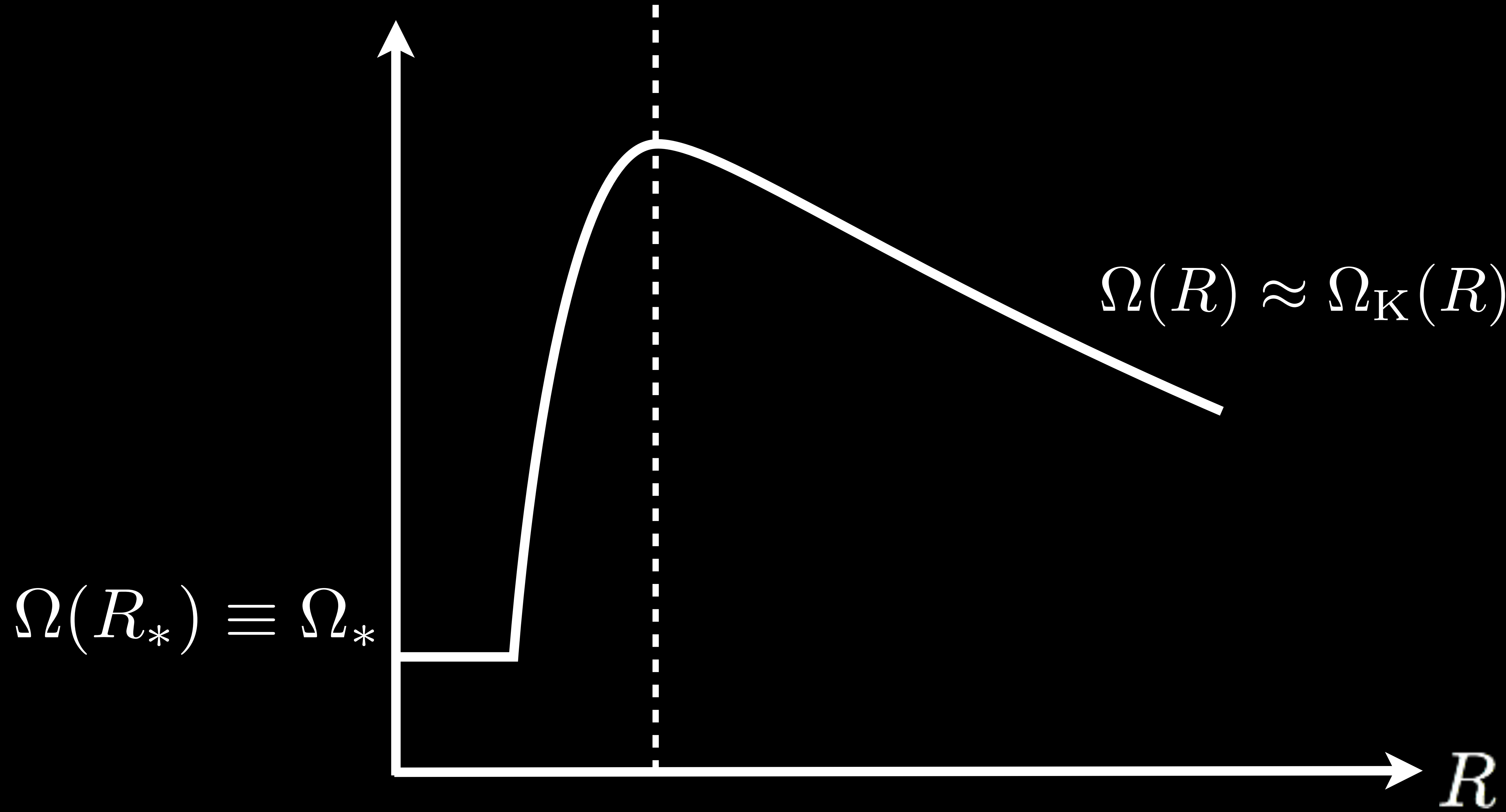
On Angular Momentum Transport in Accretion Disk Boundary Layers Around Weakly Magnetized Stars

Martin E. Pessah (NBI) + Chi-kwan Chan (NORDITA) - ApJ, 751, 48, 2012

Instabilities and Structures in Protoplanetary DIsks - September 16-20, 2012 - Marseille

Standard Picture of the BL



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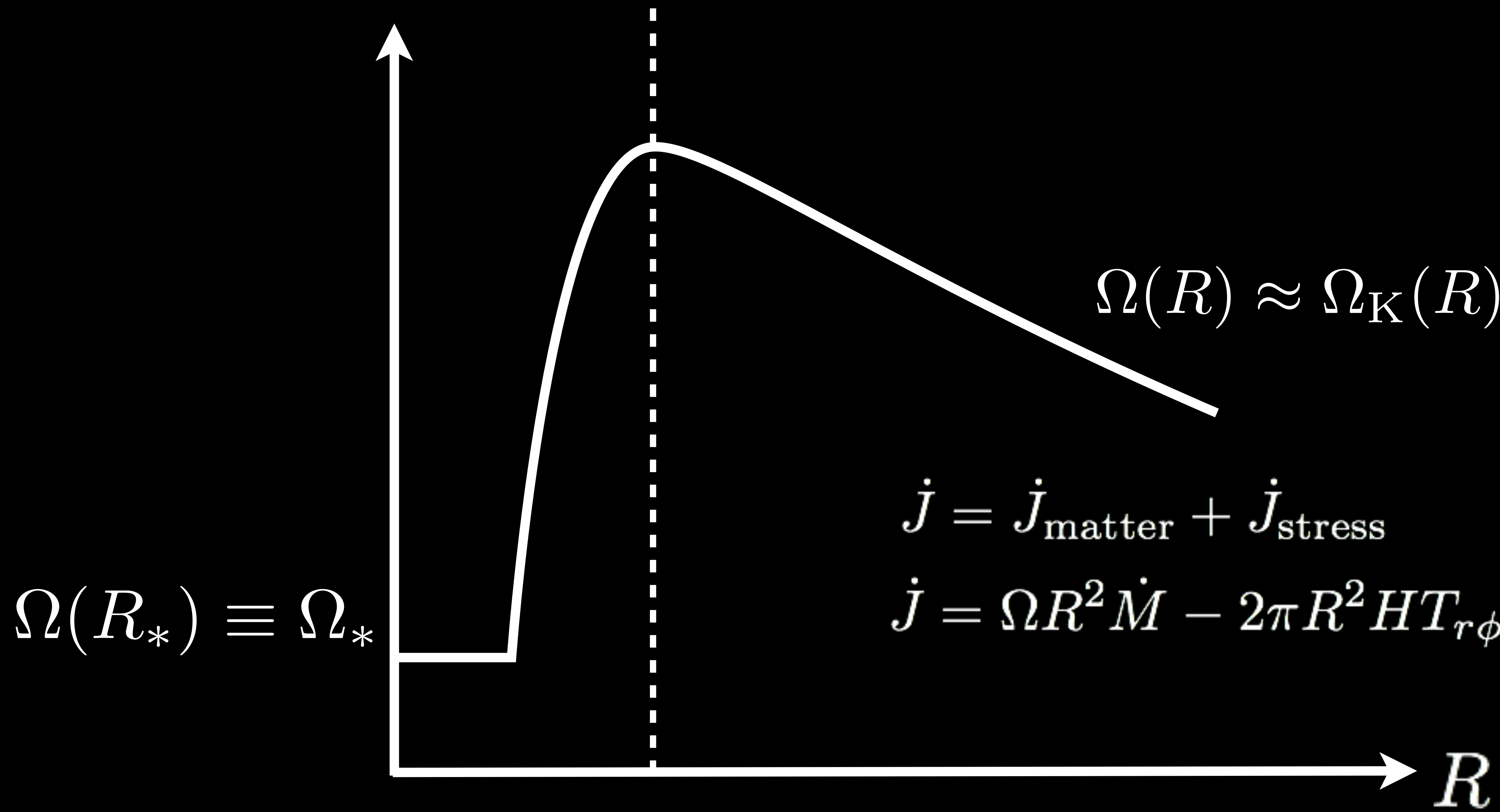

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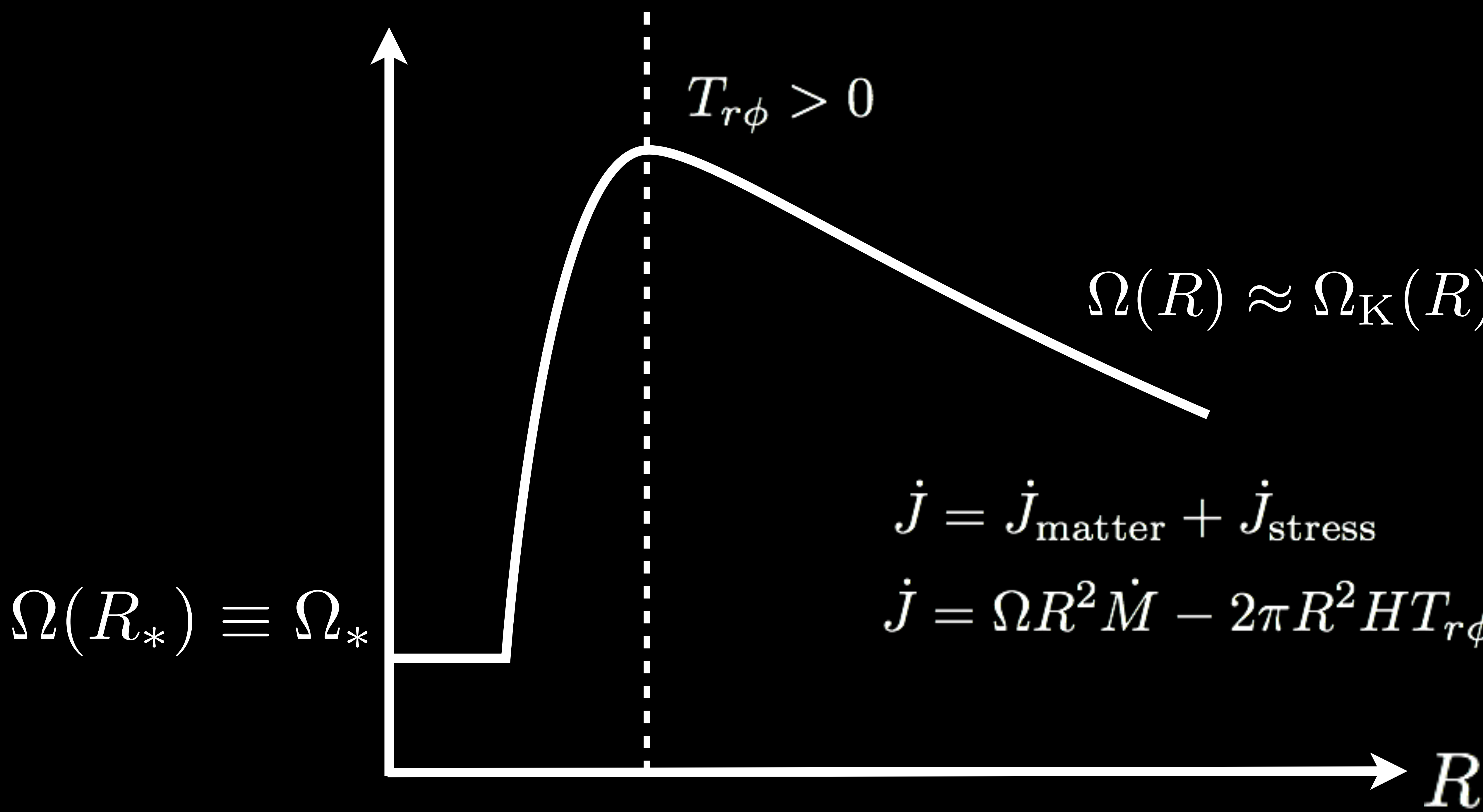

