Structure and evolution of 3-dimensional Rossby vortices

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Summary

- The system of equations
- Vortex formation by Rossby Wave Instability
- 3D structure of the resulting vortex
- Long term evolution
- Conclusion

The system of equations

$$\frac{\partial\rho}{\partial t}+\frac{1}{r}\frac{\partial r\rho u}{\partial r}+\frac{1}{r}\frac{\partial\rho v}{\partial\theta}+\frac{\partial\rho w}{\partial z}=0$$

$$\frac{\partial\rho u}{\partial t}+\frac{1}{r}\frac{\partial r(\rho u^2+P)}{\partial r}+\frac{1}{r}\frac{\partial\rho uv}{\partial\theta}+\frac{\partial\rho uw}{\partial z}=\frac{\rho v^2}{r}-\frac{\rho GMr}{(r^2+z^2)^{3/2}}+\frac{P}{r}$$

$$\frac{\partial\rho v}{\partial t}+\frac{1}{r}\frac{\partial r\rho uv}{\partial r}+\frac{1}{r}\frac{\partial\rho v^2+P}{\partial\theta}+\frac{\partial\rho vw}{\partial z}=-\frac{\rho uv}{r}$$

$$\frac{\partial\rho w}{\partial t}+\frac{1}{r}\frac{\partial r\rho uw}{\partial r}+\frac{1}{r}\frac{\partial\rho vw}{\partial\theta}+\frac{\partial\rho w^2+P}{\partial z}=-\frac{\rho GMz}{(r^2+z^2)^{3/2}}$$

$$\frac{\partial\rho e}{\partial t}+\frac{1}{r}\frac{\partial r(\rho e+P)u}{\partial r}+\frac{1}{r}\frac{\partial(\rho e+P)v}{\partial\theta}+\frac{\partial(\rho e+P)w}{\partial z}=-\rho u\frac{GMr}{(r^2+z^2)^{3/2}}-\rho w\frac{GMz}{(r^2+z^2)^{3/2}}$$

$$\rho e=\frac{P}{\gamma-1}+\frac{1}{2}\rho(u^2+v^2+w^2)$$

Solved numerically with a finite volume method using a second order MUSCL-Hancock scheme. The code is based on a 2D version specifically developed for disk studies (*Inaba et al, 2001);* it is parallelized to permit long and high resolution runs.

Initial condition

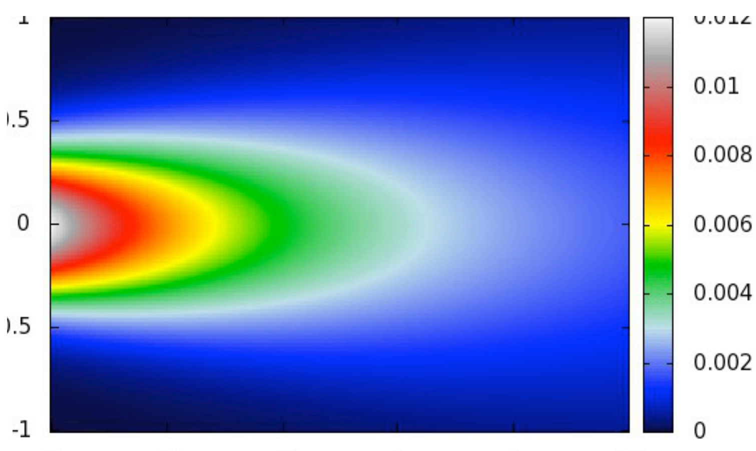
- Stable equilibrium state

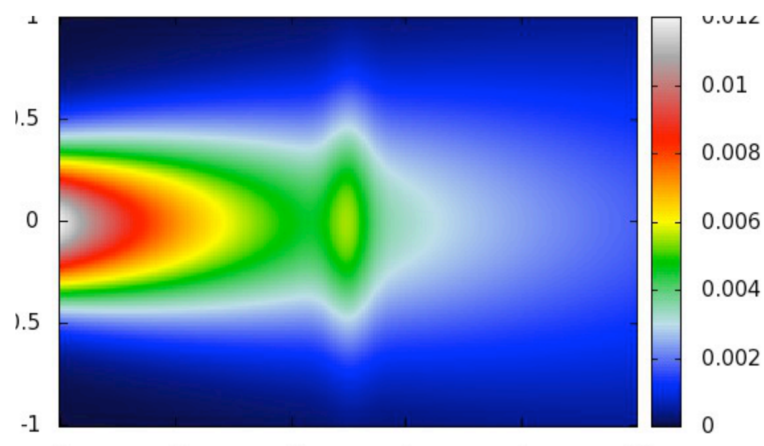
[redacted]

- Perturbed equilibriium state

[redacted]

+ noise





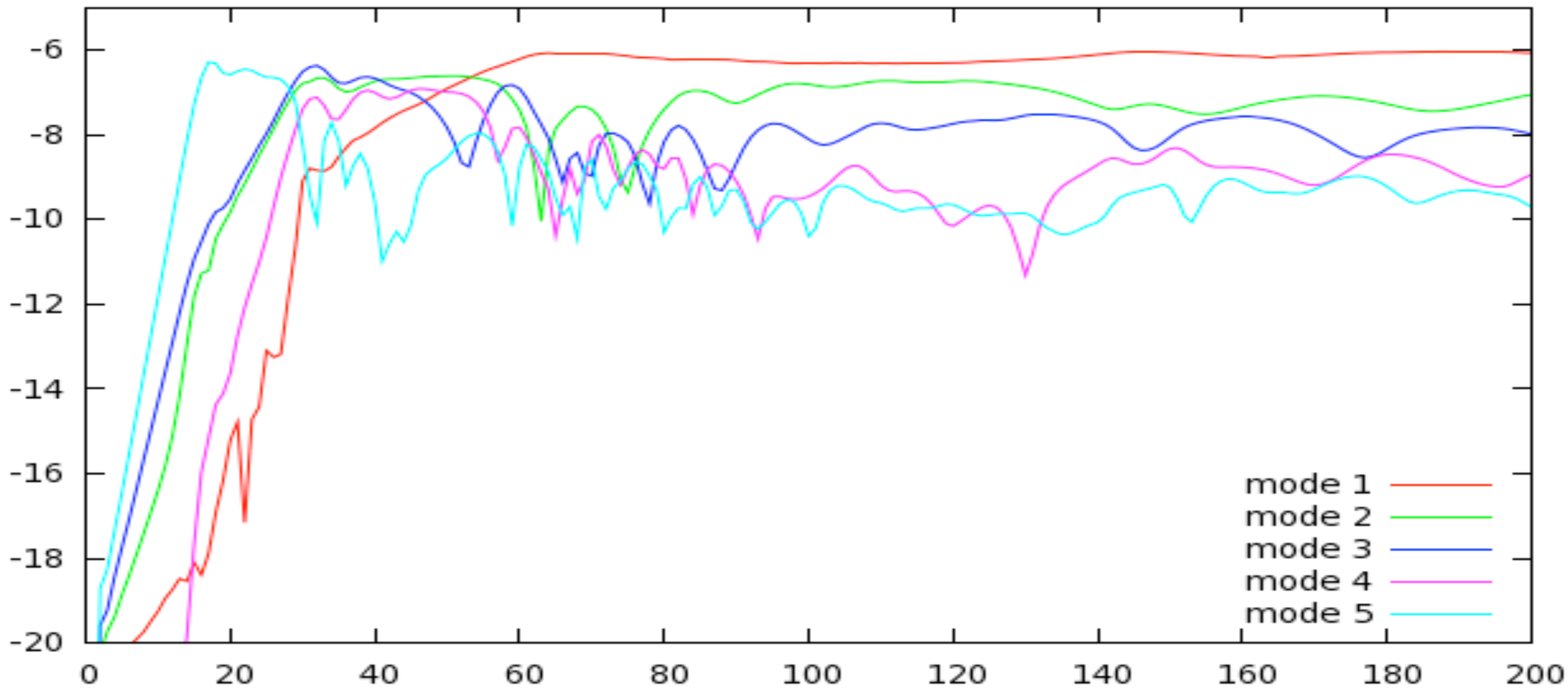
- Radial and vertical equilibrium between centrifugal force, gravity and pressure gradient are satisfied

Rossby wave Instability: 3D evolution

- Initial condition : background equilibrium + gaussian bump in density and pressure + small perturbation
- Growth of vortices with spiral waves

- Gradual merging of the vortices (mode number decreases)
- At the end: a single quasi-steady vortex that migrates toward the center in a slow time scale

