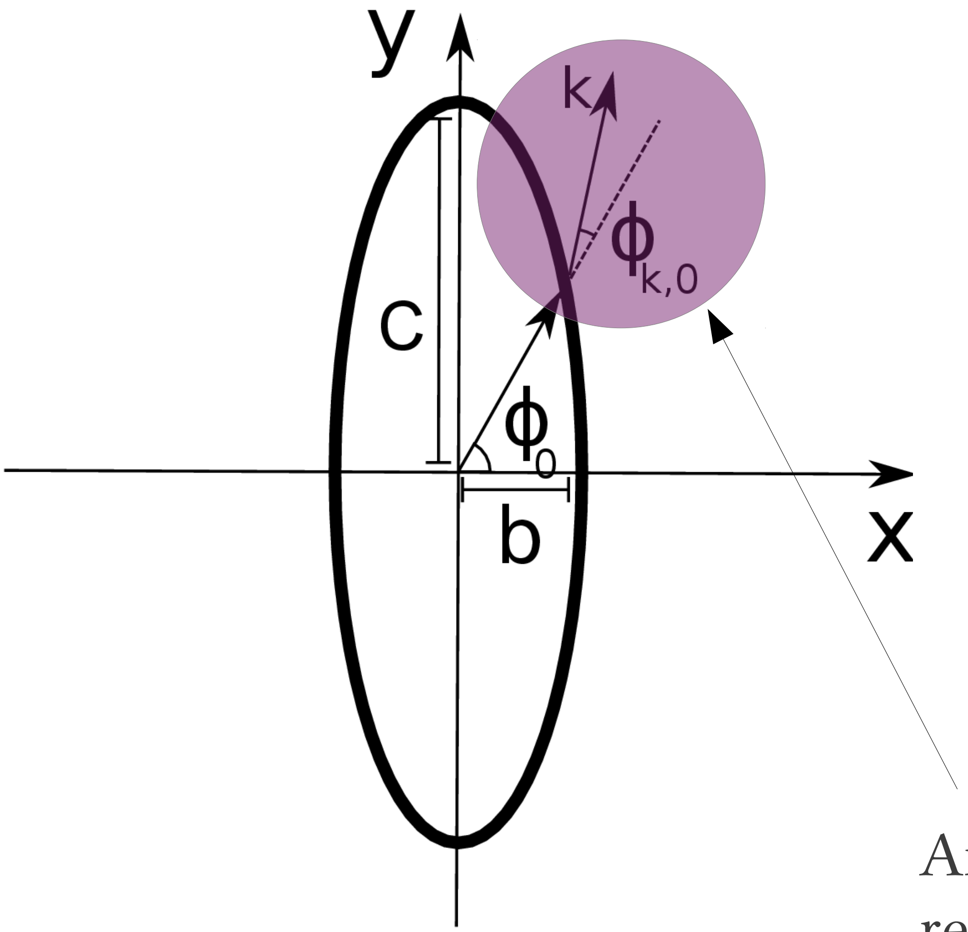
Lesur & Papaloizou (2009)

BASIC GEOMETRY



- Use *elliptical* coordinates: (b, phi)
  - b labels ellipse
  - Phi is azimuthal angle

BASIC GEOMETRY



- Use *elliptical* coordinates: (b, phi)
  - b labels ellipse
  - Phi is azimuthal angle

Angle of wavevector with respect to phase angle

Pressure Distribution in Elliptical Vortices

$$\frac{1}{\rho}\frac{\partial P}{\partial b} = \frac{3b\Omega^2}{4(\chi-1)^2}[(4\chi^2 - 8\chi + 7)\cos^2\phi + 3\chi^2\sin^2\phi - 4\chi(\chi-1)], \qquad (23)$$

$$\frac{1}{b\rho}\frac{\partial P}{\partial \phi} = -\frac{3\Omega^2}{4(\chi-1)^2}(\chi^2 - 8\chi + 7)\sin\phi\cos\phi. \qquad (24)$$

Kida vortex

$$\frac{1}{\rho}\frac{\partial P}{\partial b} = b\Omega^2\left(\frac{3\chi^2}{\chi^2-1} - 2\sqrt{\frac{3\chi^2}{\chi^2-1}}\right), \qquad (19)$$

$$\frac{1}{b\rho}\frac{\partial P}{\partial \phi} = 0. \qquad (20)$$

GNG "planets"

Much simpler!
Axisymmetric!