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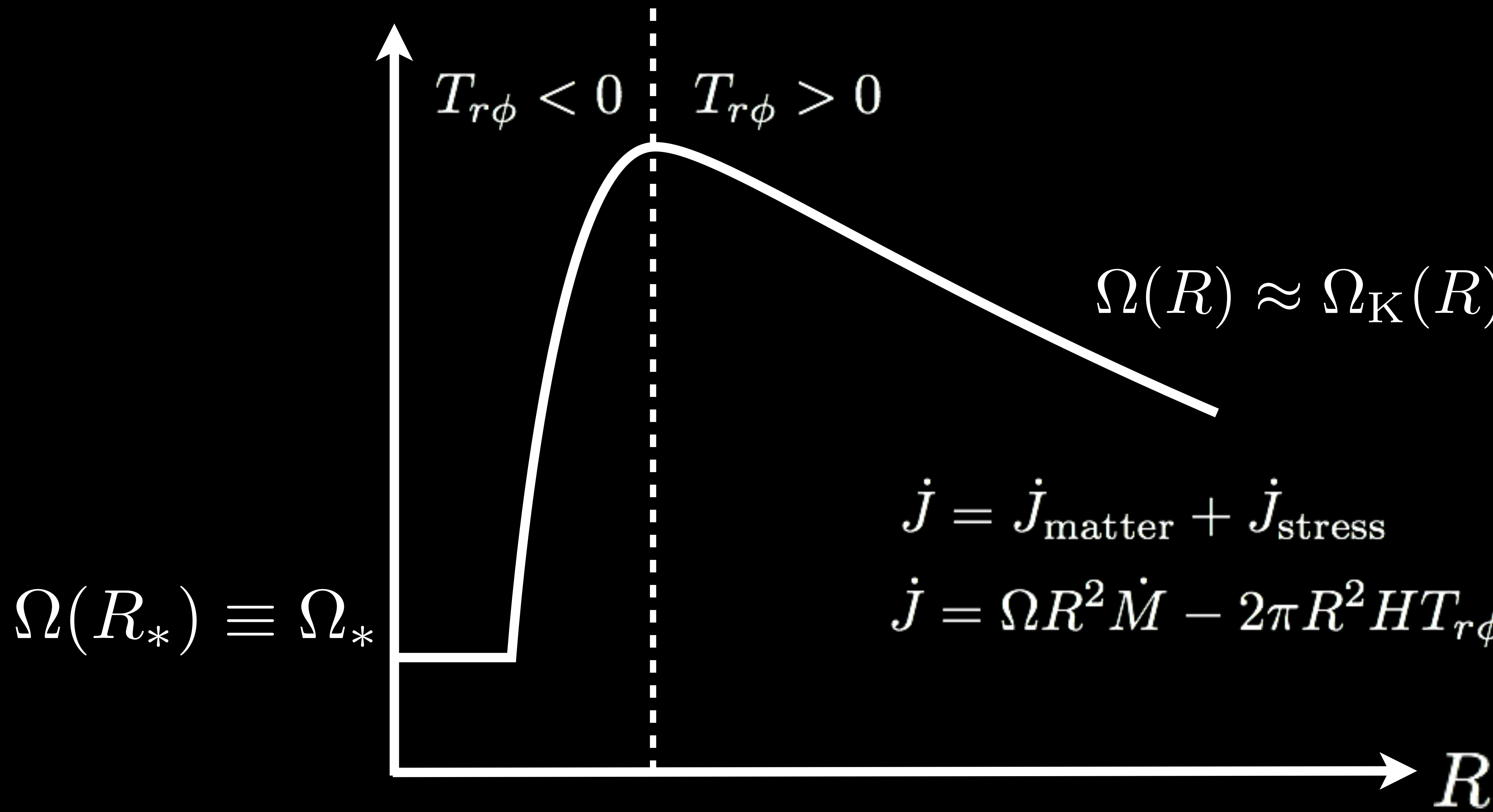
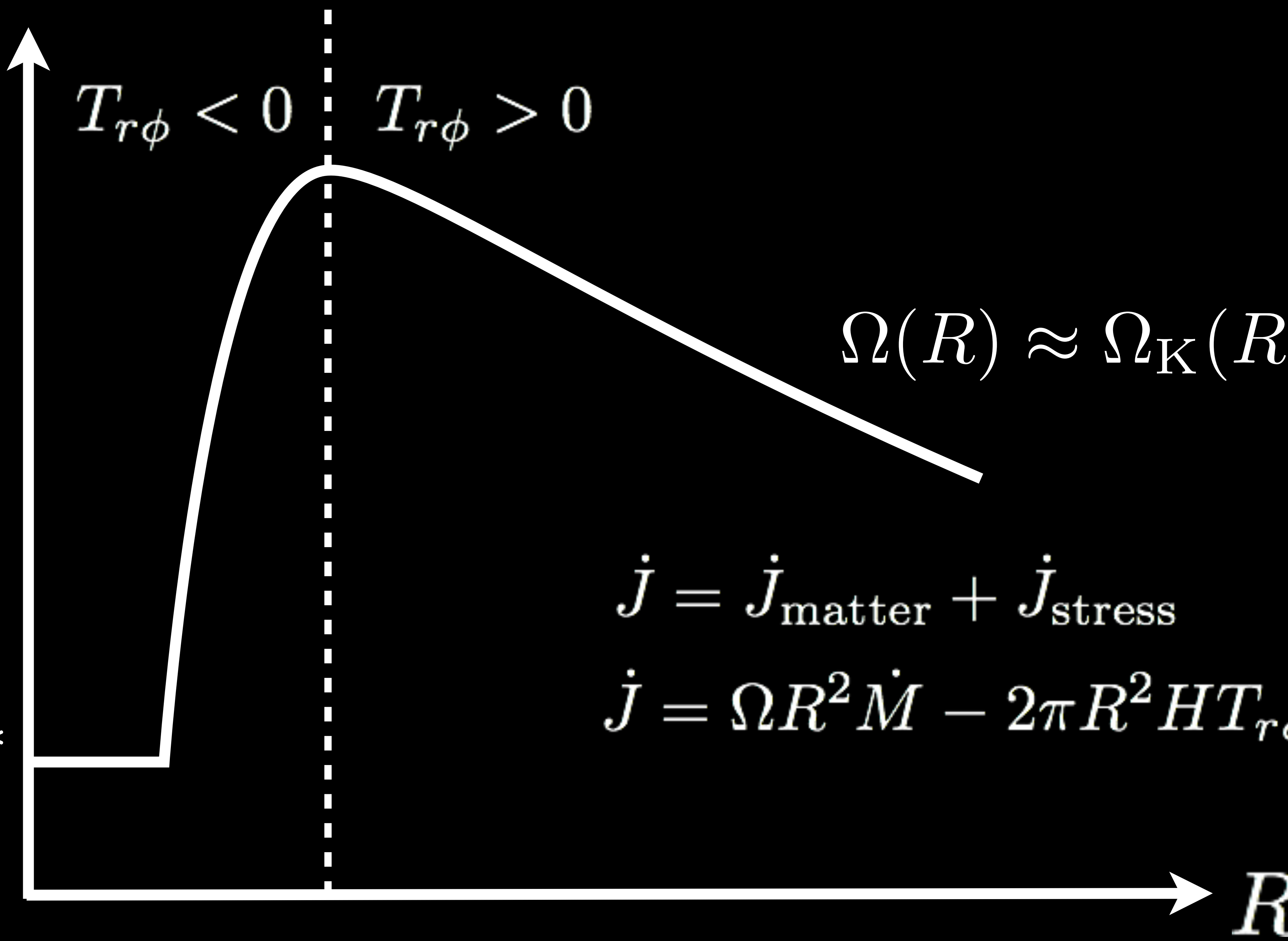


Standard Picture of the BL

"standard"

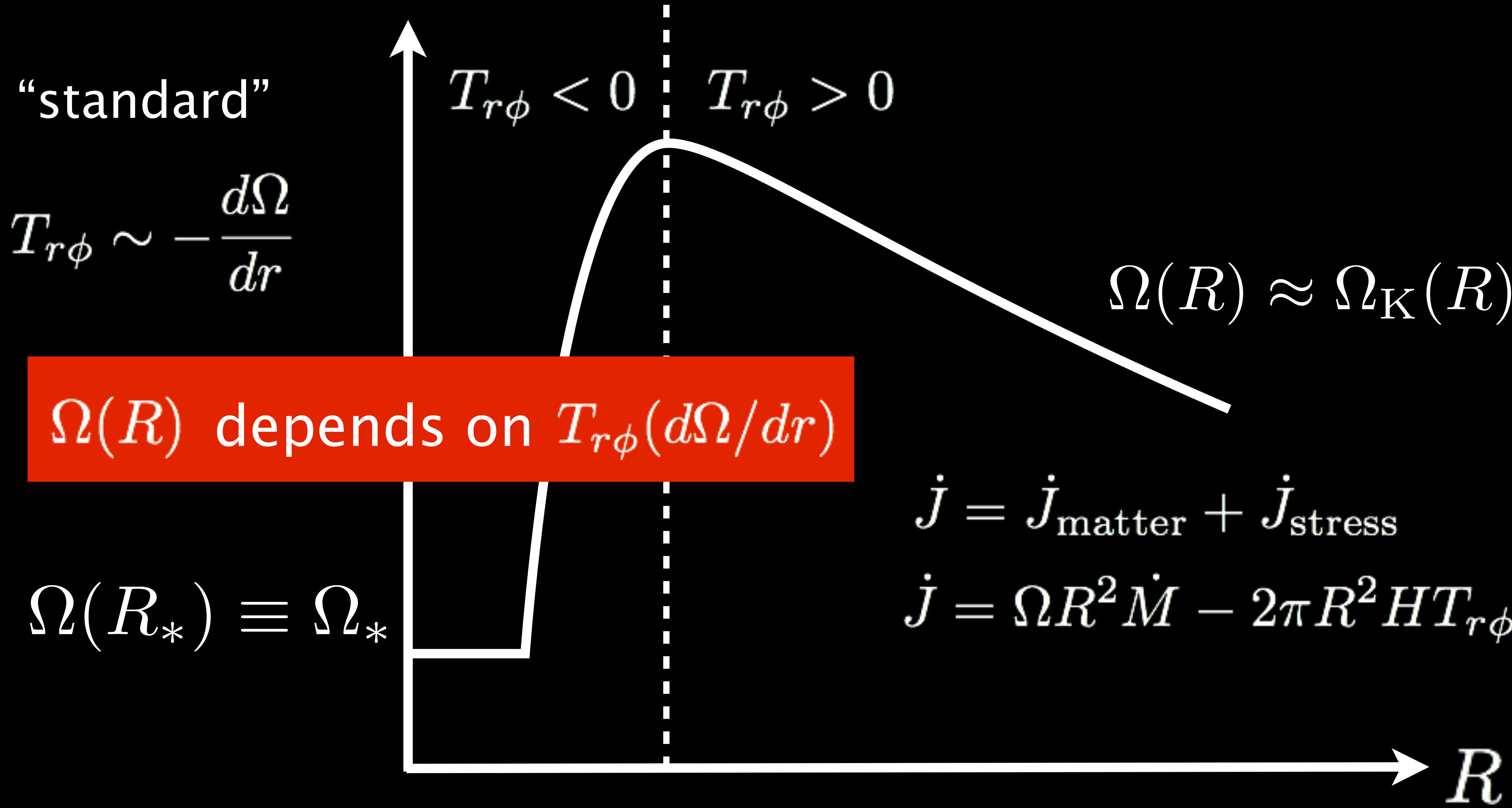
$$T_{r\phi} \sim -\frac{d\Omega}{dr}$$



$$\dot{J} = \dot{J}_{\mathrm{matter}} + \dot{J}_{\mathrm{stress}}$$

$$\dot{J} = \Omega R^2 \dot{M} - 2\pi R^2 H T_{r\phi}$$

Standard Picture of the BL



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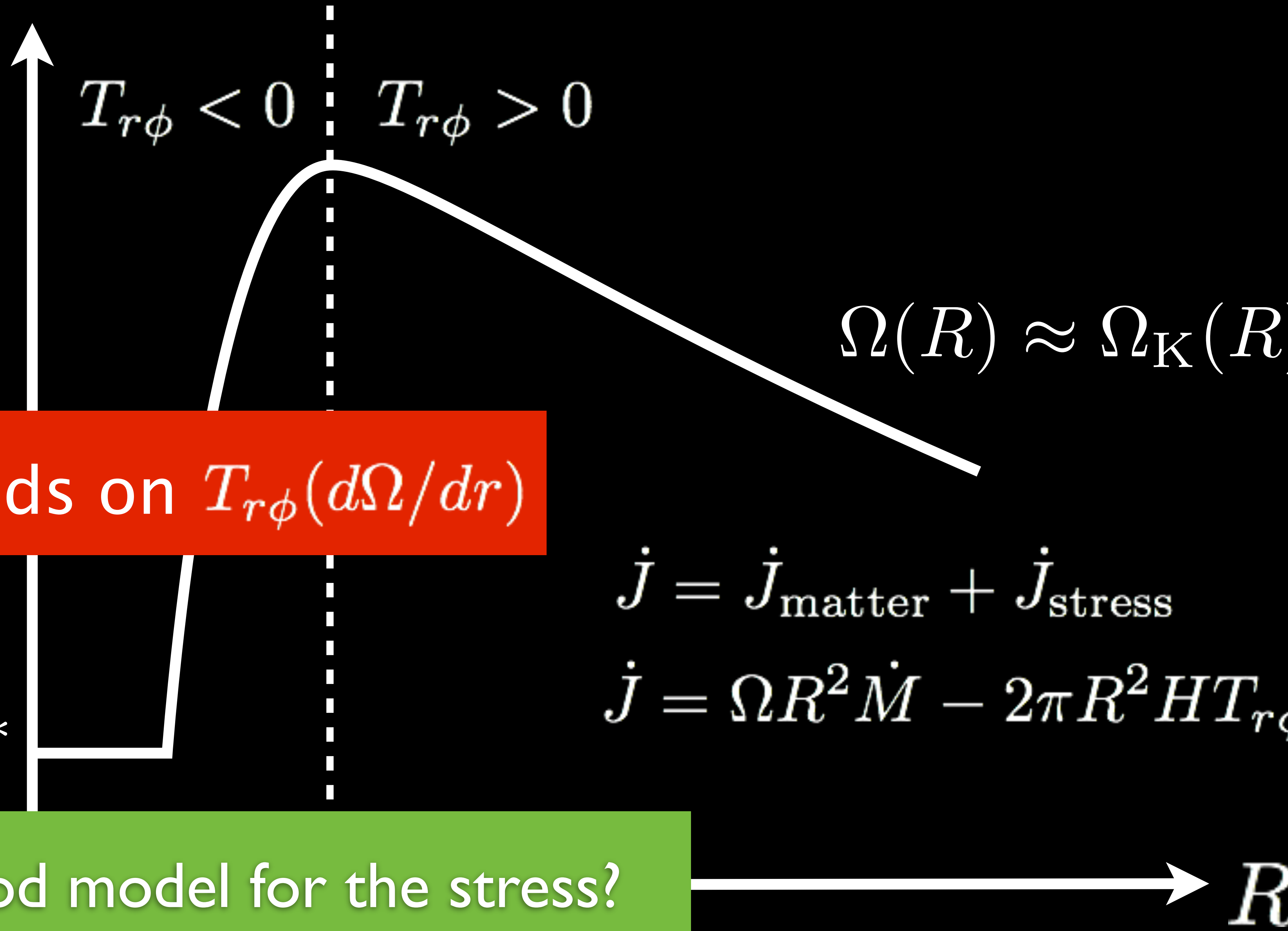


$\Omega(R)$ depends on $T_{r\phi}(d\Omega/dr)$

$$\dot{J} = \dot{J}_{\rm matter} + \dot{J}_{\rm stress}$$

$$\dot{J} = \Omega R^2 \dot{M} - 2\pi R^2 H T_{r\phi}$$

$$\Omega(R_*) \equiv \Omega_*$$

What is a good model for the stress?