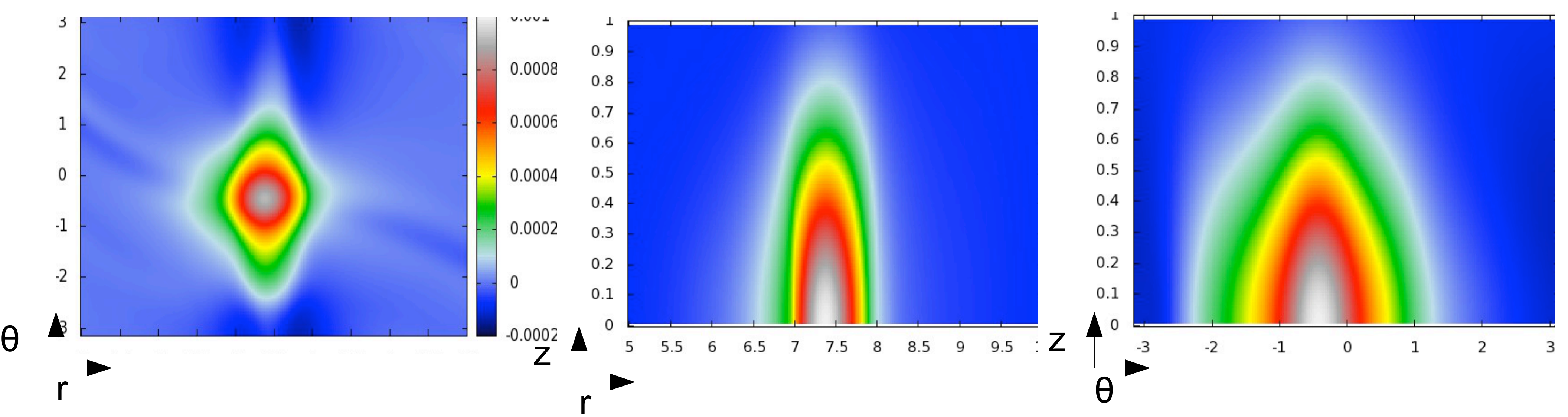
Grow of the instability



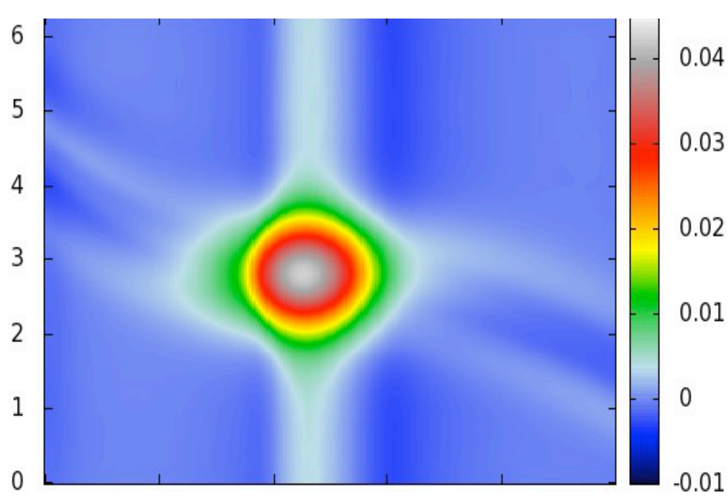
- Beginning is dominate by high mode
- Dominant mode decrease until the mode 1 is the dominant
- Amplitude of mode 1 doesn't decrease after saturation

Density once a single vortex is formed

3D



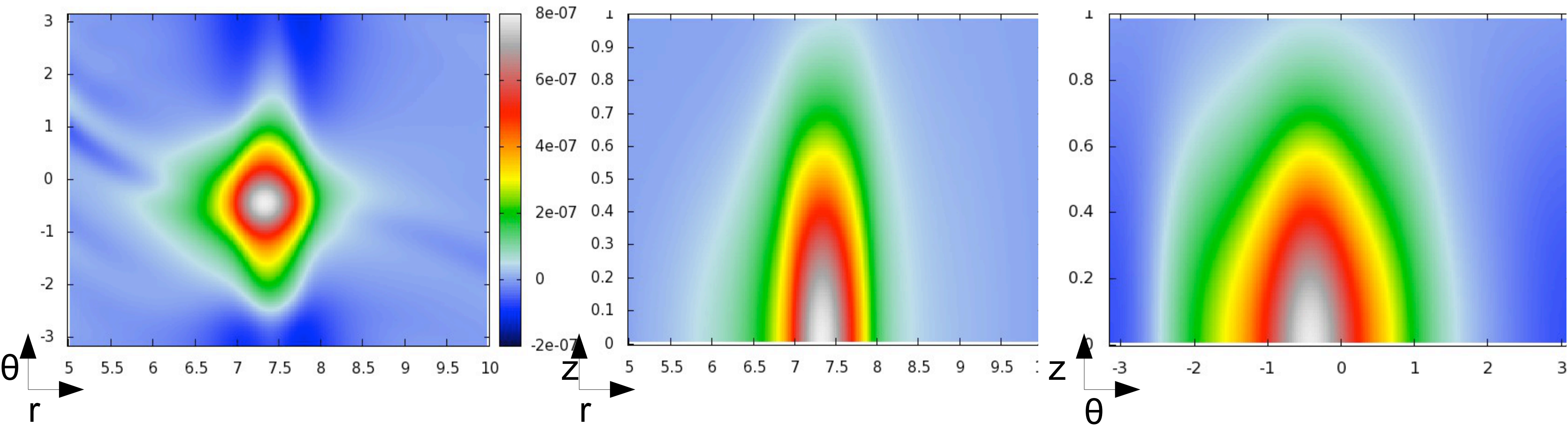
2D



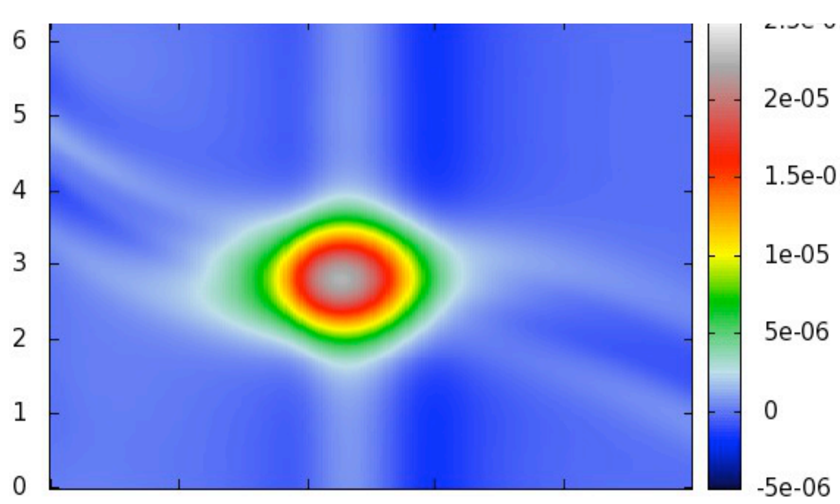
- In the mid-plane: the 3D density map looks like the 2D one
- The vertical profile is nearly gaussian

Pressure profile

3D



2D



- Pressure and density profiles have similar shapes

Vertical equilibrium

- Pressure and density have similar profile, so temperature is not modified

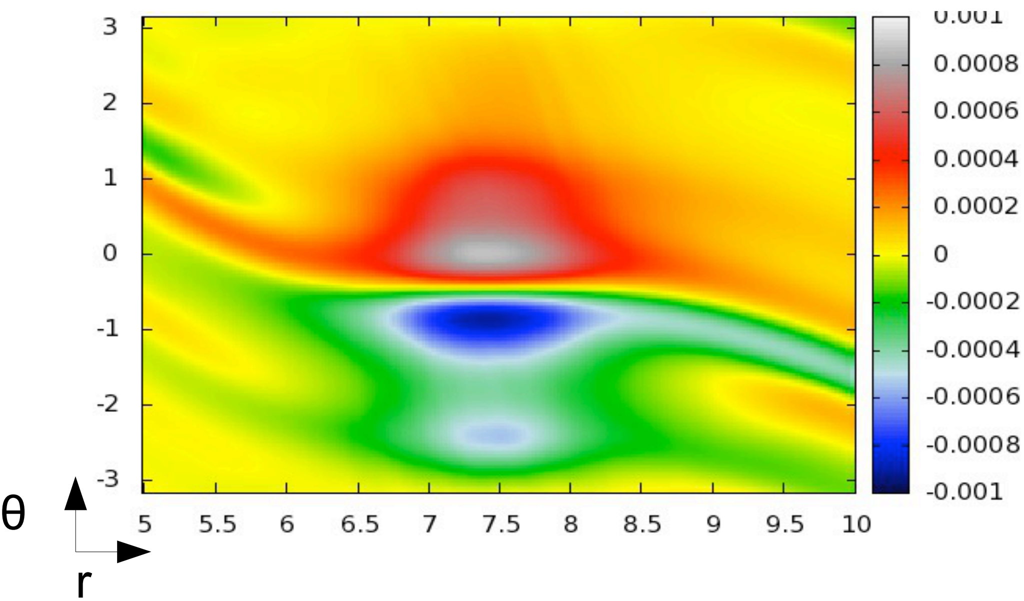
- Vertical hydrostatic equilibrium reads :

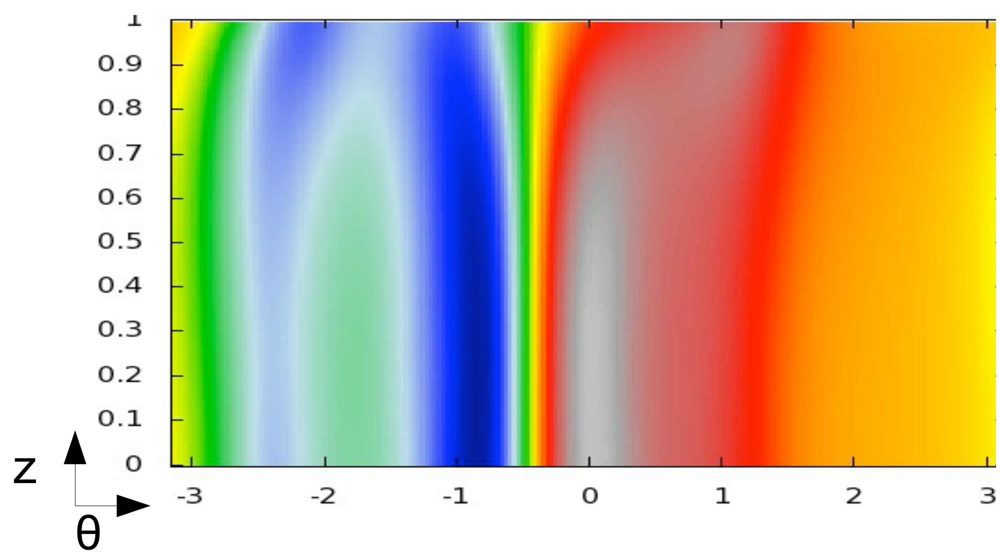
[redacted]