Use Floquet + WKB

$$\frac{d\delta\rho}{dt} + \delta\boldsymbol{u}\cdot\boldsymbol{\nabla}\rho = 0, \tag{25}$$

$$\boldsymbol{\nabla}\cdot\delta\boldsymbol{u} = 0, \tag{26}$$

$$\frac{d\delta\boldsymbol{u}}{dt} + \delta\boldsymbol{u}\cdot\boldsymbol{\nabla}\boldsymbol{u} + 2\times\delta\boldsymbol{u} = -\frac{1}{\rho}\boldsymbol{\nabla}\delta P + \frac{\delta\rho}{\rho}\frac{1}{\rho}\boldsymbol{\nabla}P, \tag{27}$$

linearize

$$+$$

$$\begin{pmatrix}\delta\boldsymbol{u}\\ \delta P\\ \delta\rho\end{pmatrix} = \exp\left(\frac{i\Phi(\boldsymbol{x},t)}{\epsilon}\right)\left[\begin{pmatrix}\tilde{\boldsymbol{u}}\\ \tilde{P}\\ \tilde{\rho}\end{pmatrix}(\boldsymbol{x},t) + \epsilon\begin{pmatrix}\tilde{\boldsymbol{u}}_\epsilon\\ \tilde{P}_\epsilon\\ \tilde{\rho}_\epsilon\end{pmatrix}(\boldsymbol{x},t)\right], \tag{28}$$

WKB(J)

$$=$$

Actually solve for amplitude a

$$\tilde{u}_x = a(b,t)\sin(\omega t + \phi_{k,0}), \tag{45}$$

$$\tilde{u}_y = a(b,t)\chi\cos(\omega t + \phi_{k,0}), \tag{46}$$

Ux, uy perturbations

End up with
Final Amplitude Equations

$$\begin{aligned}\frac{da}{dt} &= 2\Lambda^{-1}\omega(\chi^2 - 1)\cos(\omega t + \phi_{k,0})\sin(\omega t + \phi_{k,0})a \\ &\quad + \Lambda^{-1}[(\omega^2 + 3\Omega^2)\cos(\omega t + \phi_0)\sin(\omega t + \phi_{k,0}) \\ &\quad - \chi^2\omega^2\sin(\omega t + \phi_0)\cos(\omega t + \phi_{k,0}) \\ &\quad - 2\Omega\omega\chi\sin(\phi_{k,0} - \phi_0)]b\frac{\tilde{\rho}}{\rho}, \qquad (47)\end{aligned}$$

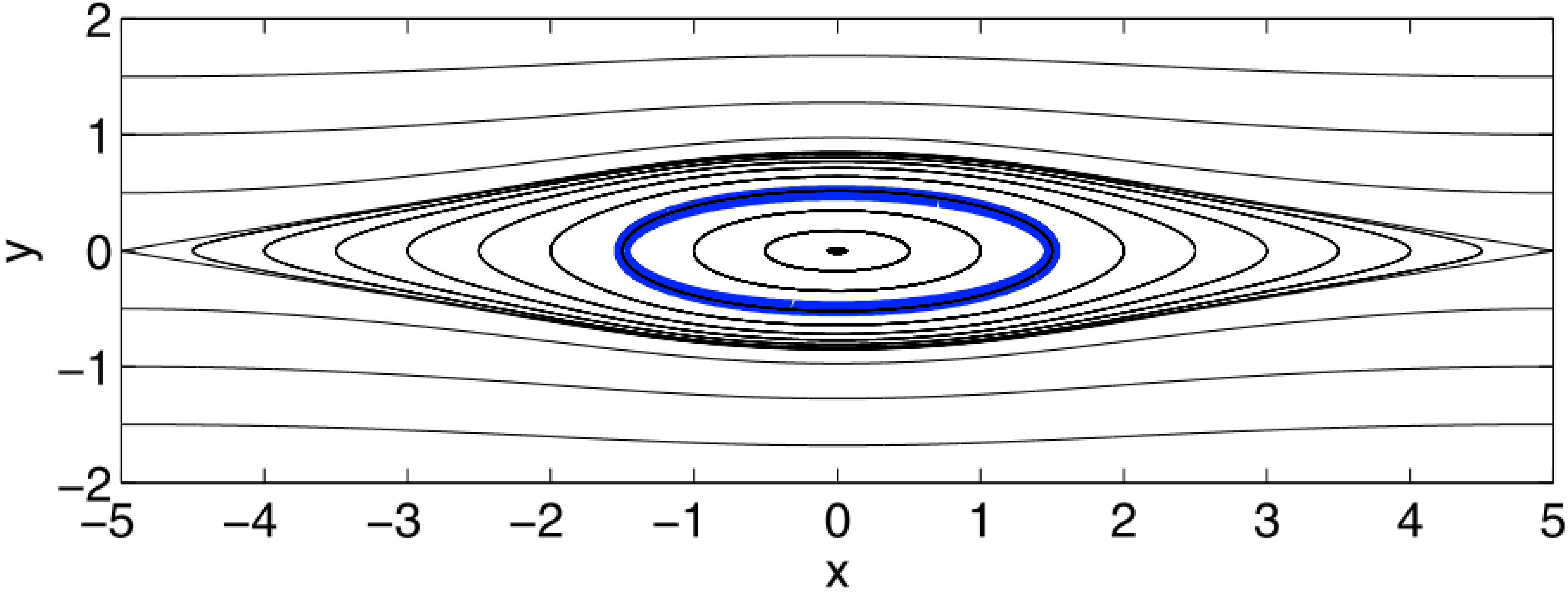
$$\frac{d\tilde{\rho}}{dt} = -a\sin(\phi_{k,0} - \phi_0)\frac{\partial\rho}{\partial b}, \qquad (48)$$