Standard Accretion Disk Model

- Shakura & Sunyaev (1973)
- "The efficiency of the mechanism of angular momentum transport is characterized by parameter"

$$\alpha = \frac{v_{\rm turb}}{c_{\rm s}} + \frac{B^2}{4\pi\rho c_{\rm s}^2}$$

- In terms of stress,

$$T_{r\phi} \equiv R_{r\phi} - M_{r\phi} \sim \alpha P$$

But...

- Shakura & Sunyaev (1973) considered Keplerian flow
- Shear-dependent turbulent viscosity - not addressed
- All later works assume Newtonian turbulent viscosity

$$\nu = \alpha c_{\rm s} H = \alpha c_{\rm s}^2 R / v_\phi$$

$$T_{r\phi} = \alpha q \Sigma c_{\rm s}^2 \qquad q \equiv -d\ln\Omega / d\ln R$$

- Is Newtonian turbulent viscosity a good assumption?

Standard Calculation

- Assume steady state
- Continuity equation ⇒ constant accretion rate

$$\dot{M} = -2\pi R \Sigma v_R = \text{constant}$$

- Angular momentum equation ⇒ flux balance

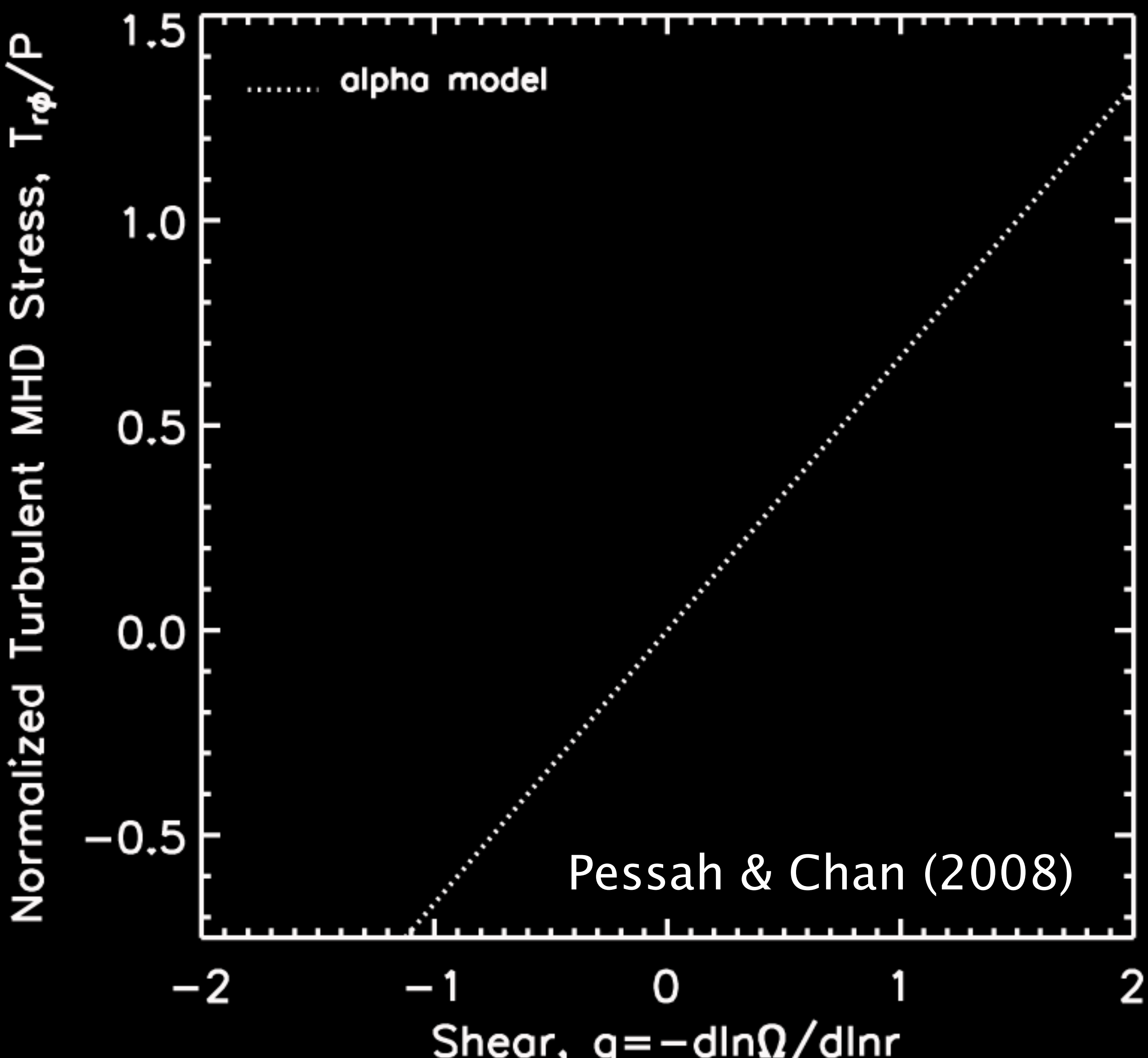
$$\dot{J} = \Omega R^2 \dot{M} - 2\pi R^2 H T_{r\phi} = \text{constant}$$

$$\dot{J} = \Omega R^2 \dot{M} + 2\pi R^2 \Sigma \nu R \Omega' = \text{constant}$$

"standard"

$$T_{r\phi} = -\nu(\Sigma/H) R \Omega'$$

MRI Turbulence $\neq \alpha$-Viscosity



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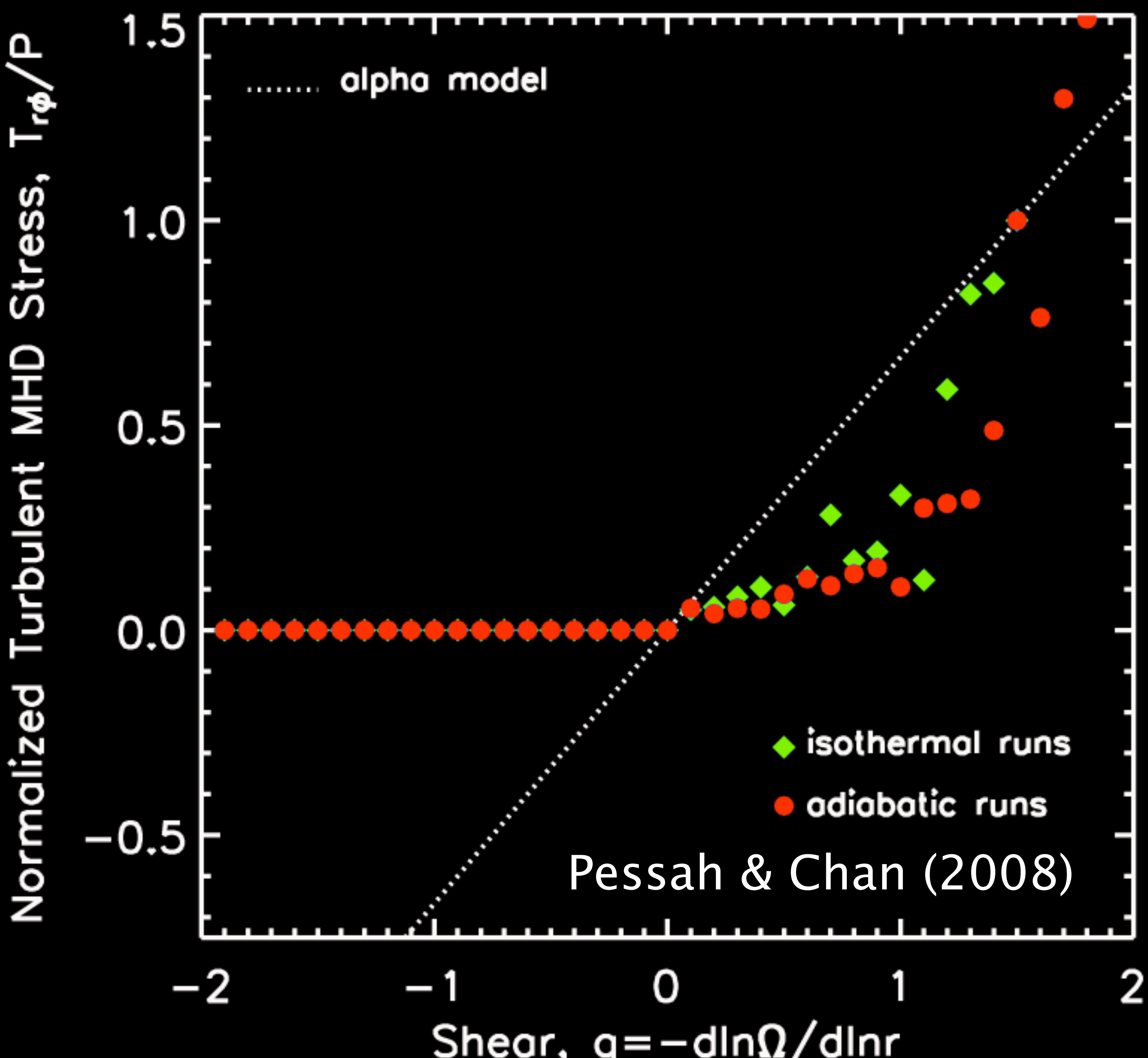

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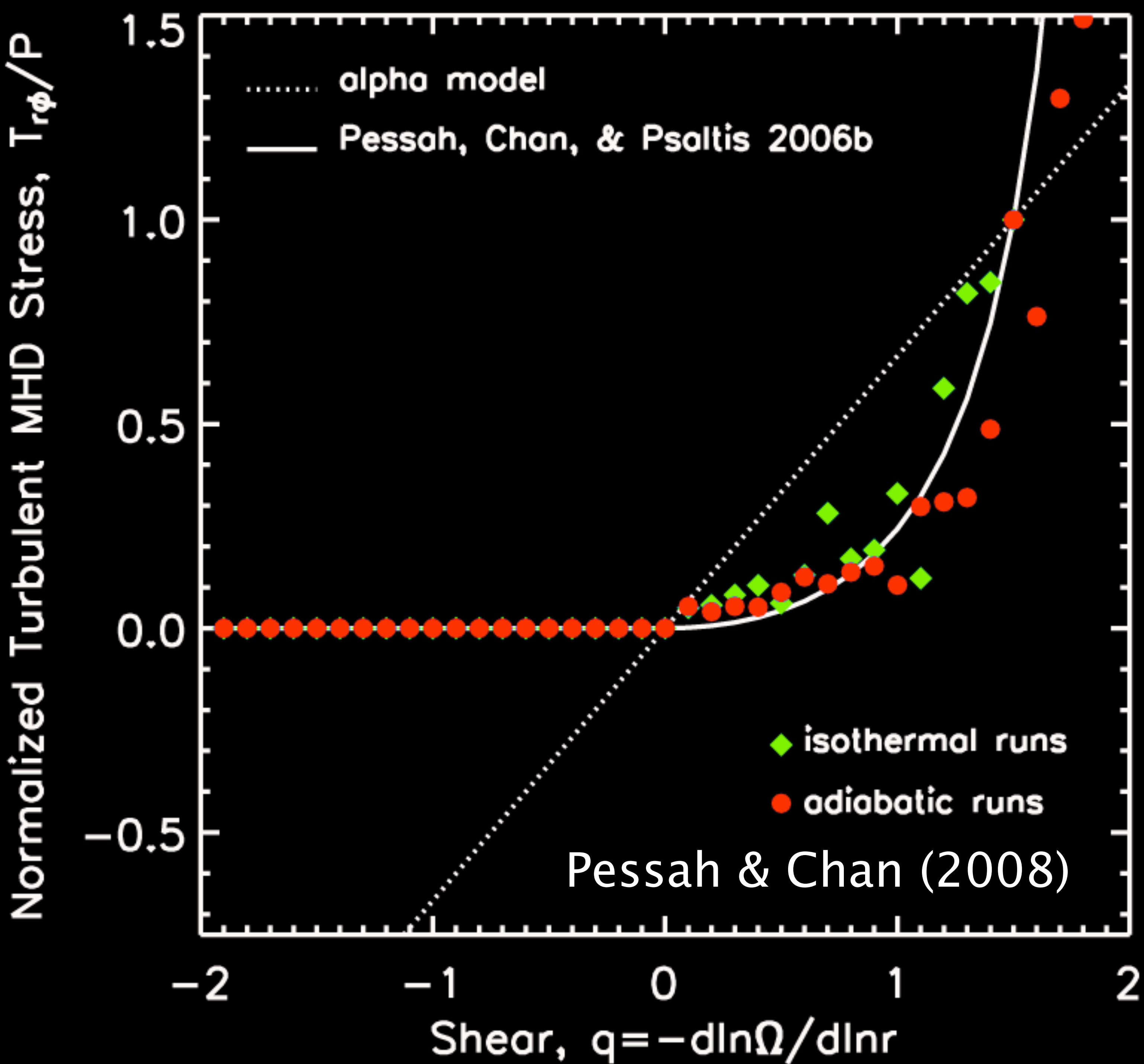