- As temperature doesn't change, the vertical equilibrium is not modified

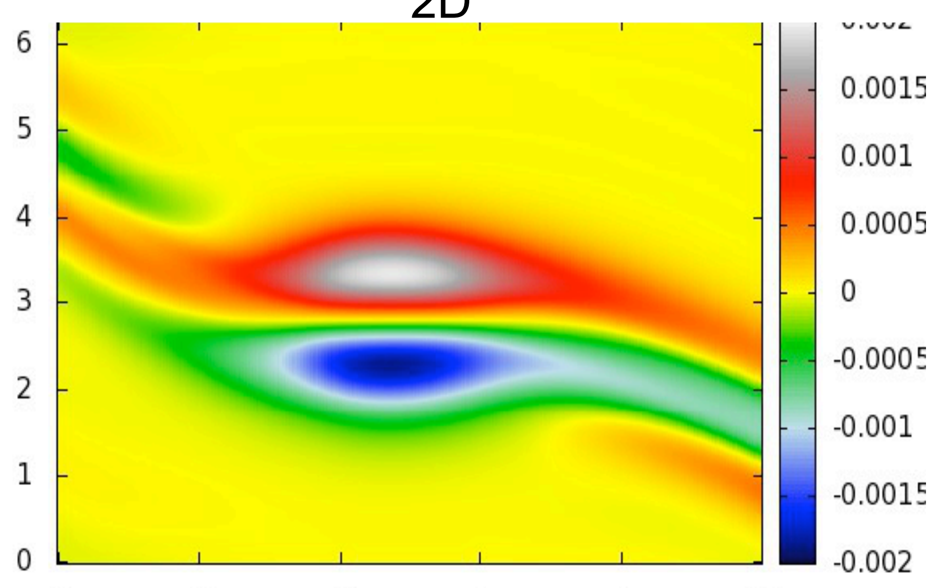
Radial velocity

3D





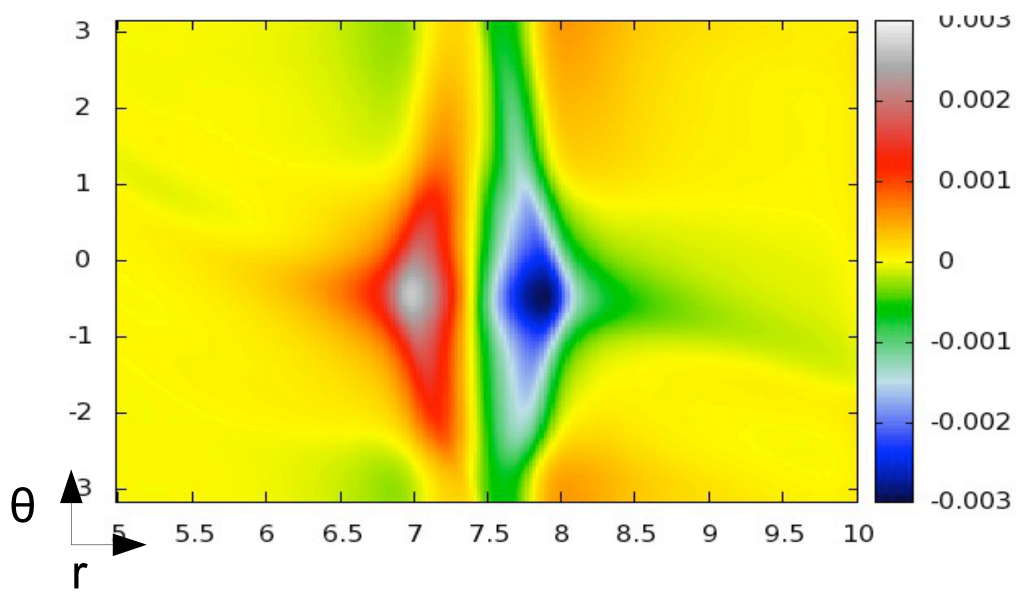
2D

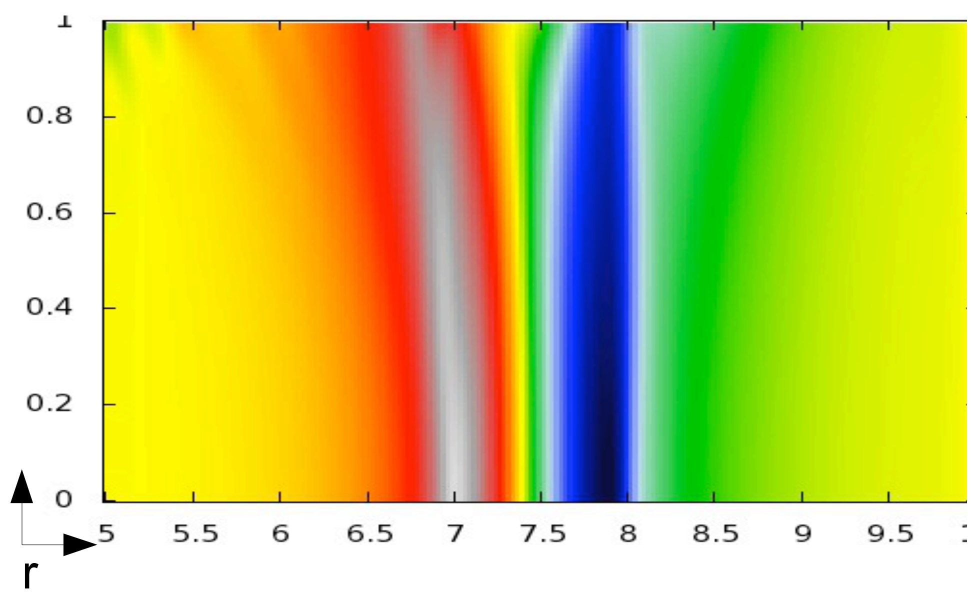


- In the midplane the velocity profile is very similar to the 2D profile
- This profile does not vary strongly with height