ODEs with periodic coefficients → Floquet Analysis

FIRST, CONSIDER *LIGHT* CORE

- Though unrealistic, it's simpler, and allows us to develop analogy

$$\frac{d\ln\rho}{d\ln b}>0$$

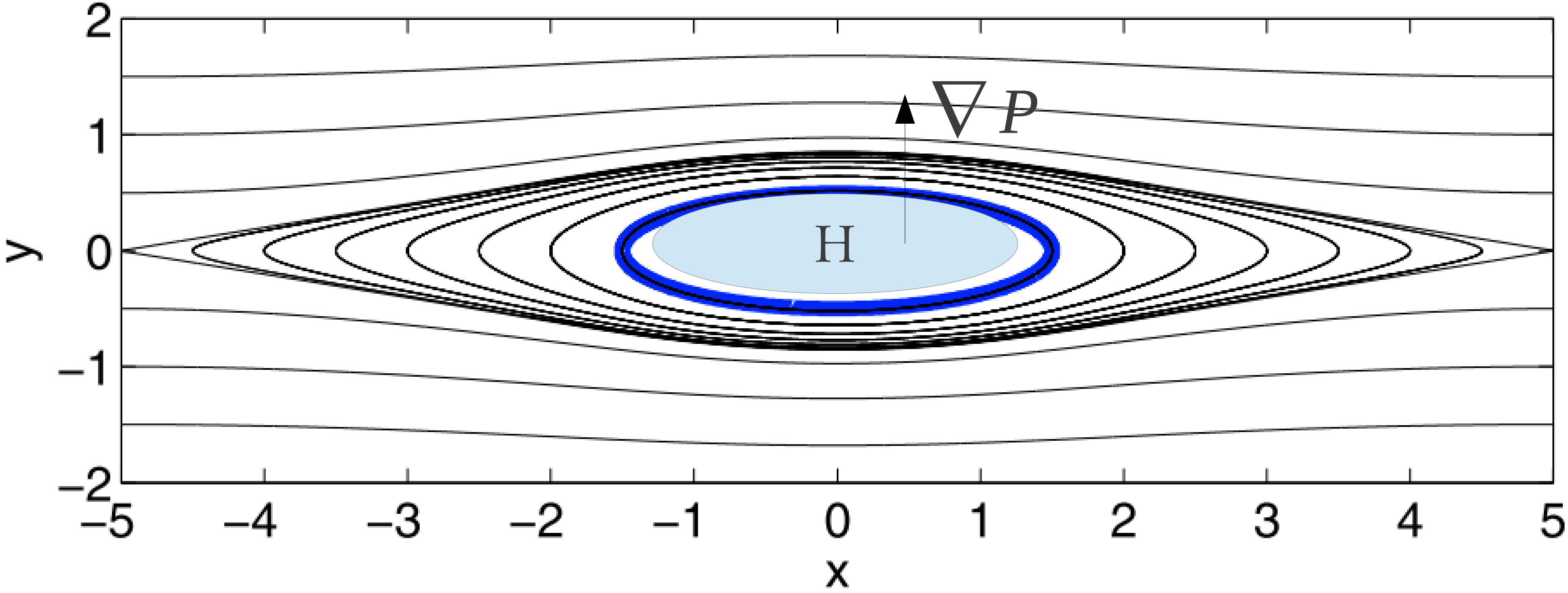


Lesur & Papaloizou (2009)