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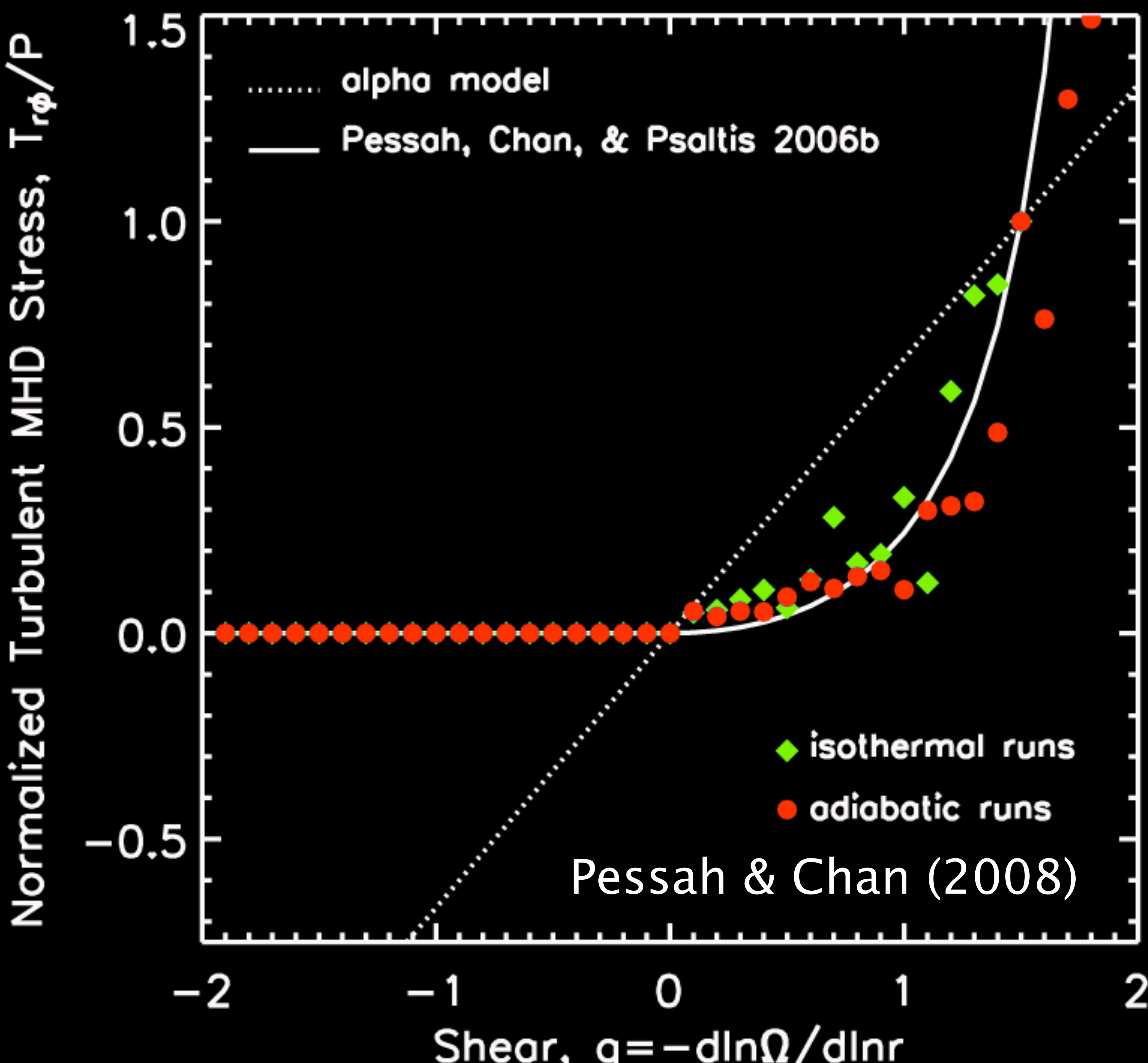

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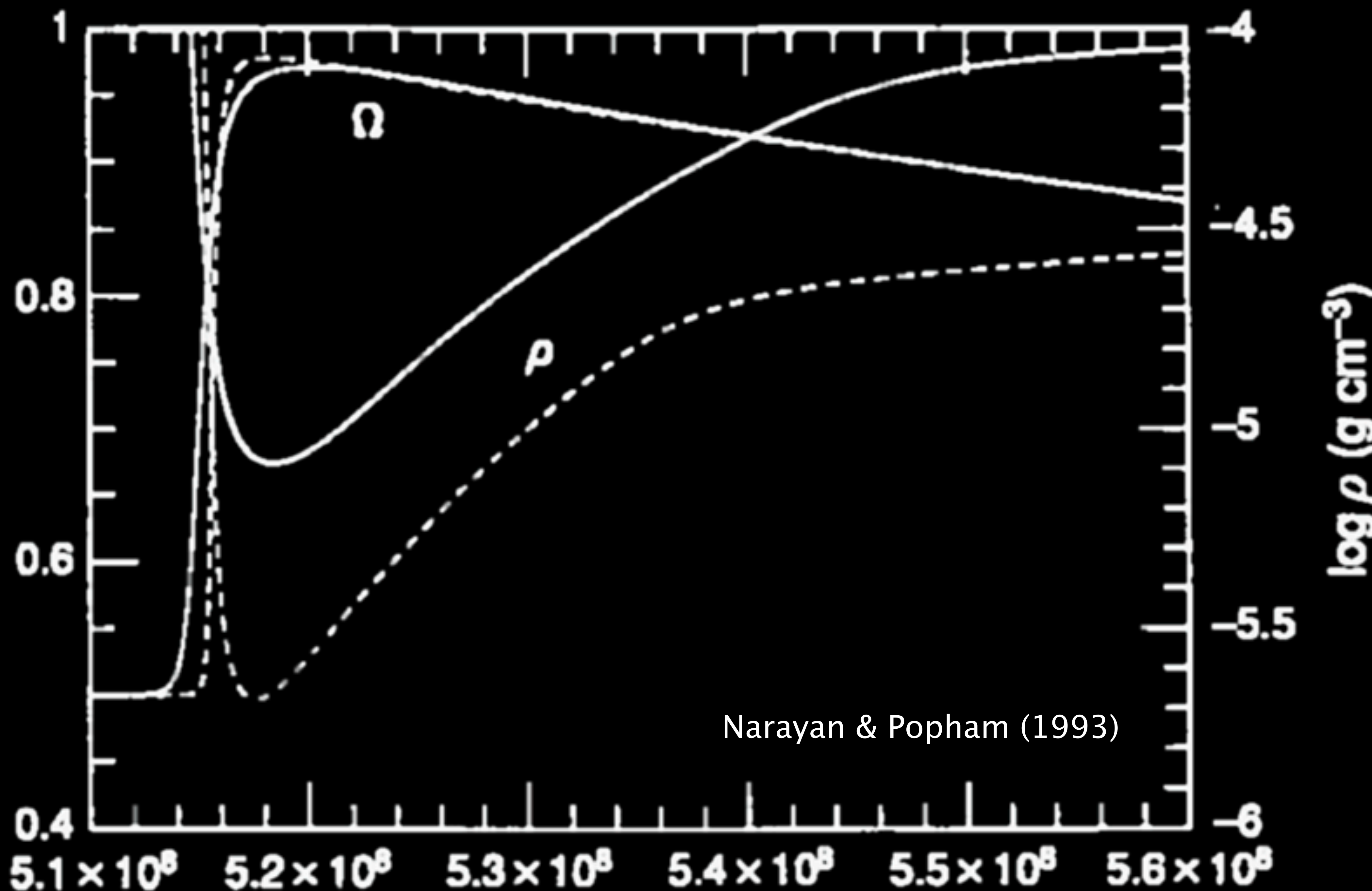
$$T_{r\phi} \sim -\frac{d\ln\Omega}{d\ln r}$$

Angular momentum transport seems rather inefficient for negative q (beware of limited range explored!)

What really happens here?



Narayan & Popham (1993)