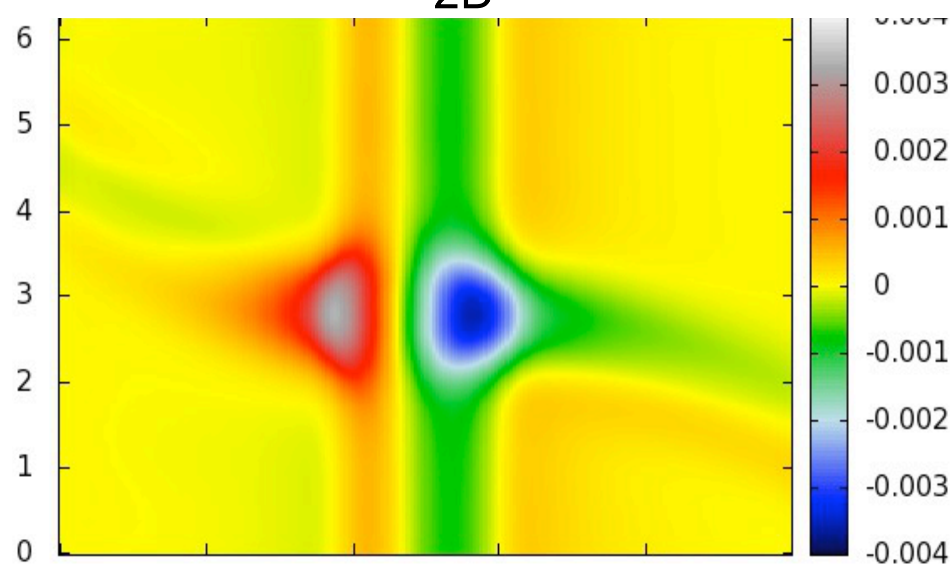
Azimuthal velocity

3D





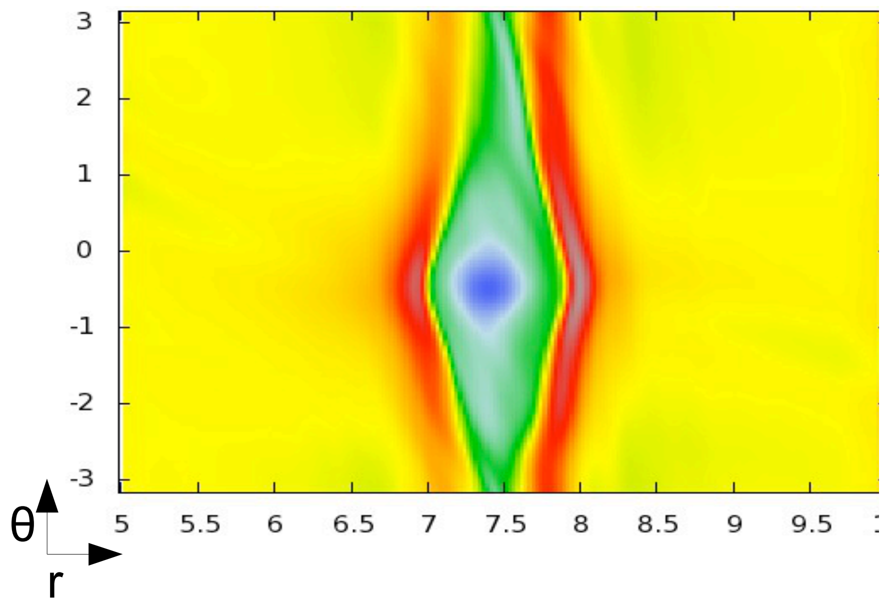
2D

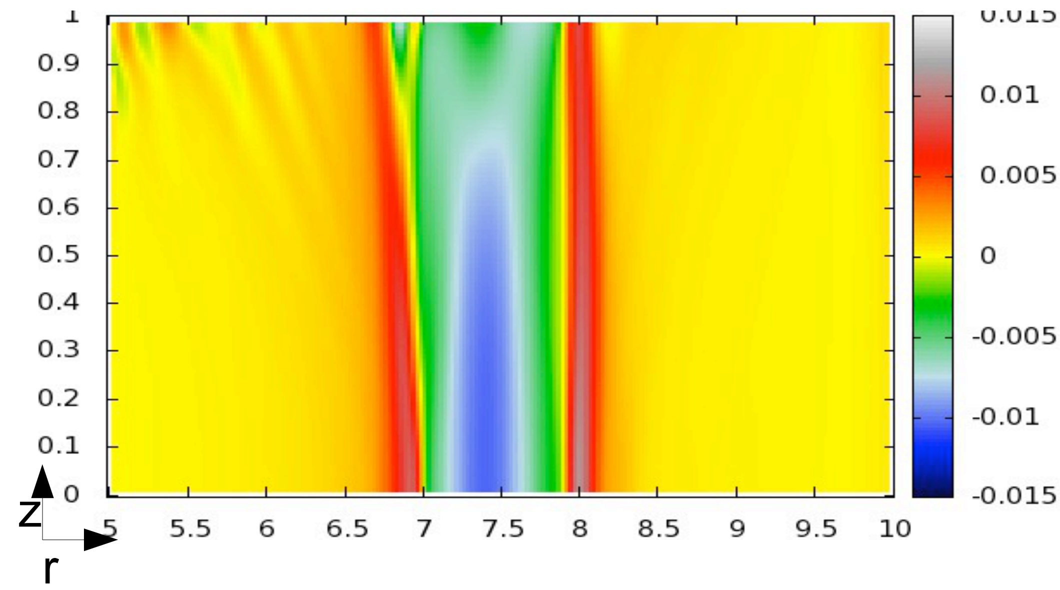


- In the midplane the profile is similar to the 2D one
- The horizontal profile varies weakly with height

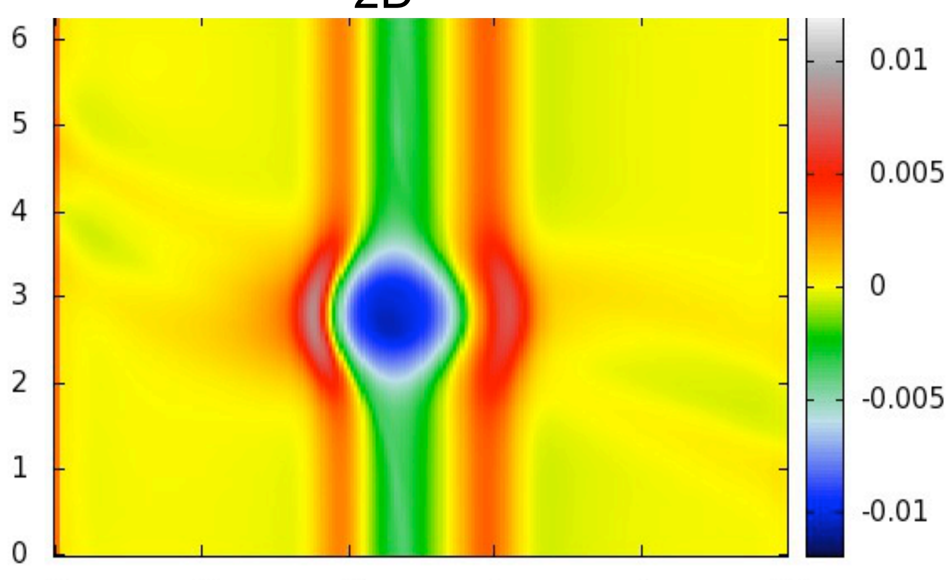
Vertical component of vorticity (ω_z)