Can Put this in terms of the VRTI

- PPD Vortex has *High Pressure* core
  - So *light cores* are unstable to VRTI

$$\frac{d\ln\rho}{d\ln b}>0$$



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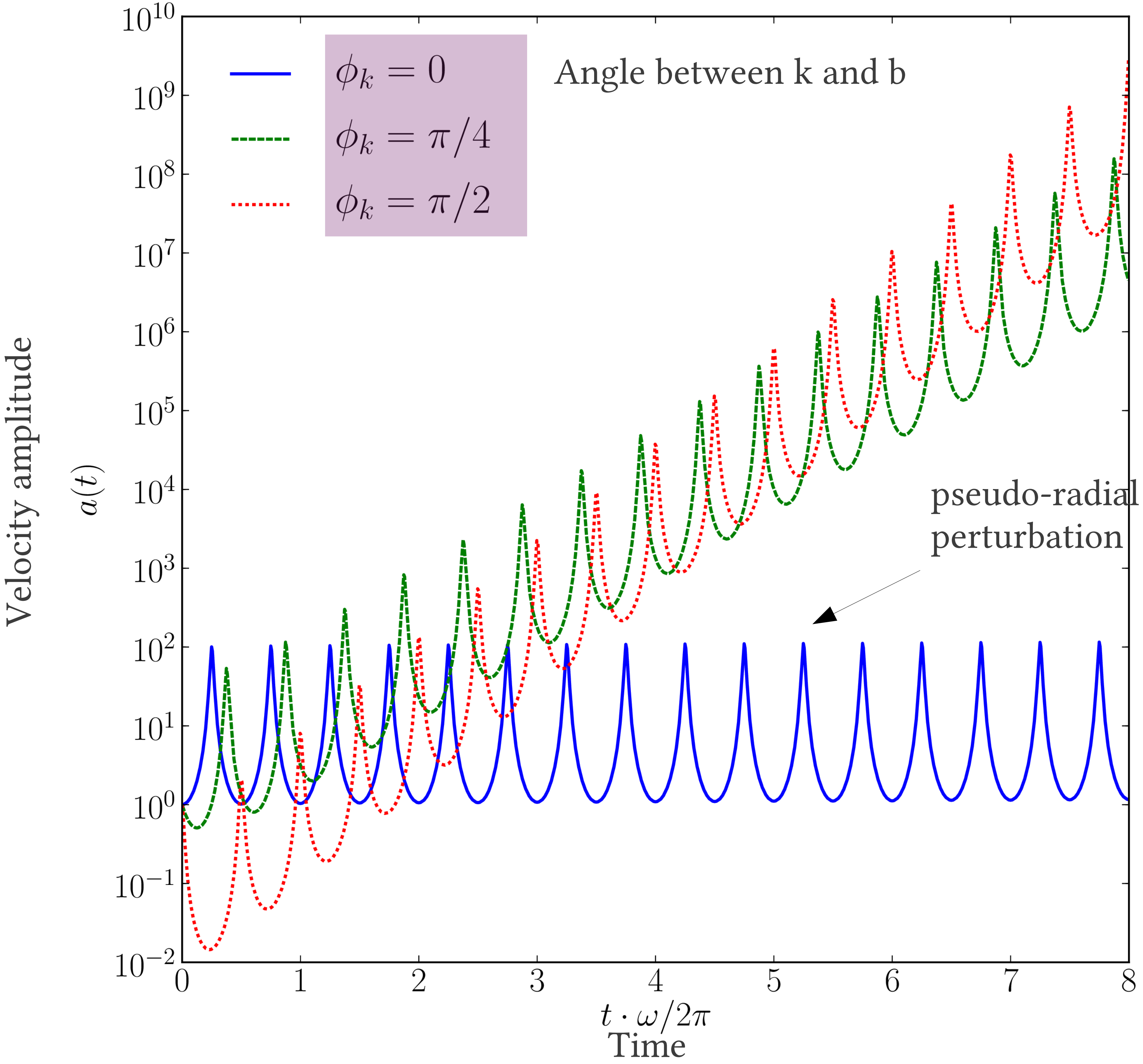