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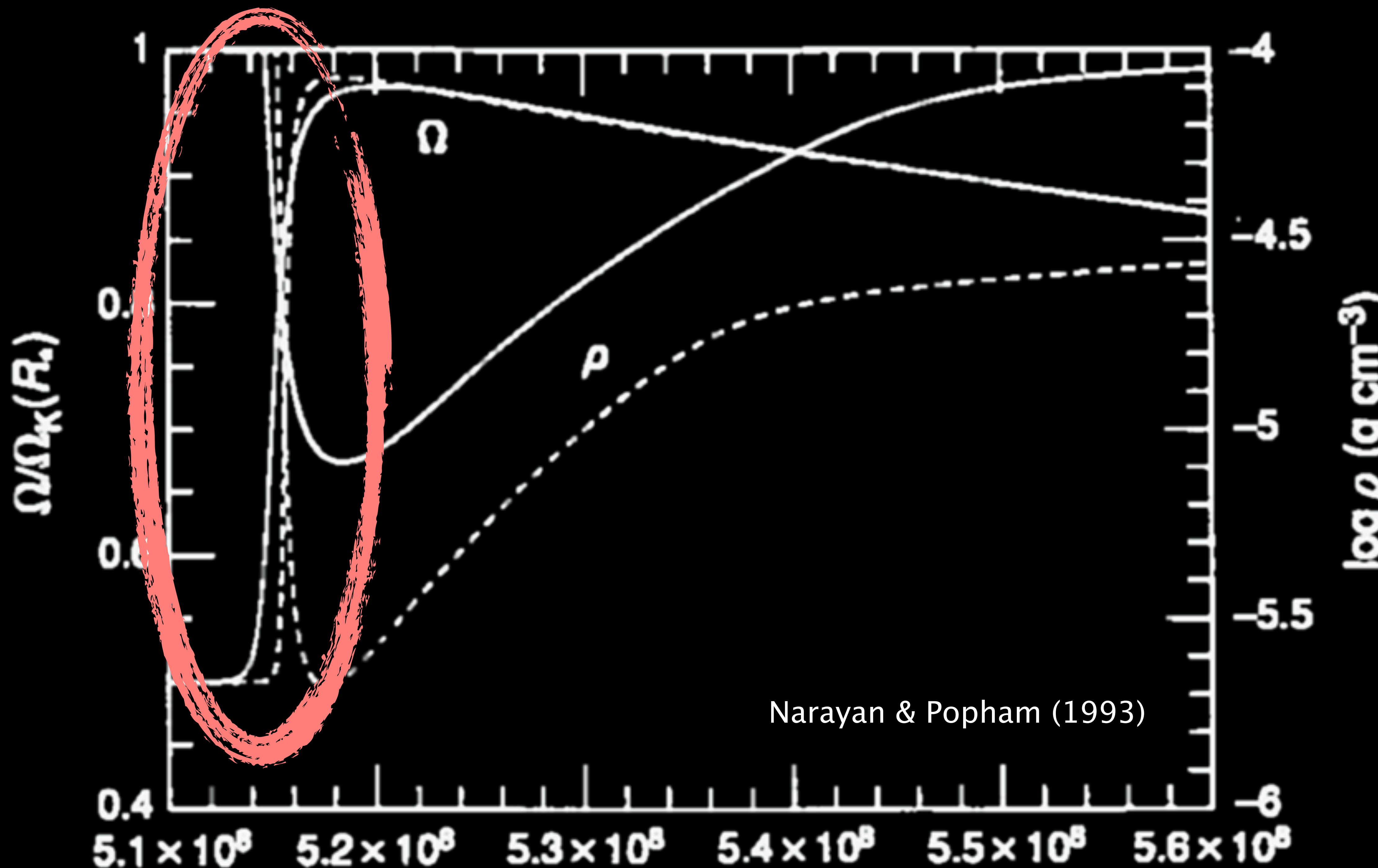
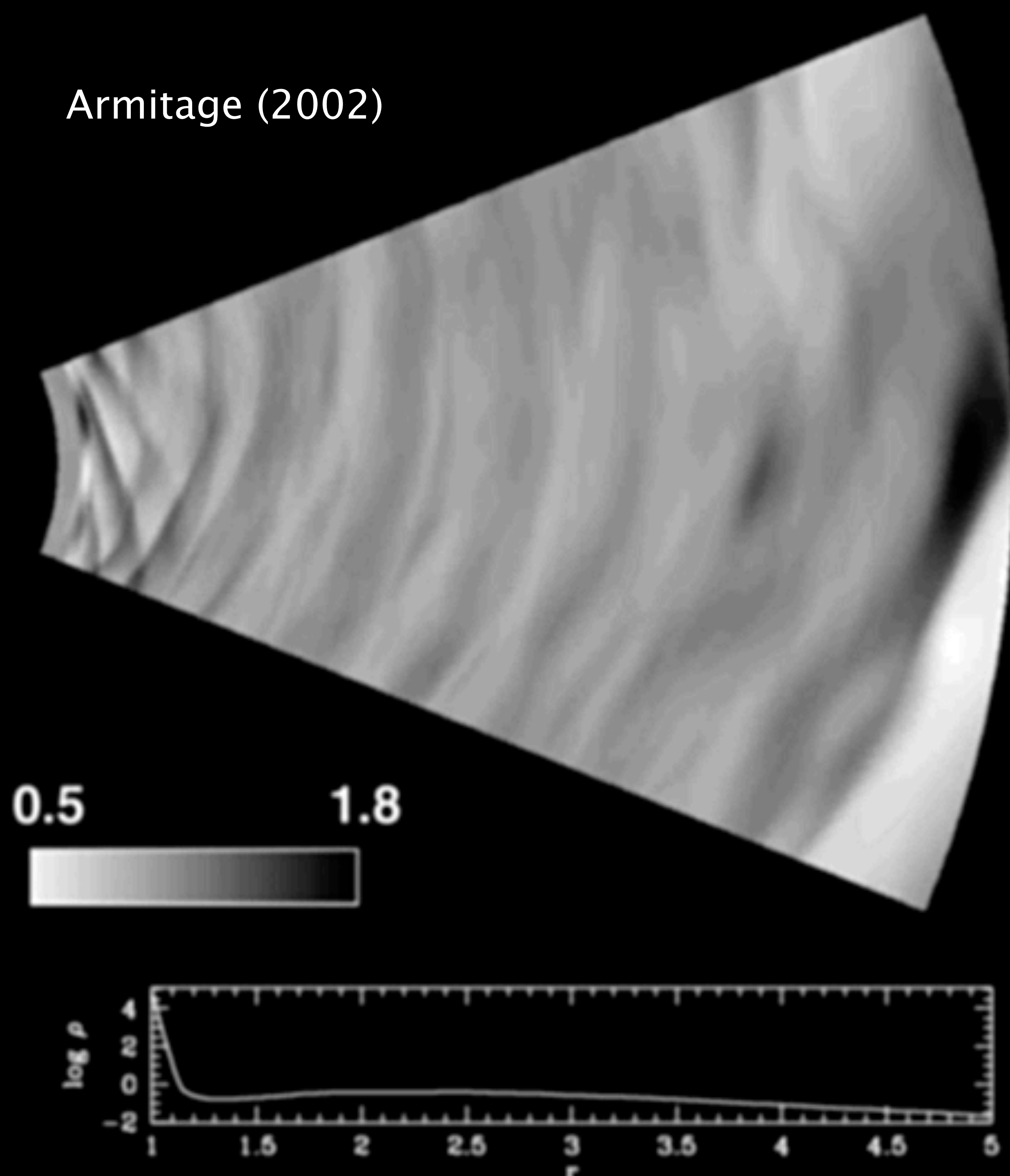
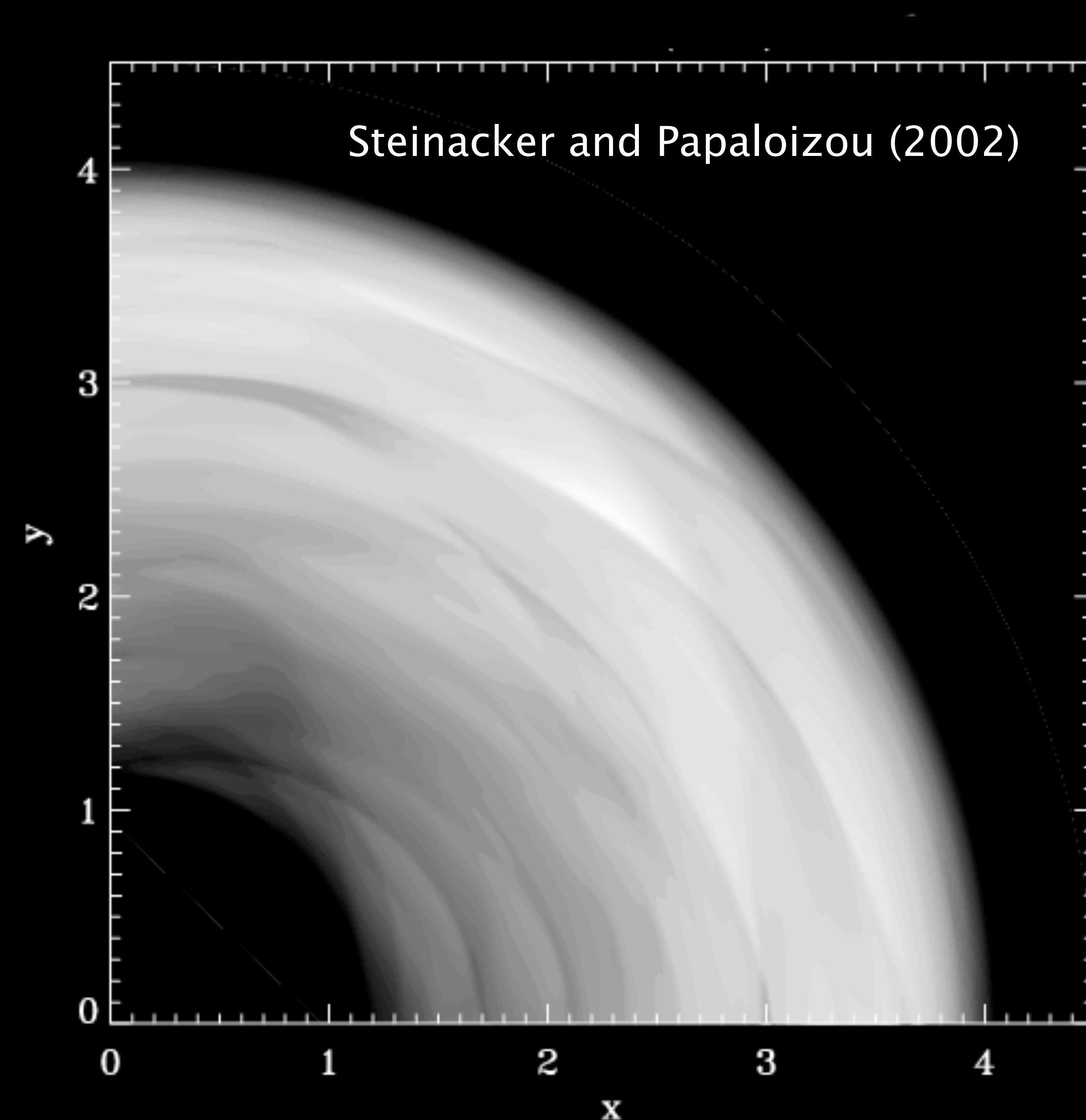
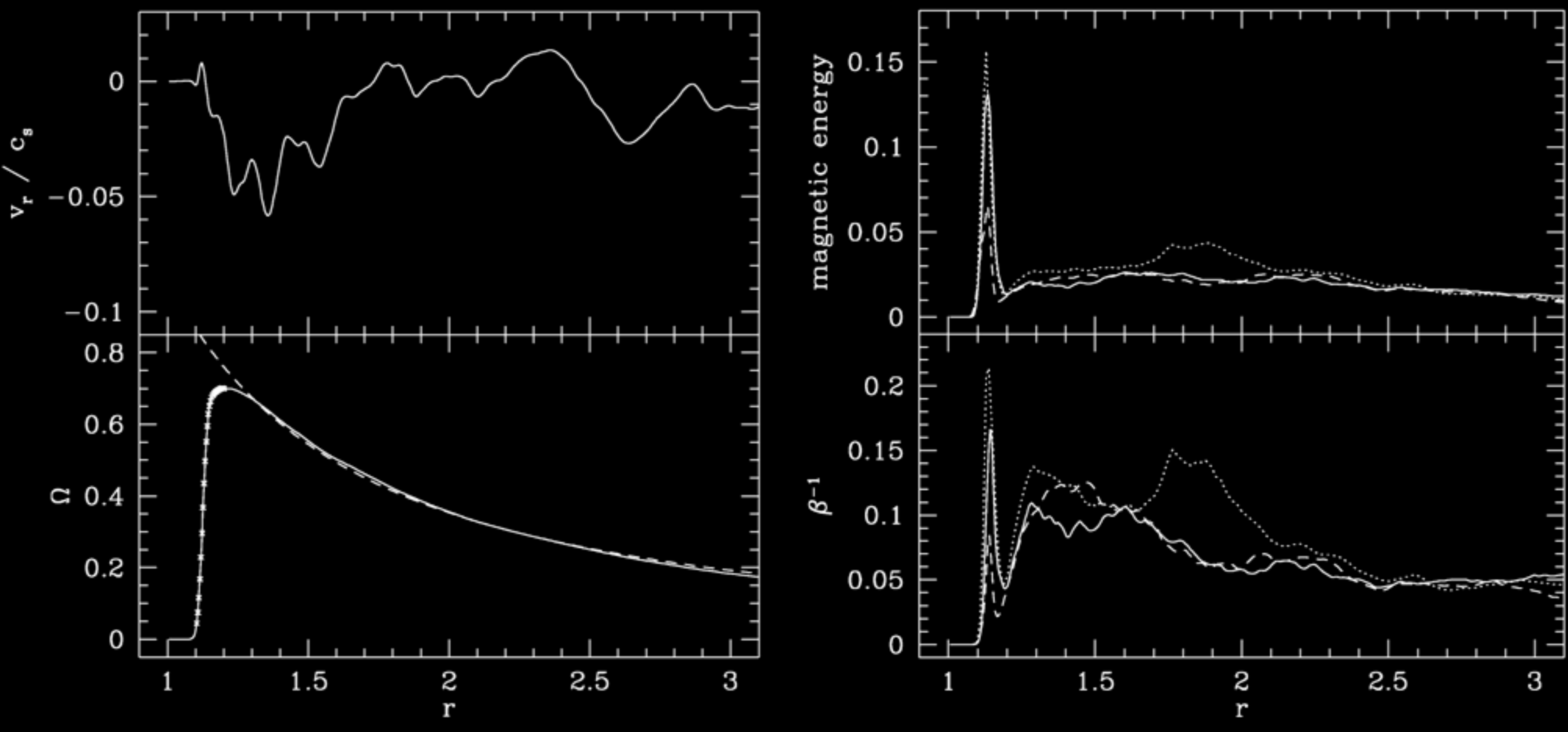


Numerical Simulations I.





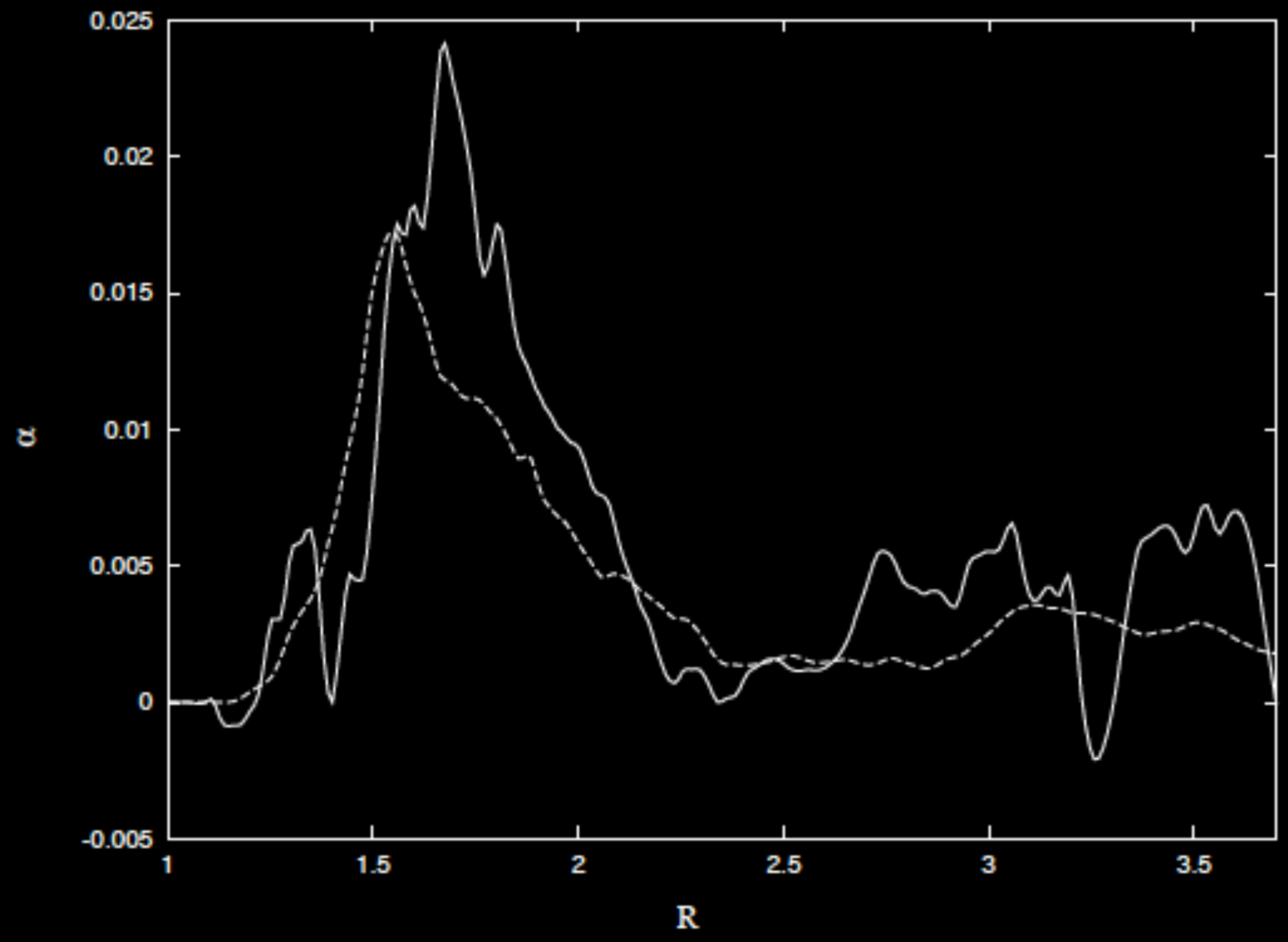
Numerical Simulations II.



Armitage (2002)

Numerical Simulations III.