3D



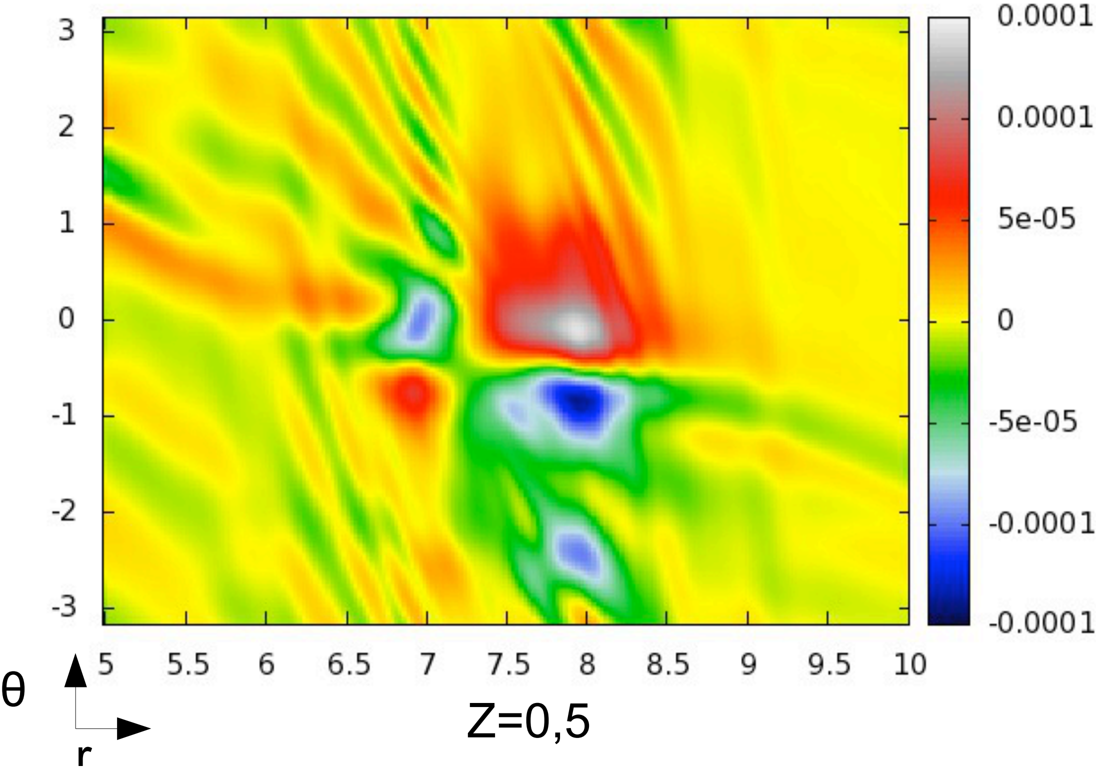


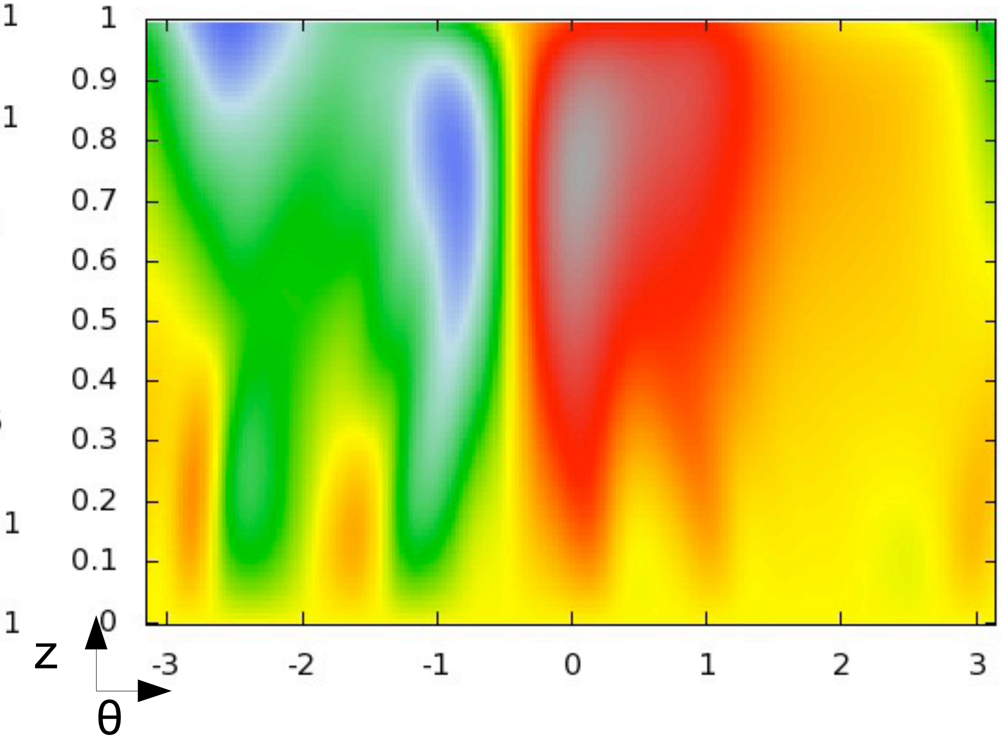
2D

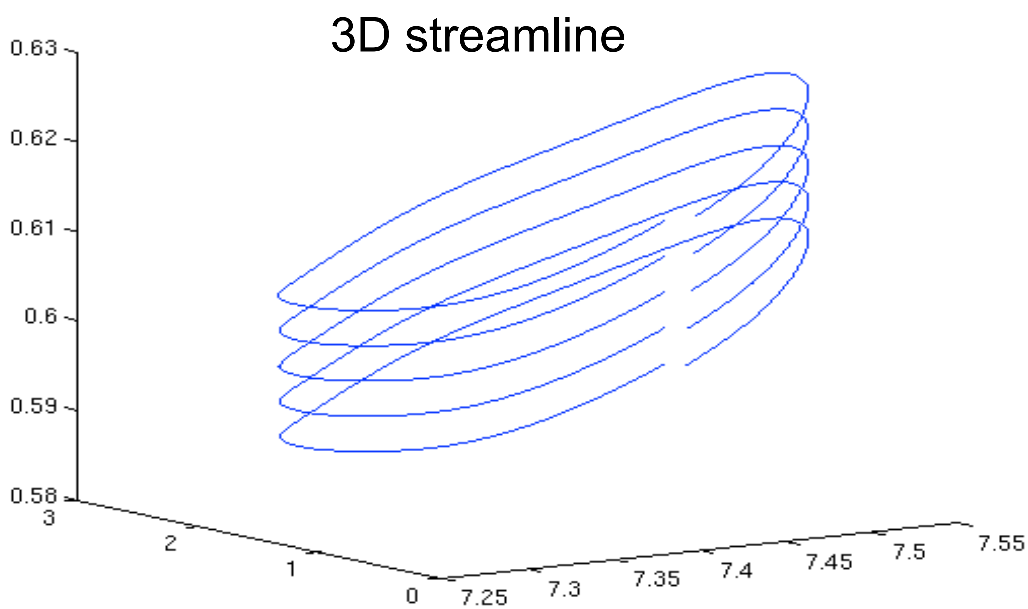


- In the mid-plane 3D vorticity ω_z looks like 2D vorticity
- Vorticity is slightly decreasing with increasing height

Vertical velocity







- The vertical velocity is zero at the equatorial plane
- It increases with height inside the disk
- The vortex follow the height of the disk

Vertical velocity

The vertical velocity comes from the entropy gradient of the disk:

- In the rotating frame, the entropy equation is :

$$\frac{\partial S}{\partial t}+u\frac{\partial S}{\partial r}+\left(\frac{v}{r}-\Omega_0\right)\frac{\partial S}{\partial \theta}+w\frac{\partial S}{\partial z}=0$$

- If Ω_0 is the angular velocity of the vortex, the flow is stationary so

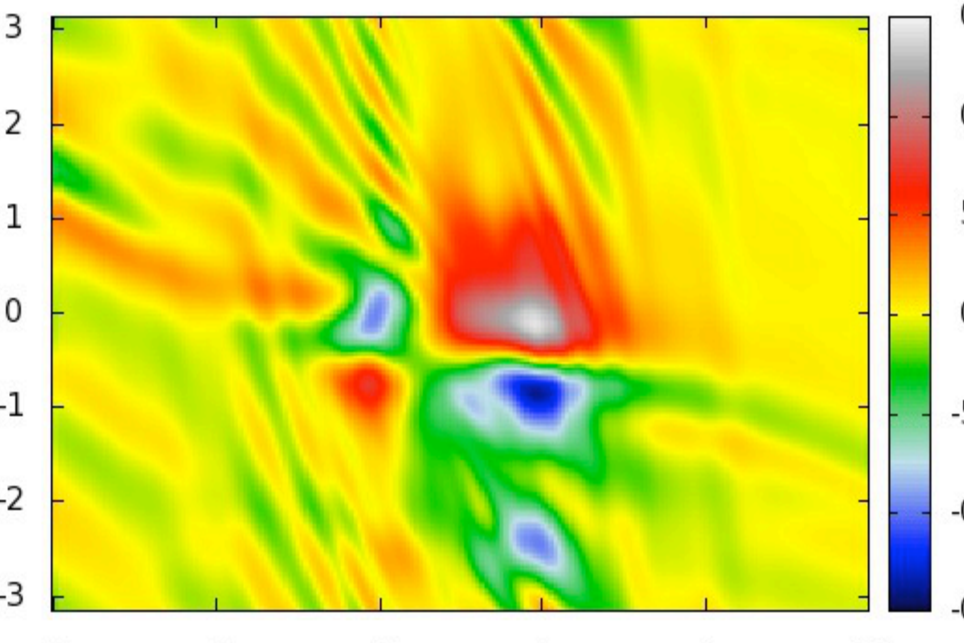
$$\frac{\partial S}{\partial t}=0$$

(i.e) fluid particles stay on surfaces of constant entropy

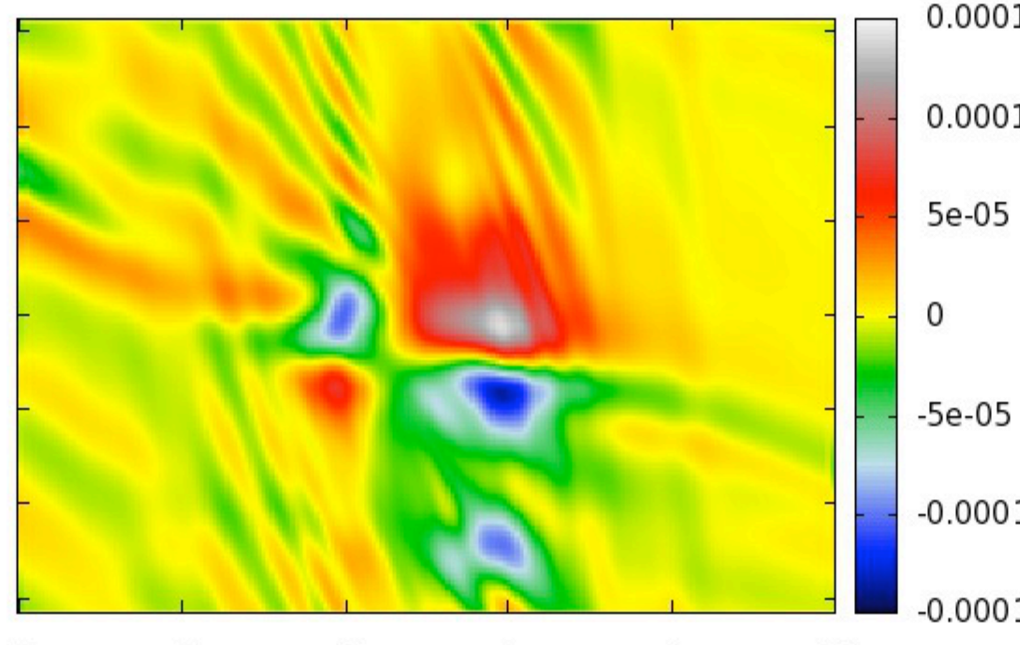
- The vertical velocity reads also :

$$w=-\frac{u\frac{\partial S}{\partial r}+\left(\frac{v}{r}-\Omega_0\right)\frac{\partial S}{\partial \theta}}{\frac{\partial S}{\partial z}}$$

Vertical velocity



w



$$-\frac{u\frac{\partial S}{\partial r}+\left(\frac{v}{r}-\Omega_0\right)\frac{\partial S}{\partial \theta}}{\frac{\partial S}{\partial z}}$$

we check with the numerical values and found good agreement

Role of stratification

- The vertical stratification is given by the Brunt-Väisälä frequency :

[redacted]

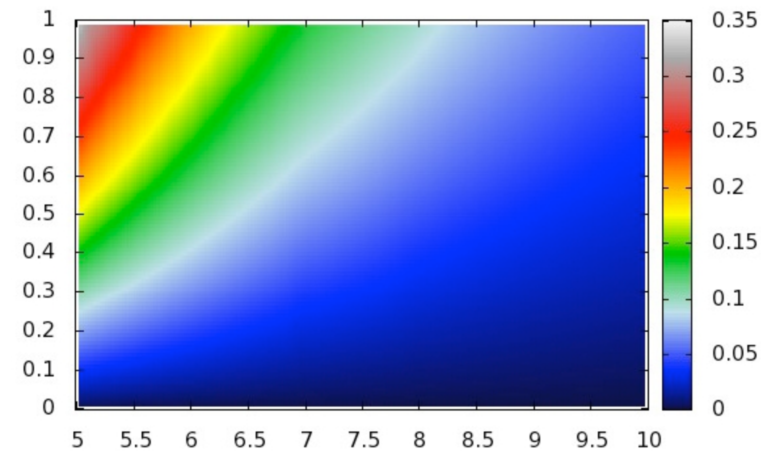
- $N^2>0$ the stratification is stable : when a fluid particle change its altitude, a restoring force appears
- $N^2<0$ the stratification is unstable : when a fluid particle change its altitude, a force which will amplify the motion will appears

Stratification

- Brunt Väisälä in our case :