$\phi_k = 0$
$\phi_k = \pi/4$
$\phi_k = \pi/2$
Angle between k and b
pseudo-radial perturbation
Velocity amplitude
$a(t)$
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
10^{5}
10^{6}
10^{7}
10^{8}
10^{9}
10^{10}
0
1
2
3
4
5
6
7
8
$t \cdot \omega/2\pi$
Time

Now go to heavy core

- Consider *effective gravity* for GNG

$$g = \sqrt{g_x^2 + g_y^2} = g_0 \sqrt{\cos^2 \phi + \frac{\sin^2 \phi}{\chi^2}}, \qquad (52)$$

where

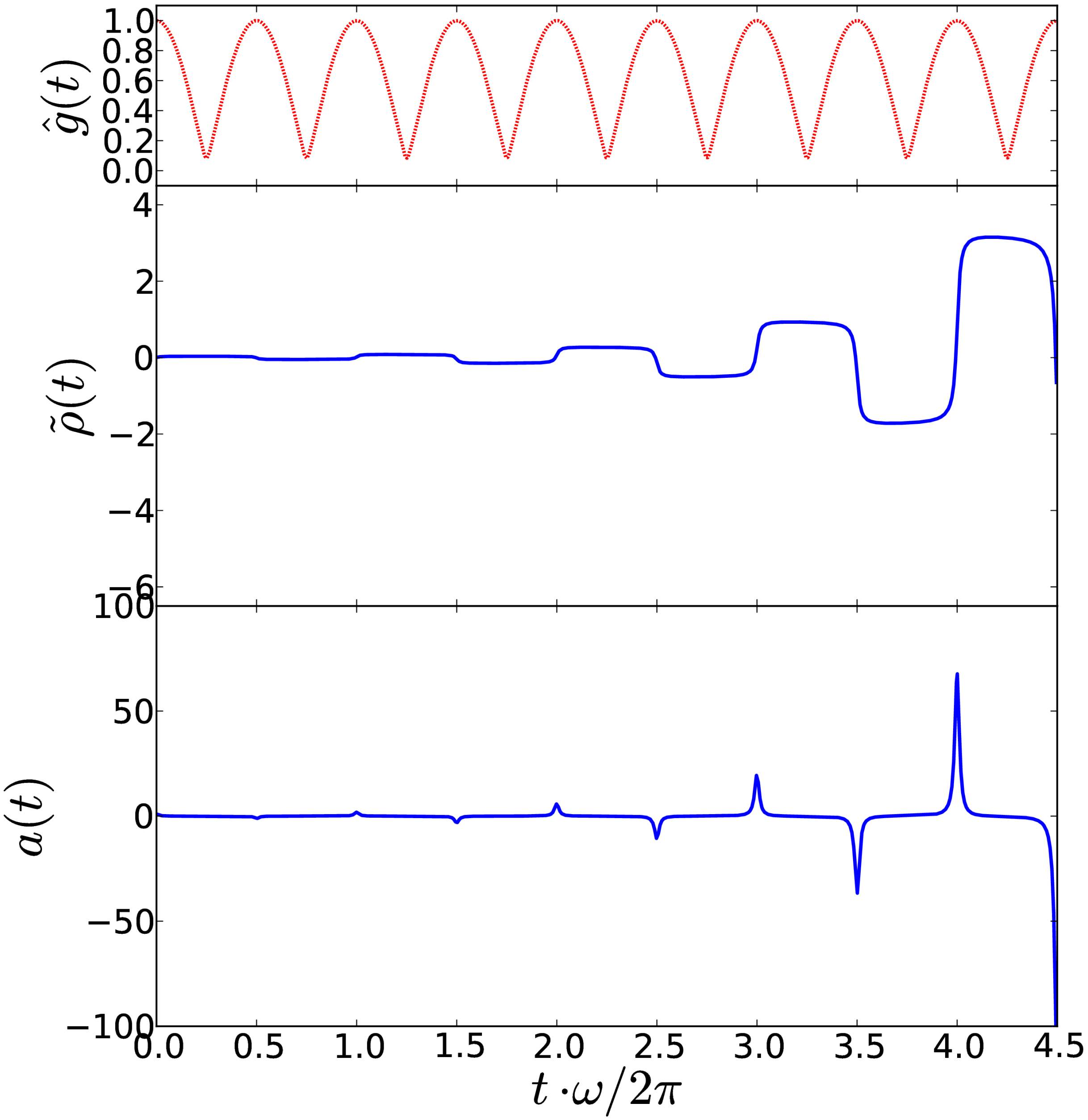
$$g_0 = b\Omega^2 \left(\frac{3\chi^2}{\chi^2 - 1} - 2\sqrt{\frac{3\chi^2}{\chi^2 - 1}} \right) = \text{const.} \qquad (53)$$