Steinacker and Papaloizou (2002)



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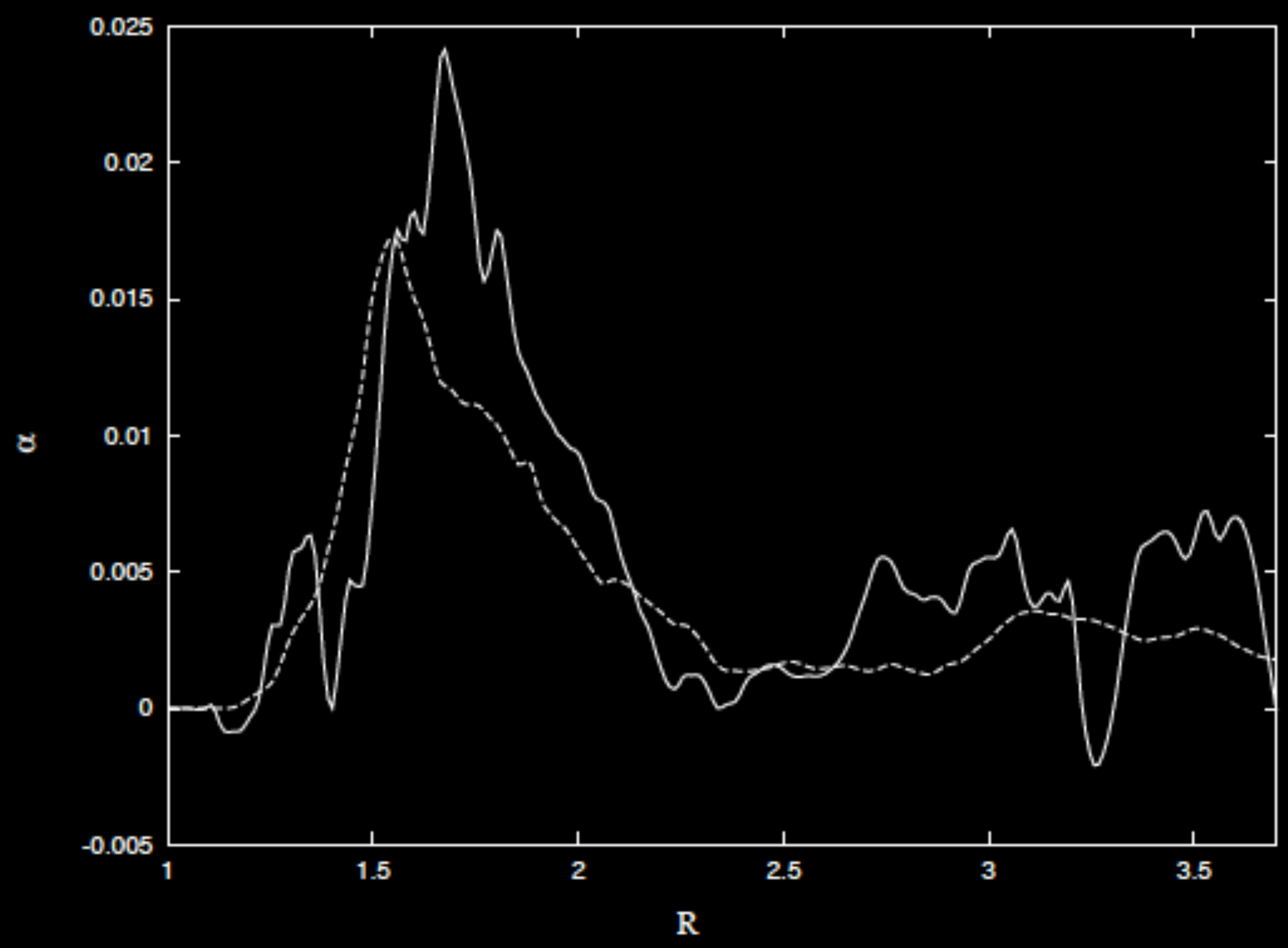

We see lots of magnetic energy but not much magnetic/Maxwell stress in simulations (but...)

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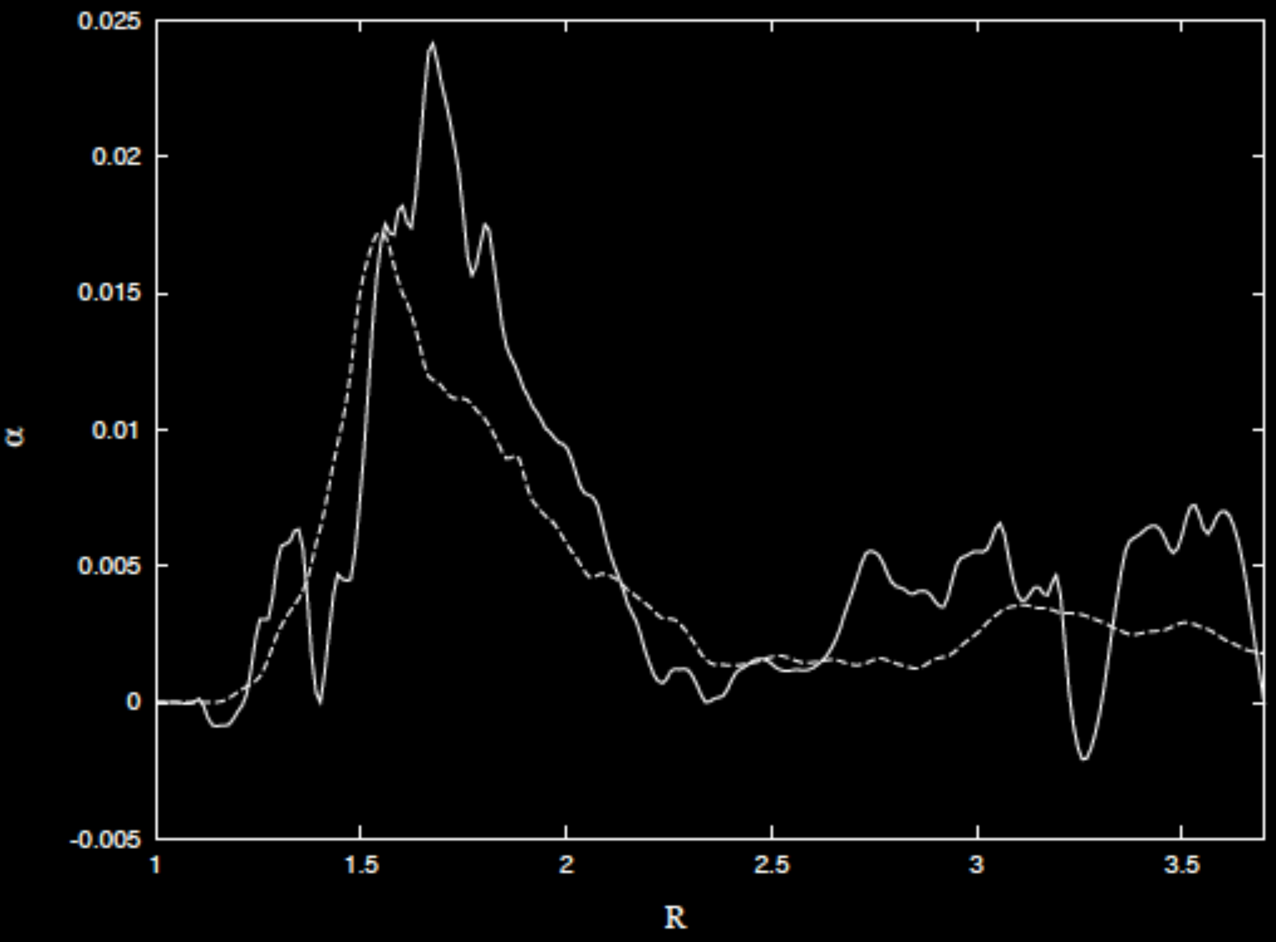

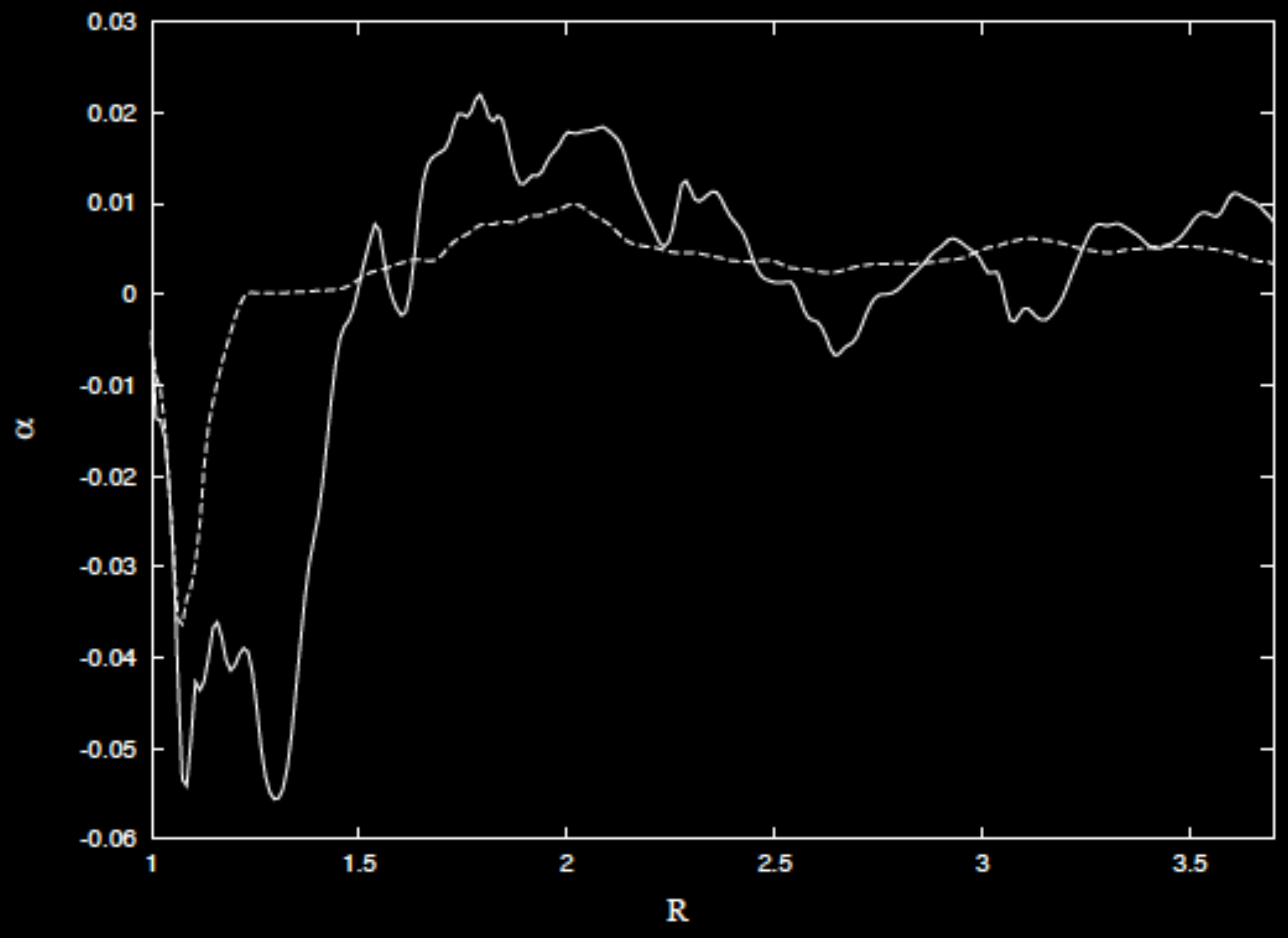

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Some Insight from a Minimal Approach

- Standard incompressible shearing box approximation
- Consider a single mode $\delta v_i, \delta b_i \sim e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$

  (non-linear terms drop out, e.g., Goodman & Xu 1994)
- Not just linearized equations!
- Isolate modes unrelated to MRI (kz = 0) [shear flow!!!]
- Explore the evolution of energy and stress as a function of the local shear parameter "q"

Equations I.

- The momentum and induction equations

$$d_t\hat{\boldsymbol{u}} - q\,\Omega_0\hat{u}_x\check{\boldsymbol{y}} = i\omega_{\mathrm{A}}\hat{\boldsymbol{b}} - \nu k^2\hat{\boldsymbol{u}} - 2\Omega_0\check{\boldsymbol{z}}\times\hat{\boldsymbol{u}} - i\boldsymbol{k}\hat{P},$$

$$d_t\hat{\boldsymbol{b}} + q\,\Omega_0\hat{b}_x\check{\boldsymbol{y}} = i\omega_{\mathrm{A}}\hat{\boldsymbol{u}} - \eta k^2\hat{\boldsymbol{b}}.$$

- Note that $\omega_{\mathrm{A}} \equiv \boldsymbol{B}(t)\cdot\boldsymbol{k}(t) = \mathrm{constant}$

  because $\partial_t\boldsymbol{B}_0 = -q\,\Omega_0 B_{0x}\check{\boldsymbol{y}}$

  $$\boldsymbol{k}'\cdot\boldsymbol{x}' = \boldsymbol{k}(t)\cdot\boldsymbol{x} = (k'_x + q\,\Omega_0 t k'_y)x + k'_y y$$

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Equations II.

- Define dimensionless quantities

$$\tau \equiv k_x(t)/k_y = q\,\Omega_0 t \qquad \omega \equiv \omega_{\mathrm{A}}/q\,\Omega_0$$

- The system of equations reduces to

$$\frac{d}{d\tau}\begin{bmatrix} \hat{u}_x \\ \hat{b}_x \end{bmatrix} = \begin{bmatrix} -\Gamma(\tau) & i\omega \\ i\omega & 0 \end{bmatrix}\begin{bmatrix} \hat{u}_x \\ \hat{b}_x \end{bmatrix}$$

$$\Gamma(\tau) \equiv 2\tau/(\tau^2+1)$$

Results

- Reduce the coupled equations to one 2nd order ODE

$$\frac{d^2}{d\tau^2}\hat{b}_x + \Gamma(\tau)\frac{d}{d\tau}\hat{b}_x + \omega^2\hat{b}_x = 0$$