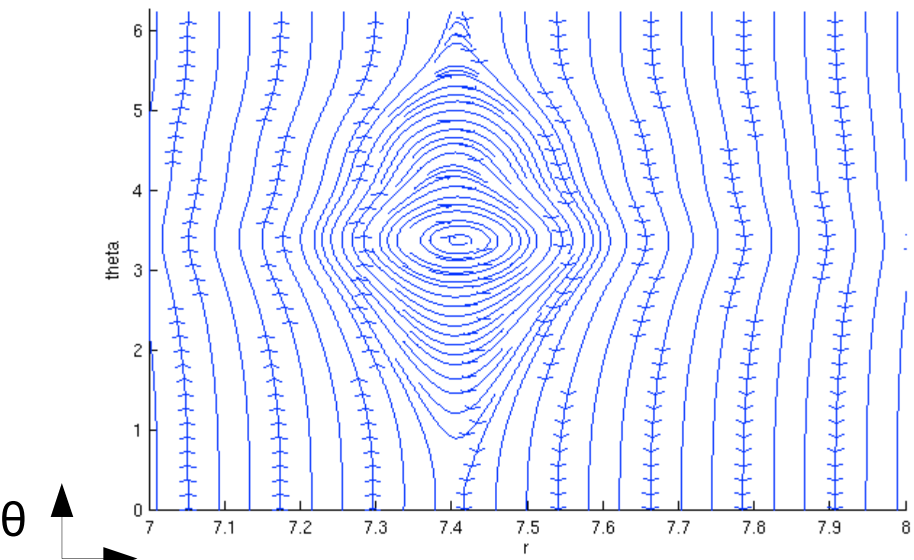
  Always positive :

  stable vertical stratification

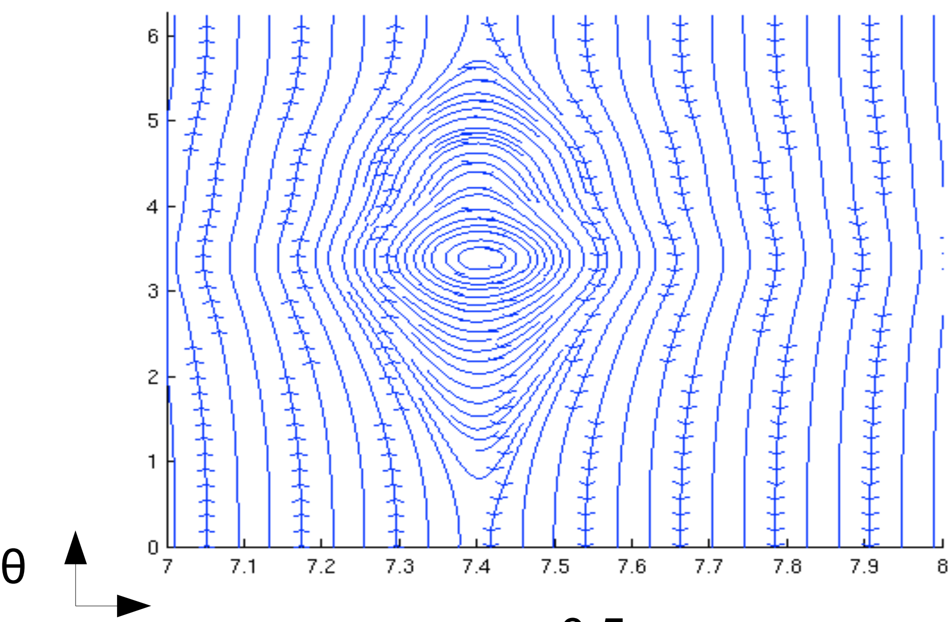


- Fluid particle can't move vertically without be affected by a restoring force, it's why vertical motions are small.

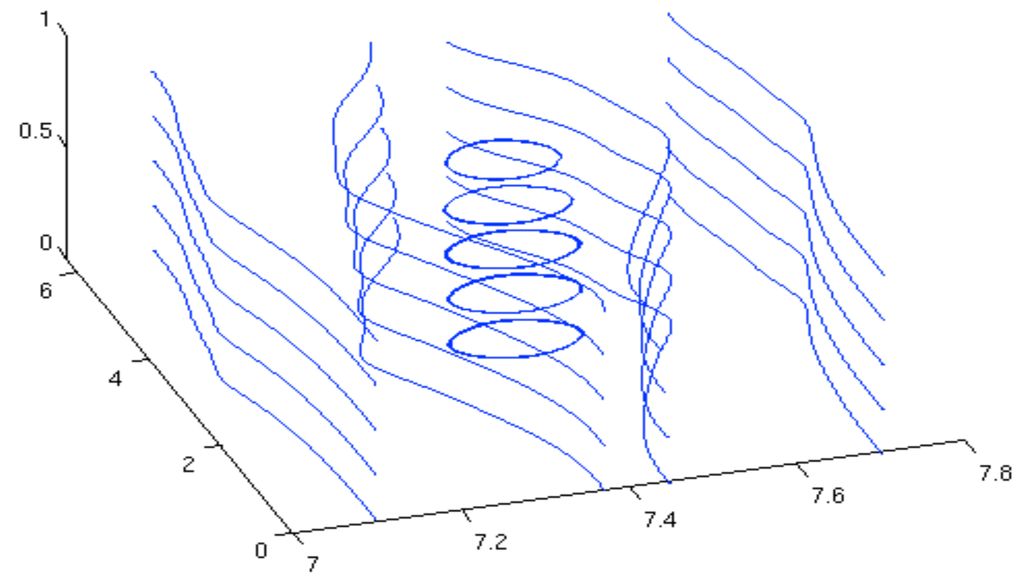
Streamline in the rotating frame



z=0

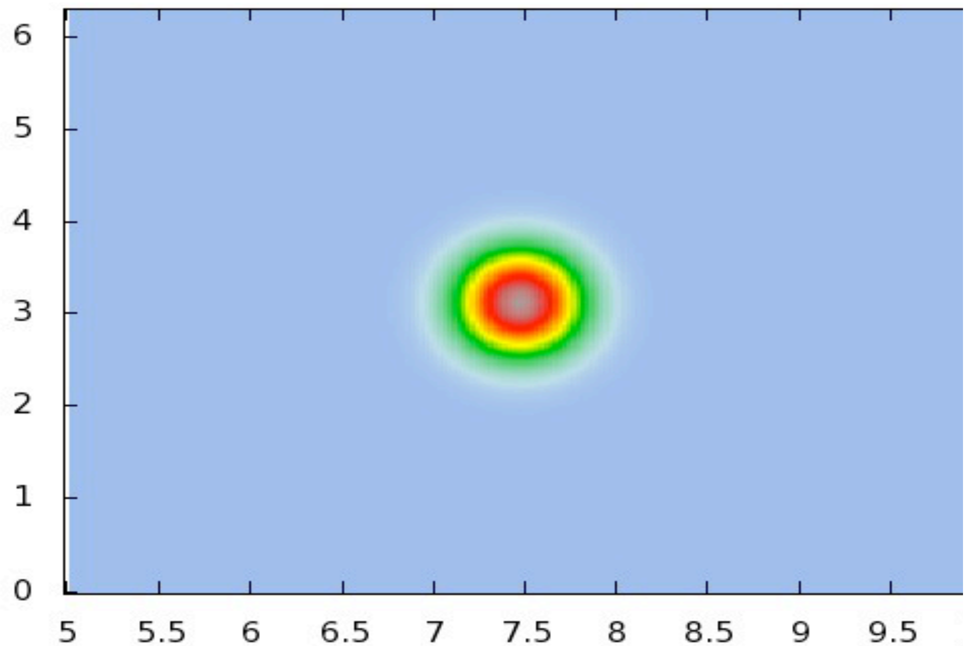


z=0,5

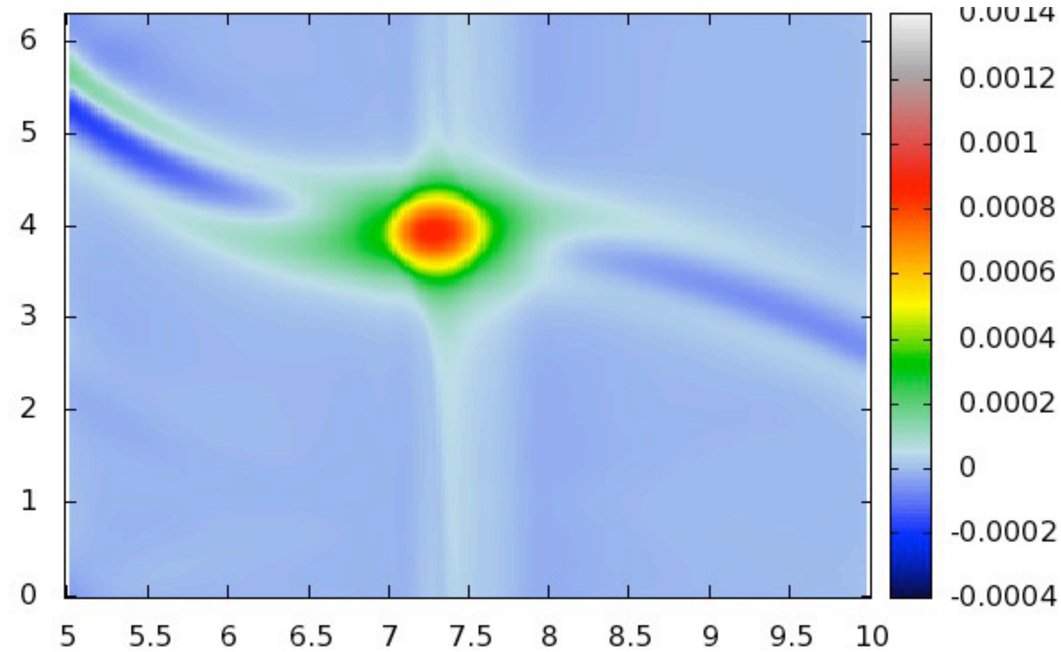


- The streamline are plotted in the rotating frame, when the flow is steady, so the streamline coincide with the trajectory of fluid particles
- The rotation of the gas is mainly in the horizontal plane
- Columnar anticyclonic vortices