Heavy Cores

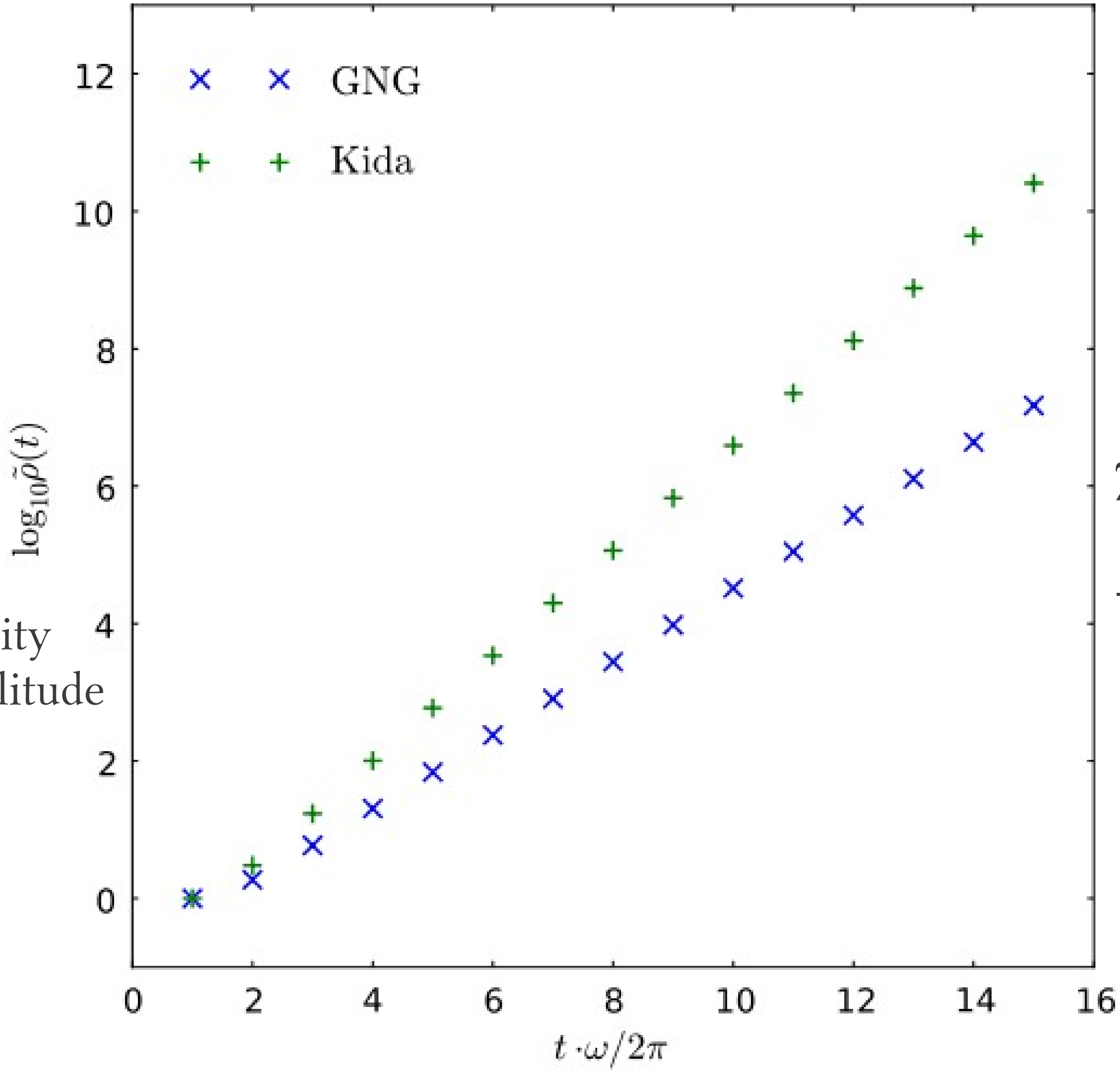
Effective gravity

Density perturbation

Velocity perturbation



Density Amplitude

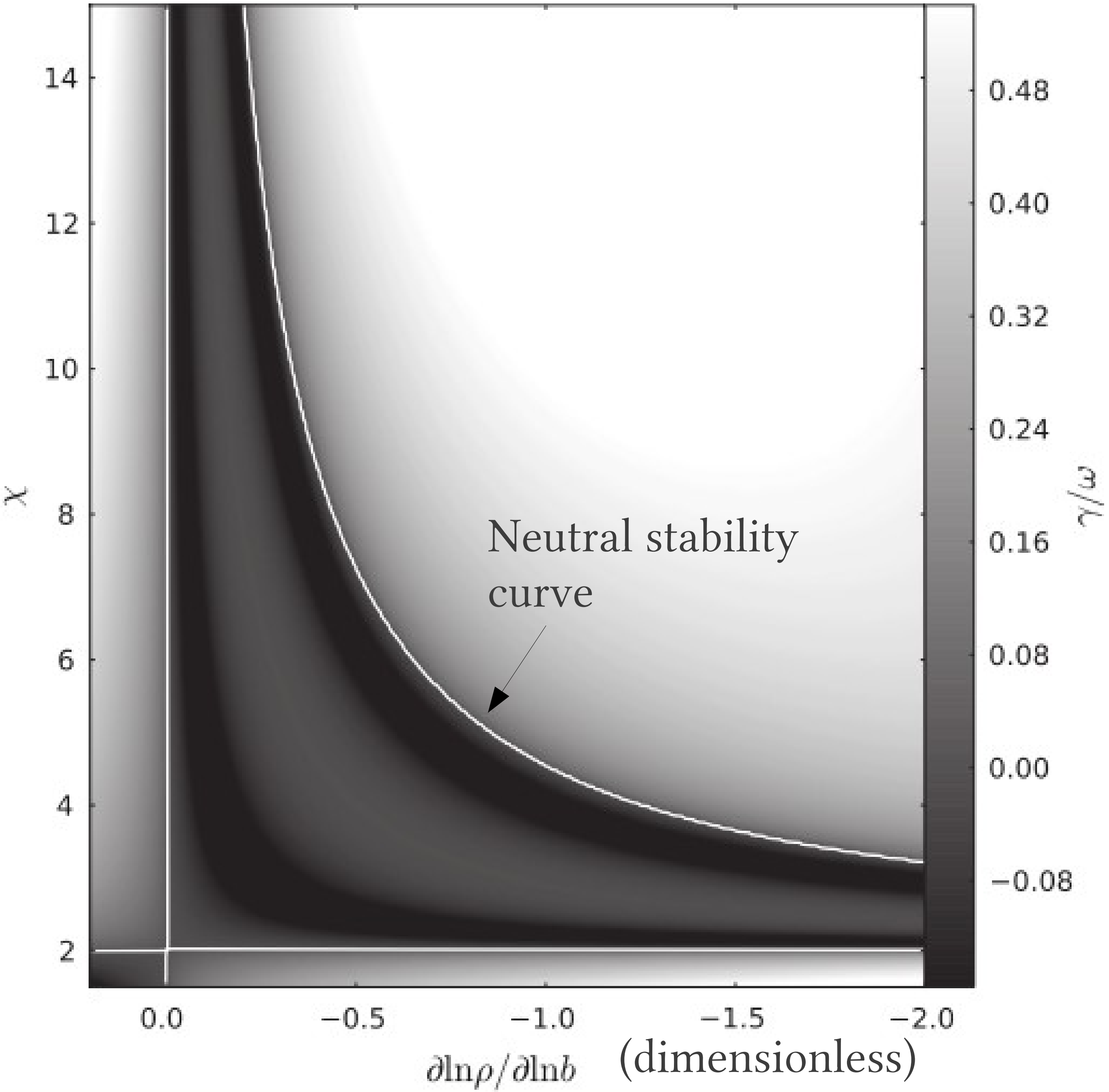


time

$\chi = 12$

$$\frac{d \ln \rho}{d \ln b} = -0.3$$

Survey Parameter Space

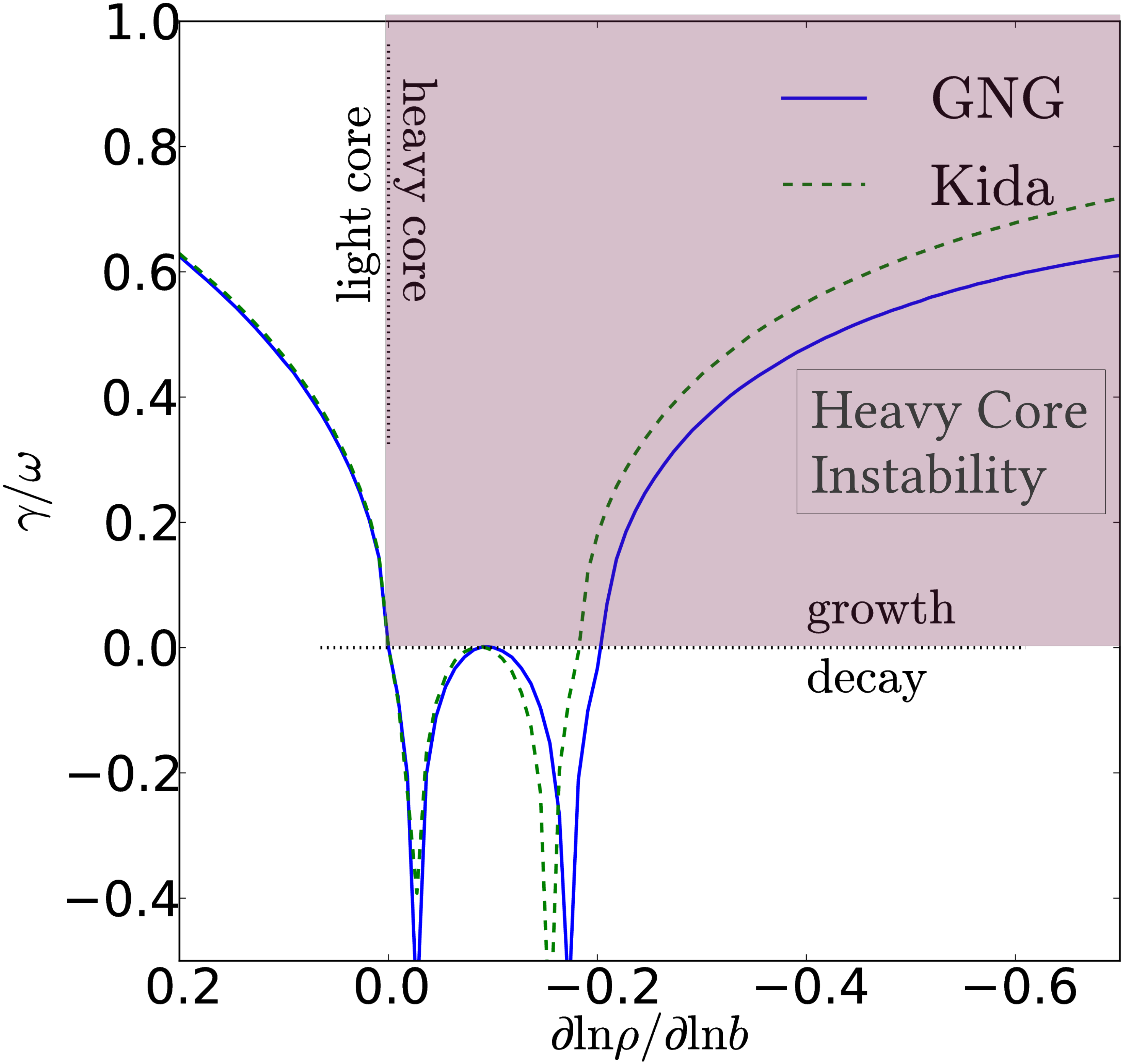


Aspect ratio

Neutral stability curve

Growth rate

(dimensionless) Density gradient



Growth Rate

Initial Vortex Elliptical Density Gradient

OPEN ISSUES

- Why has this not been seen in simulations?
  - Could be one-fluid approximation
  - Could be resolution—may require more points across vortex gradients
- Is the equilibrium (Kida + dln rho /d ln b) reasonable?

CONCLUSIONS

- The Heavy Core Instability grows on vortices with a steep enough density gradient and a big enough ellipticity
- It is a purely 2D instability, unlike the elliptic instability
- This effect is robust for at least two different vortex equilibria
- The non-linear saturation of the instability is unknown