- For $\tau^2 \gg 1$, we get a spherical Bessel equation

$$\hat{u}_x = \quad Sj_1(\omega\tau) + \ Cy_1(\omega\tau)$$

$$\hat{b}_x = -iSj_0(\omega\tau) - iCy_0(\omega\tau)$$

$$\hat{u}_y = -\tau\hat{u}_x$$

$$\hat{u}_y = -\tau\hat{b}_x$$

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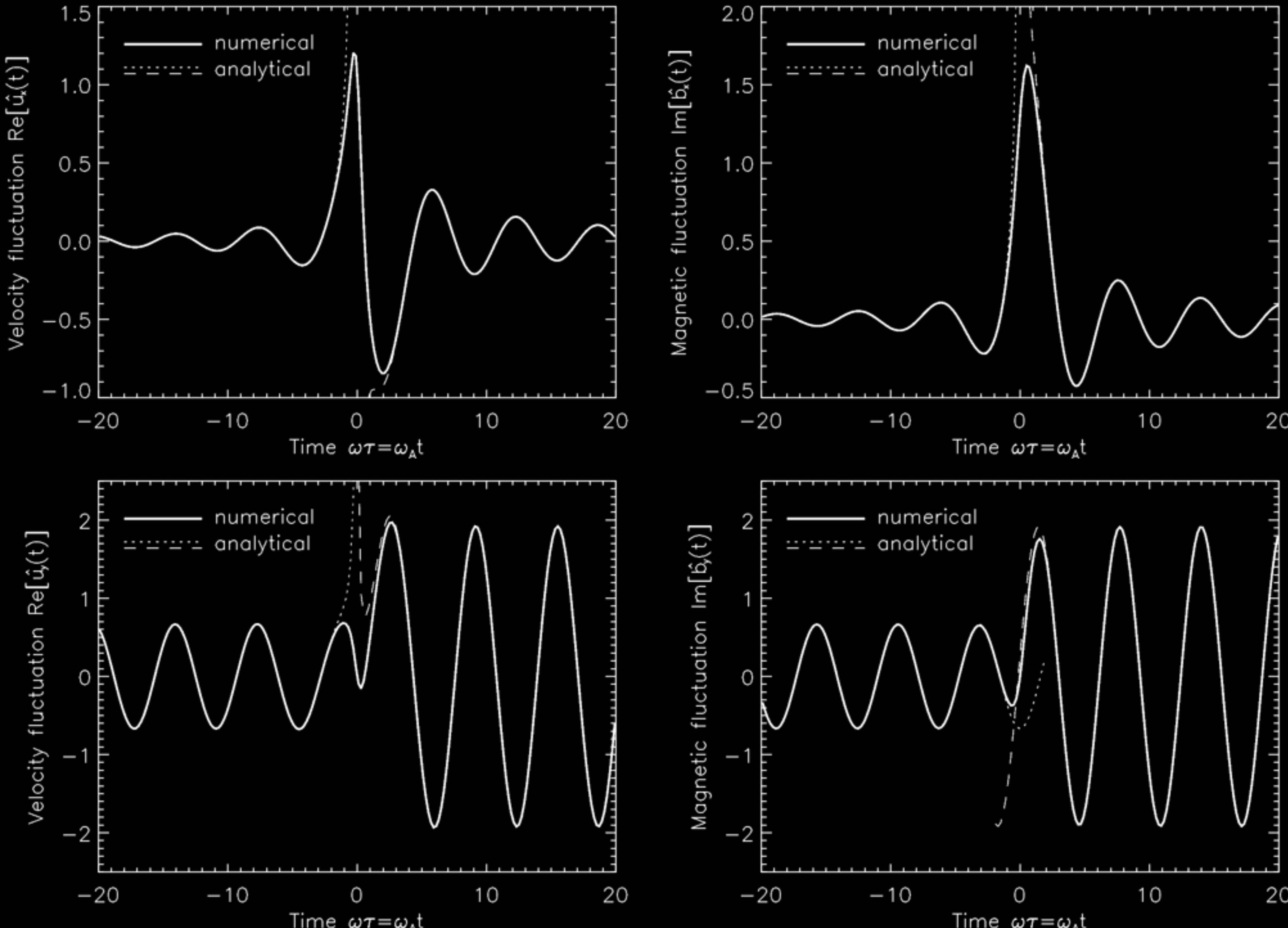
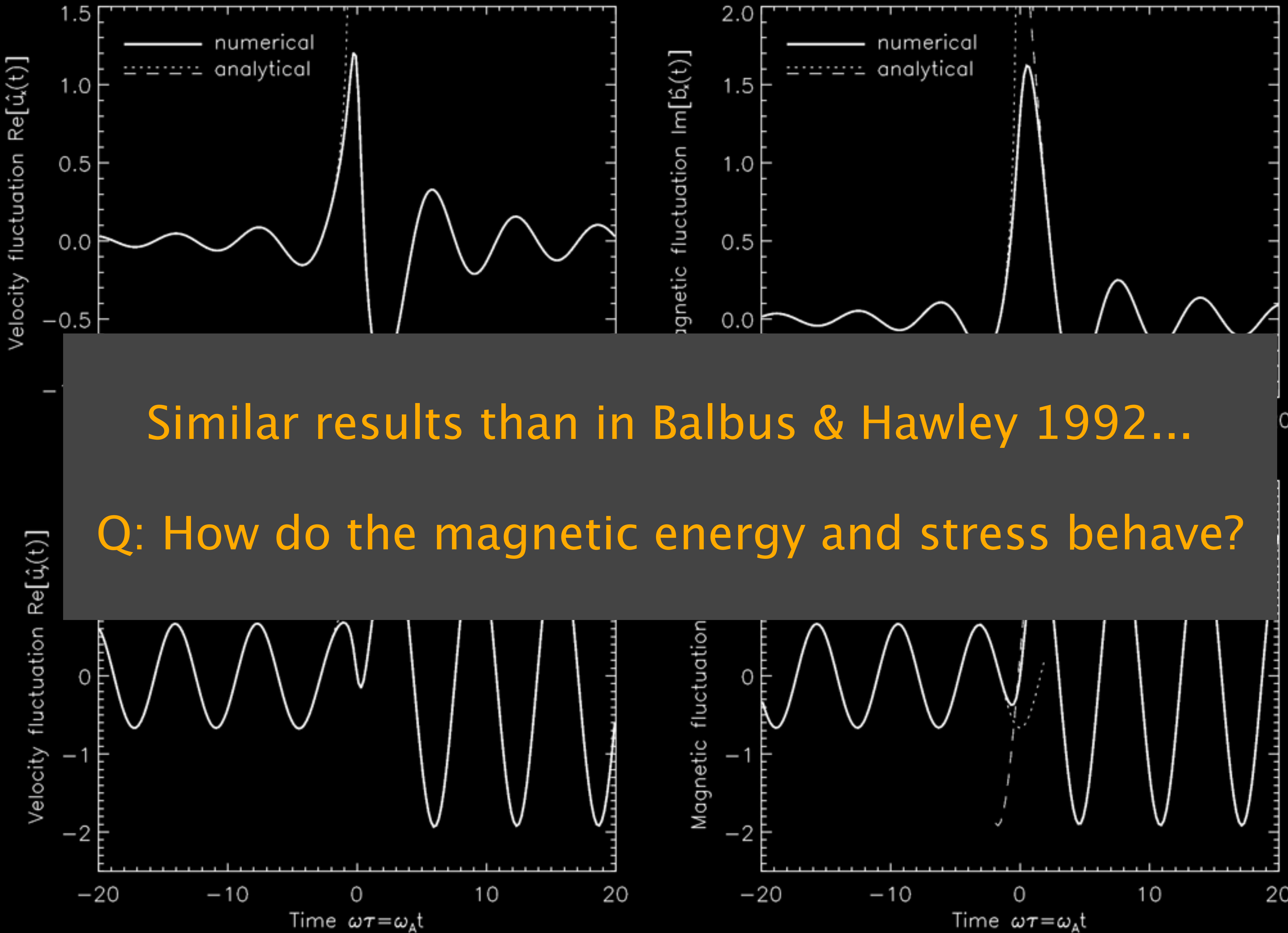


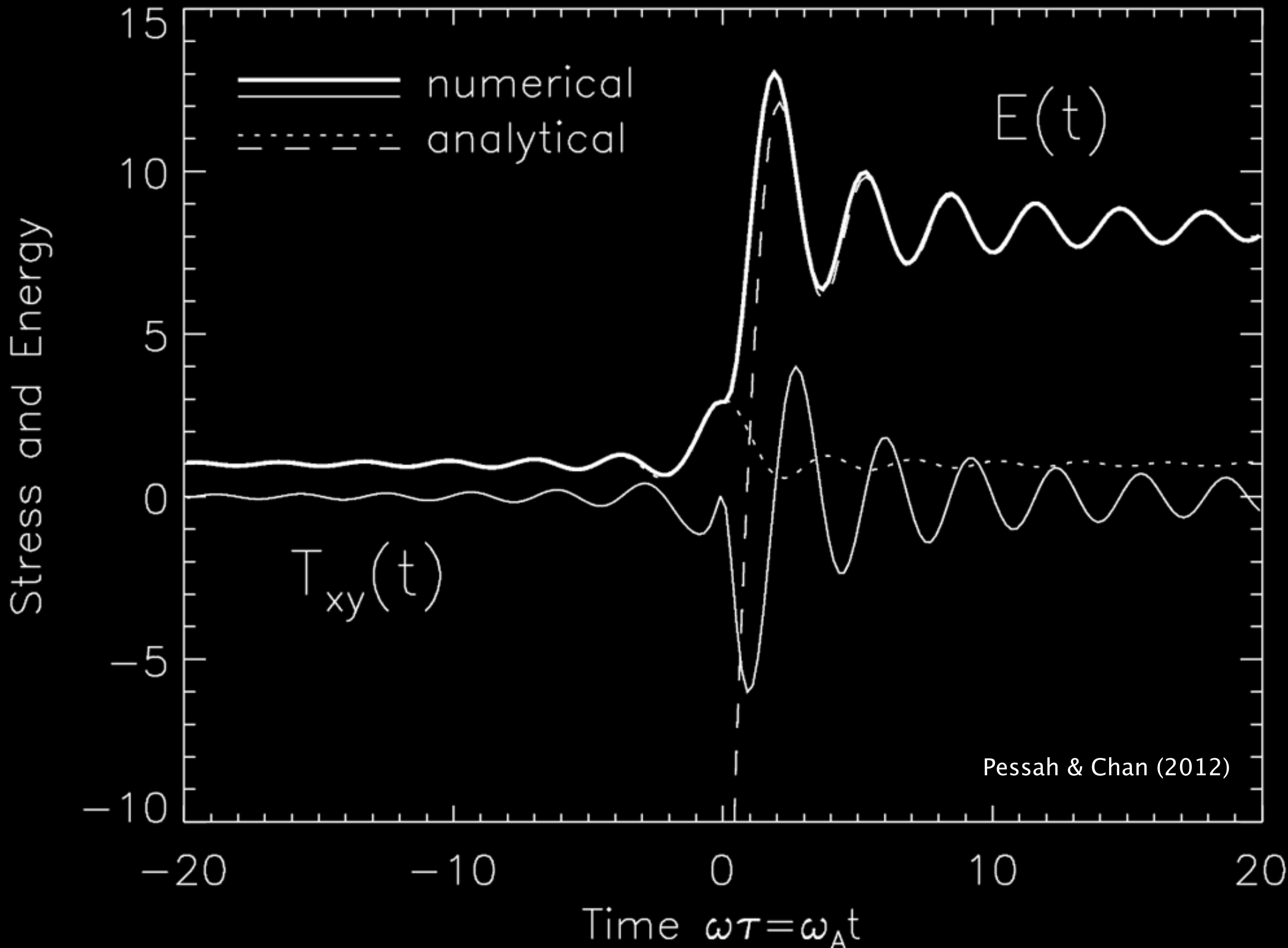
Pessah & Chan (2012)



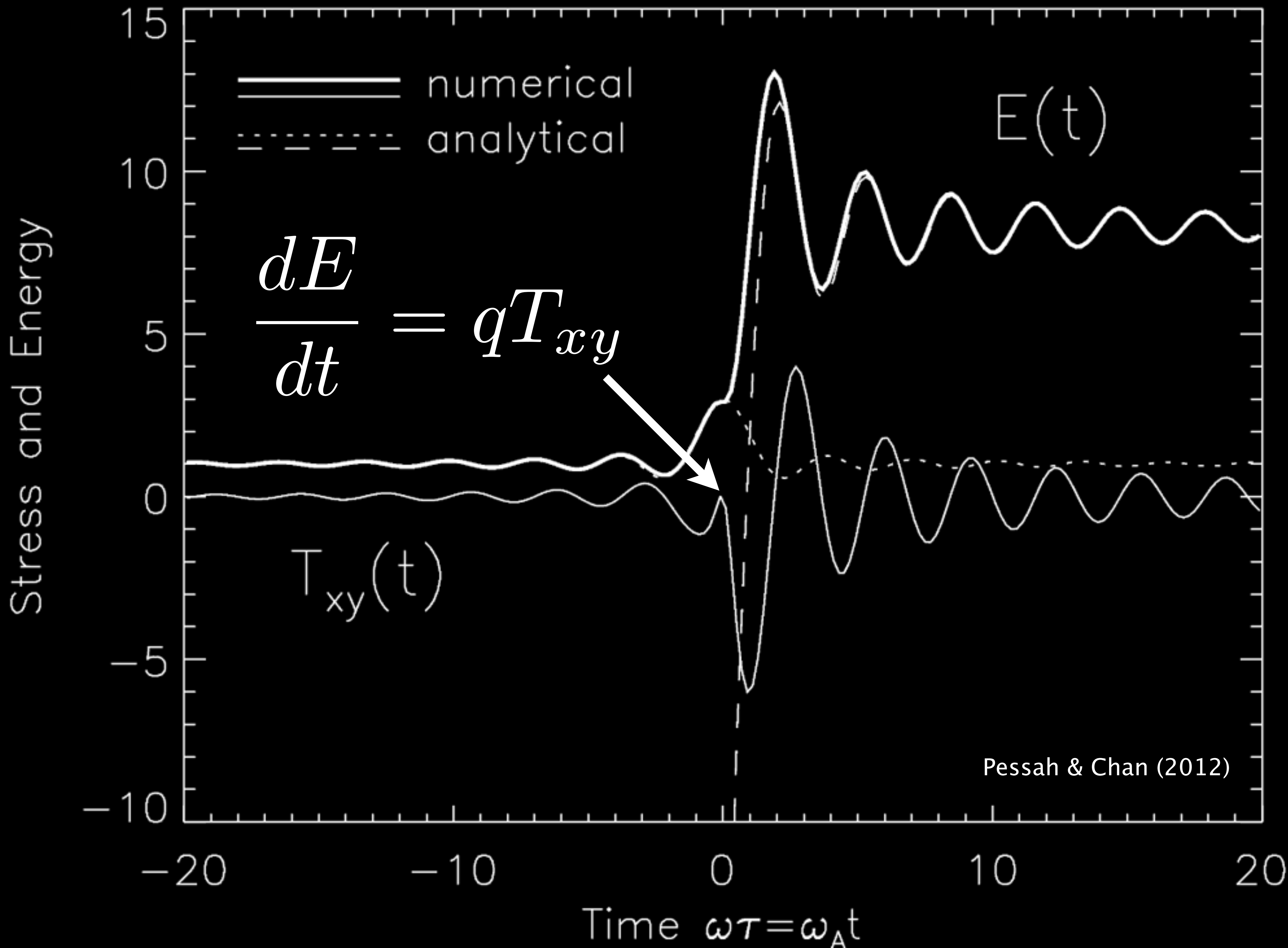
Similar results than in Balbus & Hawley 1992...