Long time evolution



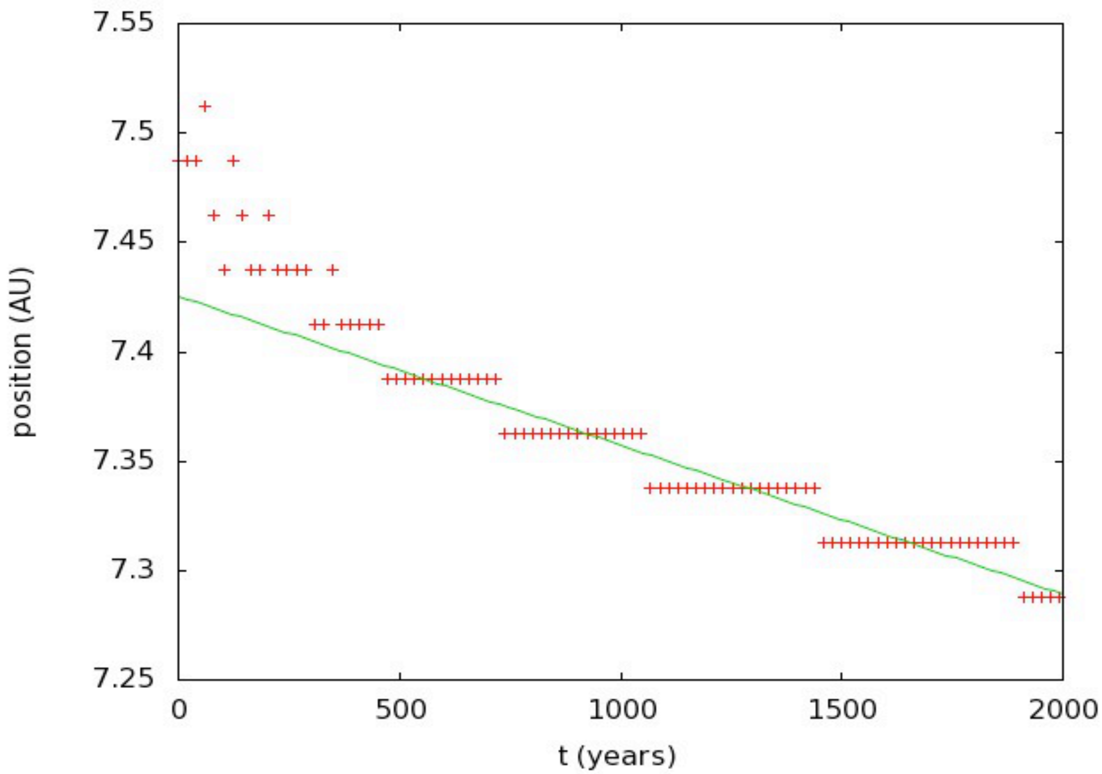
Initial condition : model with gaussian density profile



Density map after 100 rotations

- We start from a vortex model to study its long time evolution
- After a few rotations, it relaxes to a quasi stationary vortex that survives more than 100 rotation periods
- The structure of the vortex looks like the vortex obtained by the Rossby wave instability

Migration of the vortex



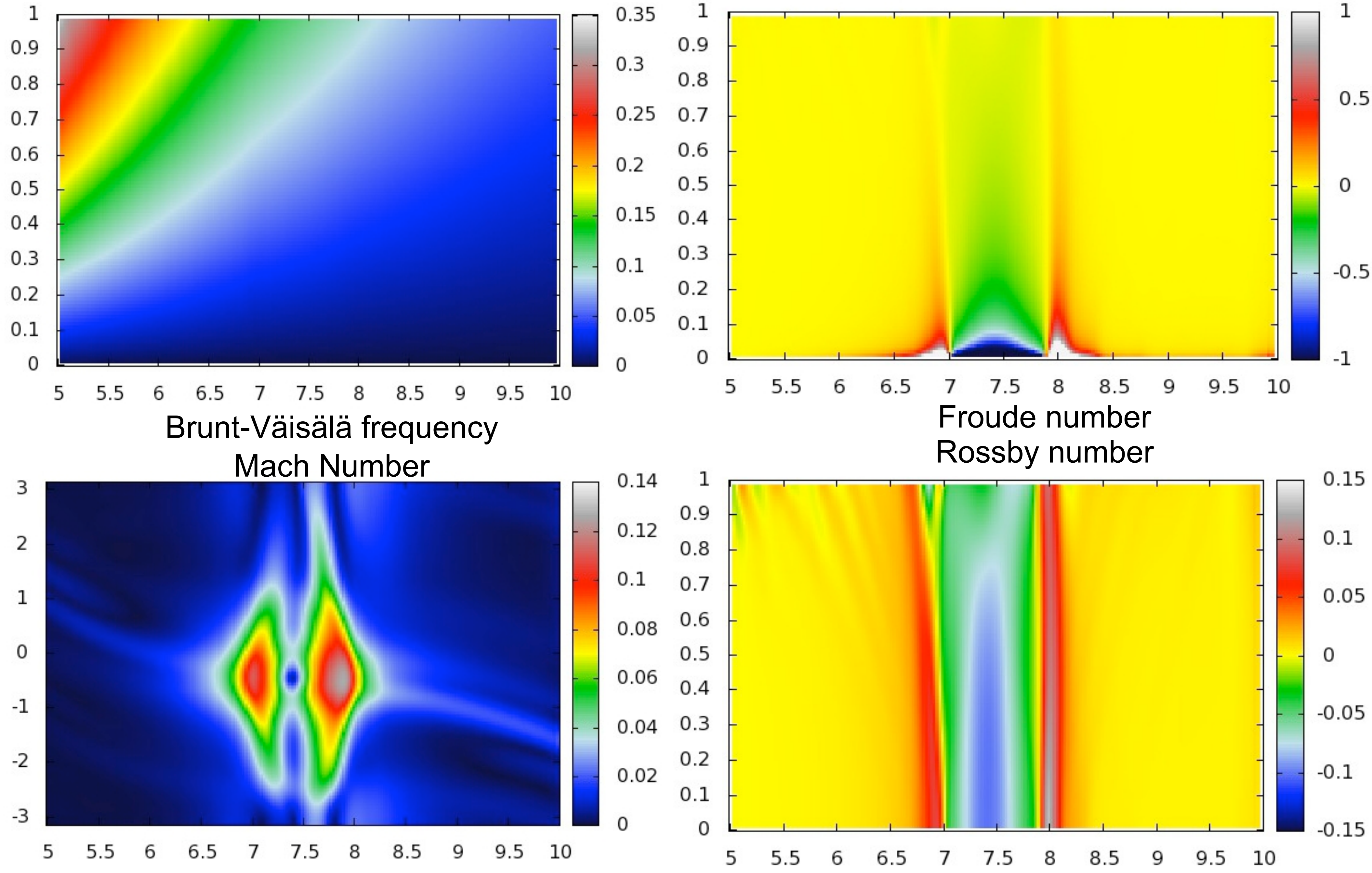
[redacted]

- After the relaxation phase, the migration rate is constant like in 2D
- The migration rate $6,8.10^{-2}$ AU / 1000 years or 0,126 AU/ 100 rotations (consistent with the 2D result)

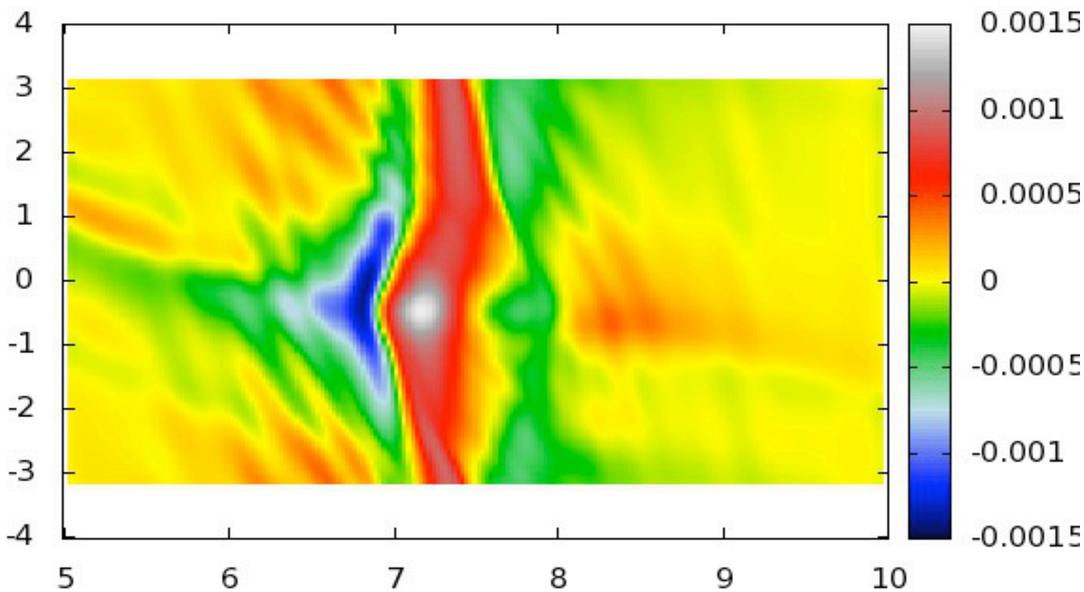
Conclusion

- We confirm that Rossby instability occurs in 3D as in 2D.
- Formed vortices are columnar quasi-steady structures that slowly migrate toward the central star
- A simple vortex model obtained as an approximate solution of the steady state fluid equations is found to relaxes to such vortices.

$$N=\sqrt{-\frac{g_z}{\rho}\frac{\partial\rho}{\partial z}} \qquad Fr=\frac{\omega_z}{2N} \qquad Ro=\frac{\omega_z}{2\Omega}$$

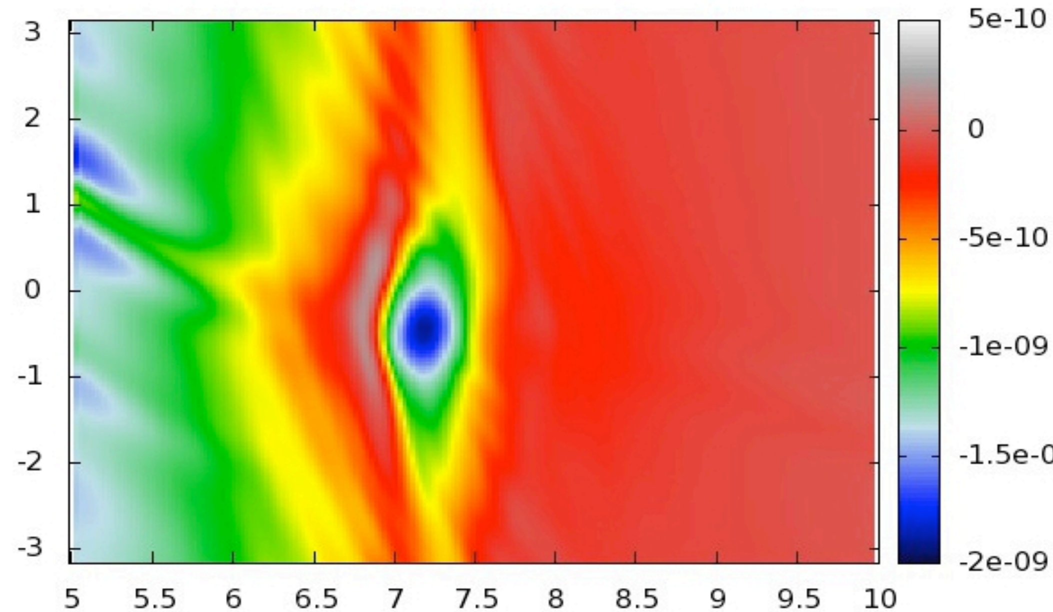


In the radial direction

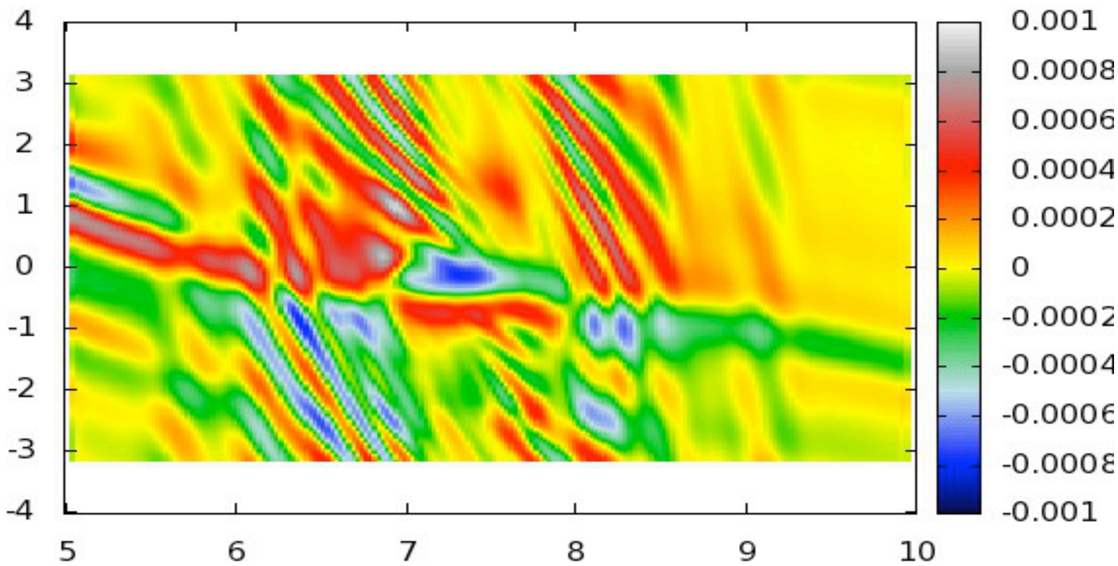


Vorticity

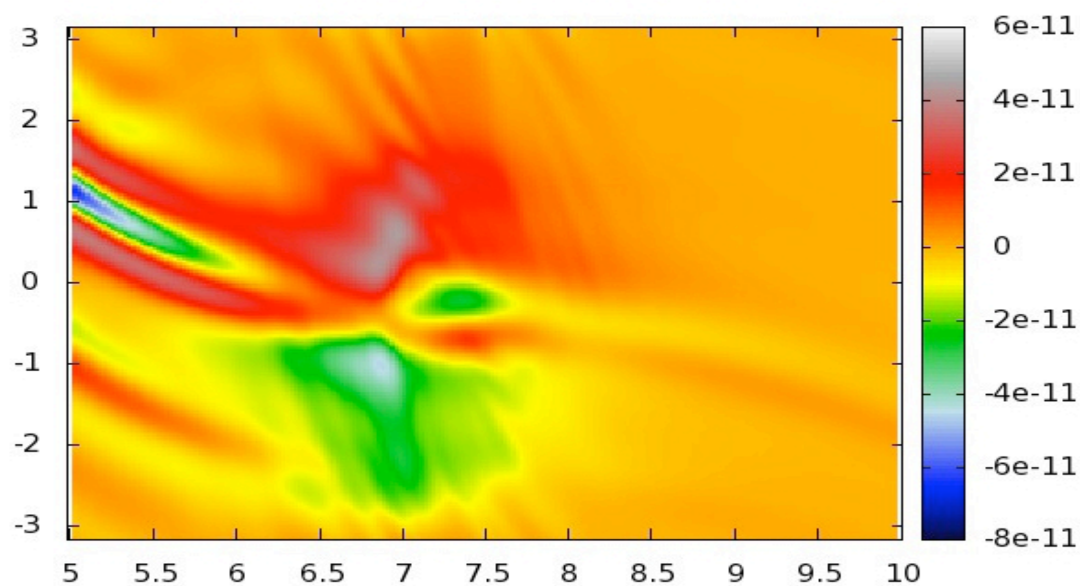
baroclinic term



In the azimuthal direction

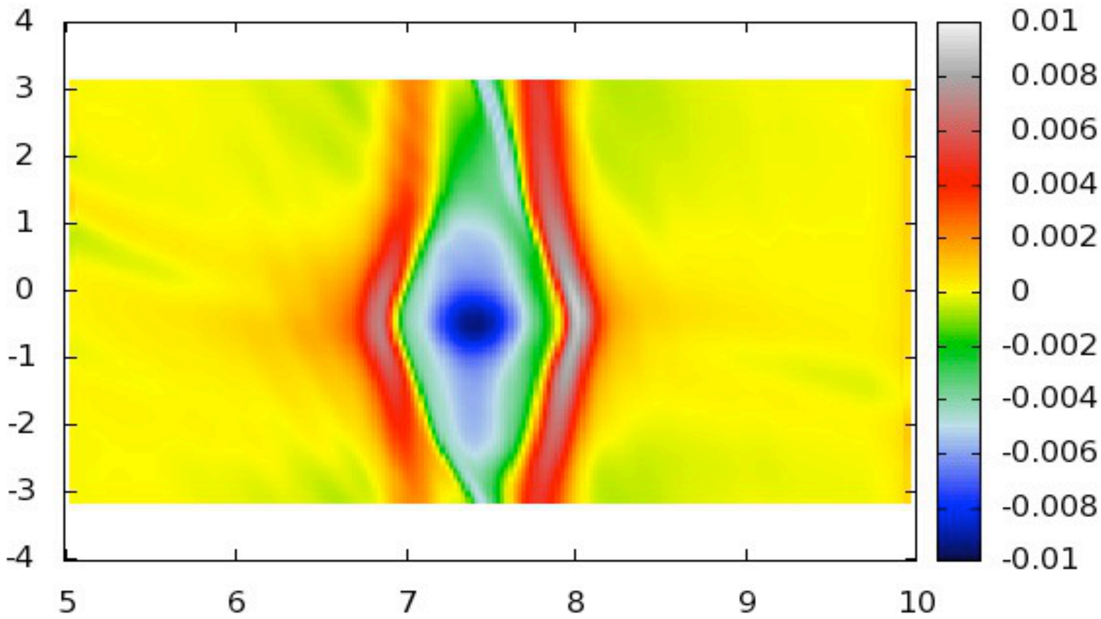


Vorticity



Baroclinic term

In the vertical direction



vorticity

Baroclinic term