Q: How do the magnetic energy and stress behave?

Pessah & Chan (2012)



15
10
5
0
−5
−10
−20
−10
0
10
20
numerical
analytical
E(t)
T_xy(t)
Stress and Energy
Time ωτ=ω_A t
Pessah & Chan (2012)



numerical
analytical
E(t)
$$\frac{dE}{dt} = qT_{xy}$$
$T_{xy}(t)$
Stress and Energy
Time $\omega\tau = \omega_A t$
15
10
5
0
−5
−10
−20
−10
0
10
20
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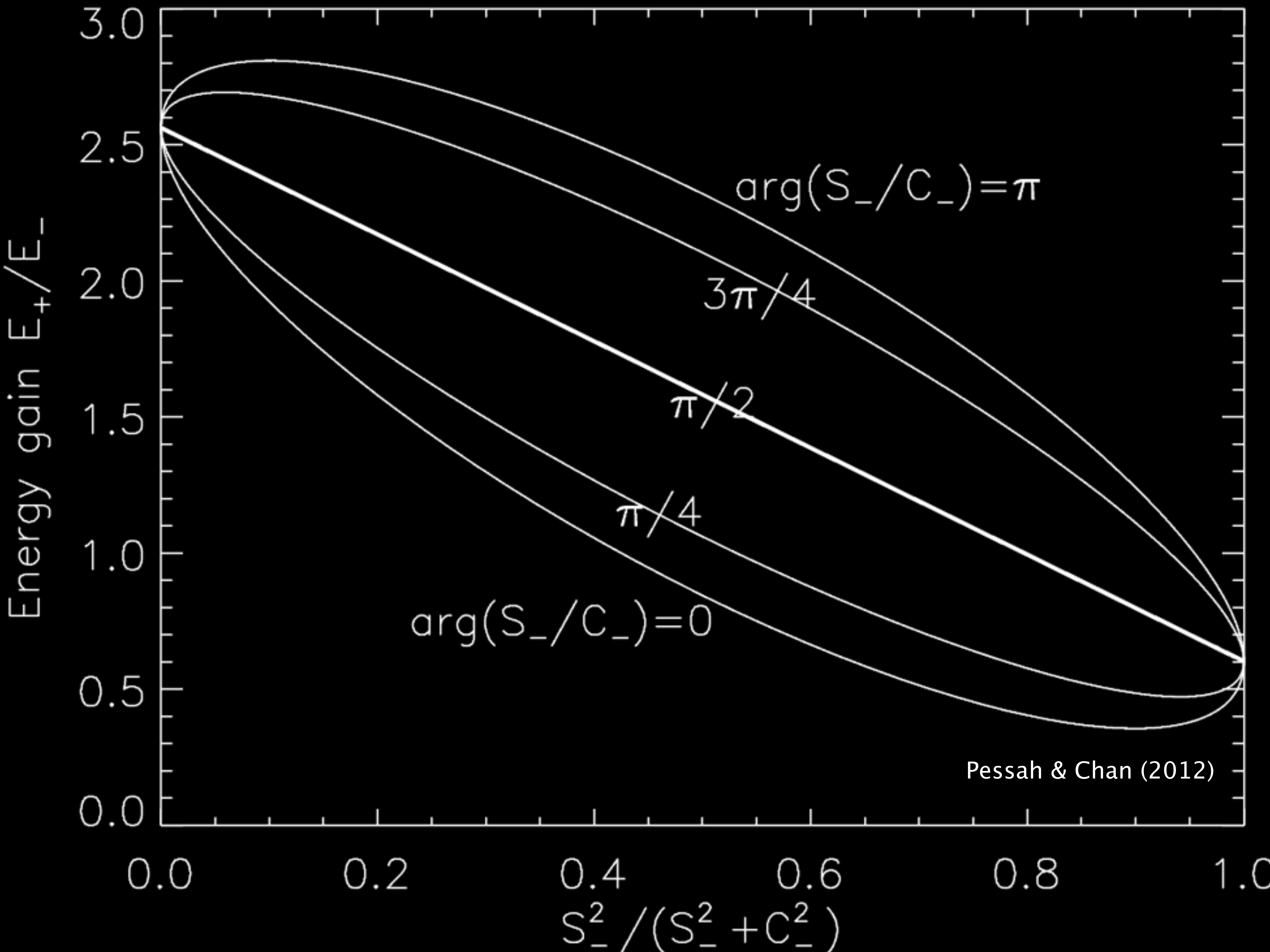
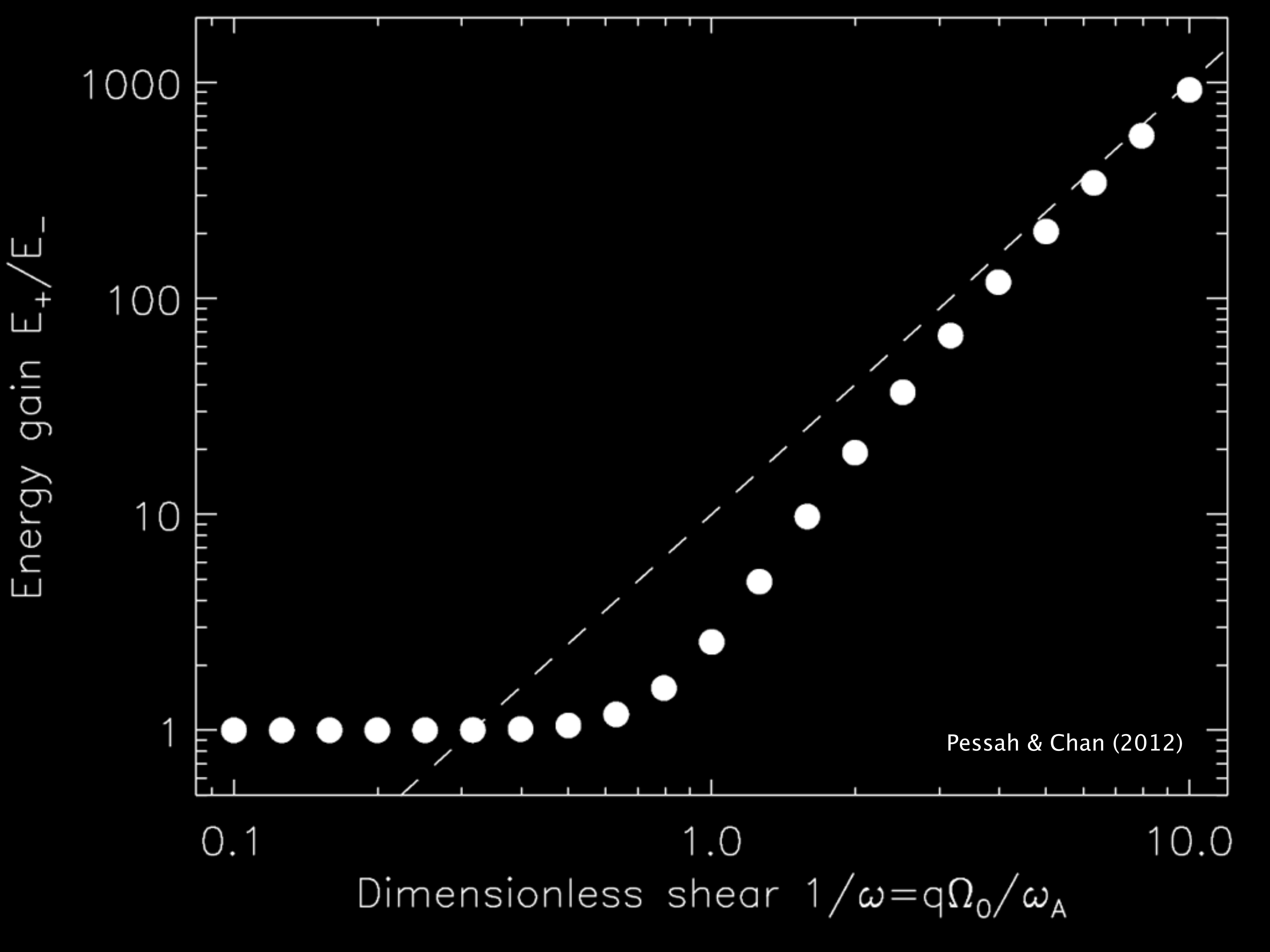
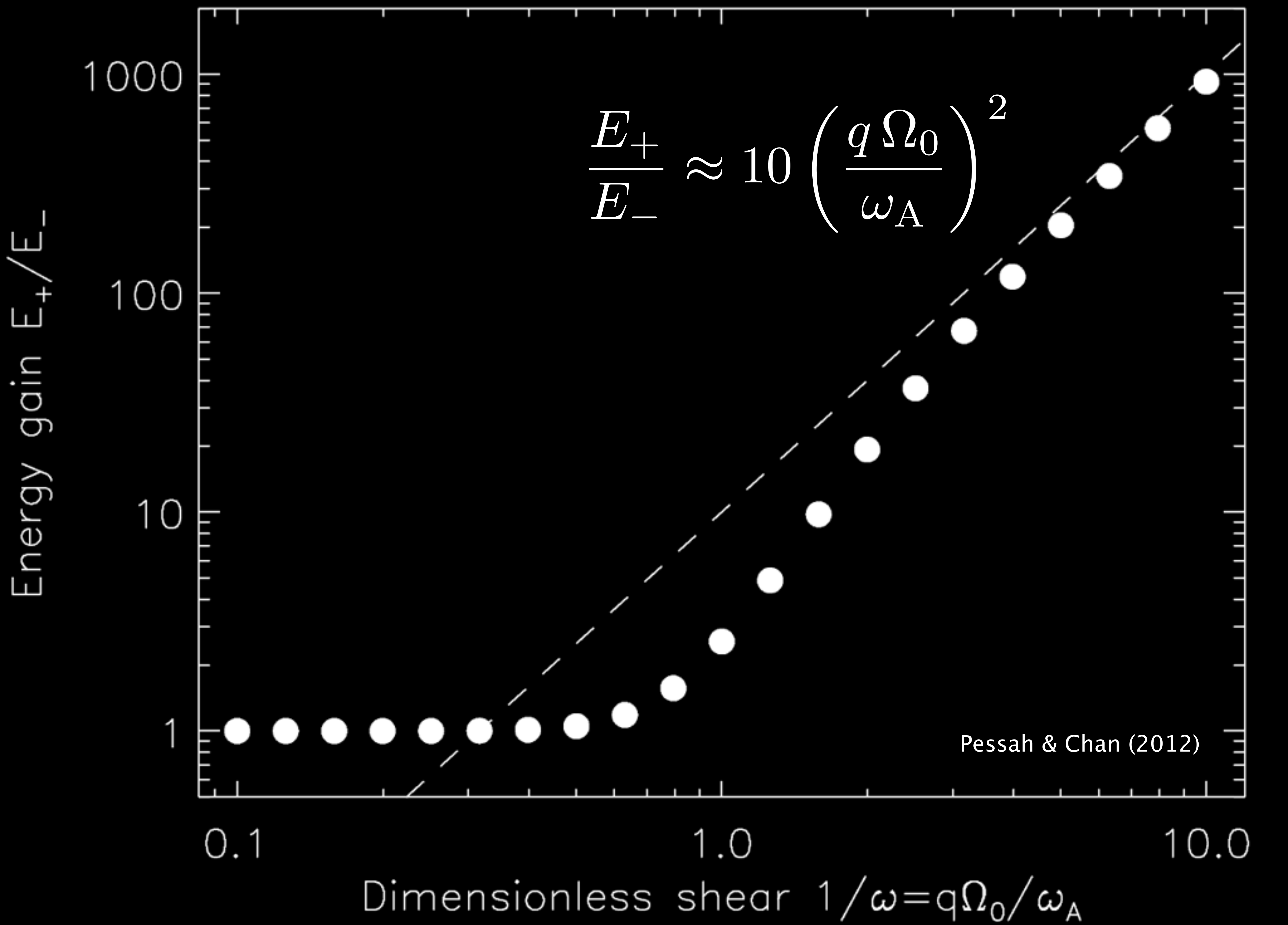


3.0
2.5
2.0
1.5
1.0
0.5
0.0
Energy gain E+/E-
arg(S_/C_)=π
3π/4
π/2
π/4
arg(S_/C_)=0
Pessah & Chan (2012)
0.0
0.2
0.4
0.6
0.8
1.0
S²_/(S²_+C²_)


1000
100
10
1
0.1
1.0
10.0
Energy gain E_+/E_-
Dimensionless shear $1/\omega = q\Omega_0/\omega_A$
Pessah & Chan (2012)



1000
100
10
1
0.1
1.0
10.0
Energy gain E_+/E_-
Dimensionless shear $1/\omega = q\Omega_0/\omega_A$
$$\frac{E_+}{E_-} \approx 10 \left(\frac{q\,\Omega_0}{\omega_\mathrm{A}}\right)^2$$
Pessah & Chan (2012)

Conclusions & Pending Issues

- It is possible to generate magnetic energy without generating much stress in the boundary layer
- The energy gain is described by $$\frac{E_+}{E_-} \approx 10 \left(\frac{q\,\Omega_0}{\omega_{\rm A}} \right)^2$$
- "Consistent" with simulations
- Something important remains to be understood!
- What is the mechanism for angular momentum transport when the MRI is not active? Belyaev, Rafikov, & Stone, 2012
- How does this translate into a model for $T_{r\phi}(d\Omega/dr)$?