Numerical simulation of compressible Baroclinic instability

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Summary

- Introduction
- The role of stratification
- Cooling time versus aspect ratio
- Baroclinic instability with thermal diffusivity
- 3D simulation
- Conclusion

The baroclinic Instability

- The baroclinic instability originates from the production of vorticity by the baroclinic term :

$$\vec{\nabla}\rho \wedge \vec{\nabla} P$$

- In the context of protoplanetary disk it is a way to obtain vortices which may be able to concentrate solid material.

Solved equations

$$\frac{\partial \sigma}{\partial t}+\frac{1}{r}\frac{\partial r \sigma u}{\partial r}+\frac{1}{r}\frac{\partial \sigma v}{\partial \theta}=0$$

$$\frac{\partial \sigma u}{\partial t}+\frac{1}{r}\frac{\partial r\left(\sigma u^2+P\right)}{\partial r}+\frac{1}{r}\frac{\partial \sigma uv}{\partial \theta}=\frac{\sigma v^2}{r}-\frac{\sigma GM}{r^2}+\frac{P}{r}$$

$$\frac{\partial \sigma v}{\partial t}+\frac{1}{r}\frac{\partial r \sigma uv}{\partial r}+\frac{1}{r}\frac{\partial \sigma v^2+P}{\partial \theta}=-\frac{\sigma uv}{r}$$

$$\frac{\partial \sigma e}{\partial t}+\frac{1}{r}\frac{\partial r\left(\sigma e+P\right)u}{\partial r}+\frac{1}{r}\frac{\partial\left(\sigma e+P\right)v}{\partial \theta}=-\frac{\sigma uGM}{r^2}+Q$$

$$\sigma e=\frac{P}{\gamma-1}+\frac{1}{2}\sigma\left(u^2+v^2\right) \qquad P=\sigma RT$$

Two dimensional Euler equations in cylindrical coordinates for a perfect gas in a central potential with cooling rate Q.

Initial condition

- Equilibrium state

$$\sigma_e(r)=\sigma_0\left(\frac{r}{r_0}\right)^{-p}$$

$$T_e(r)=T_0\left(\frac{r}{r_0}\right)^{-q}$$

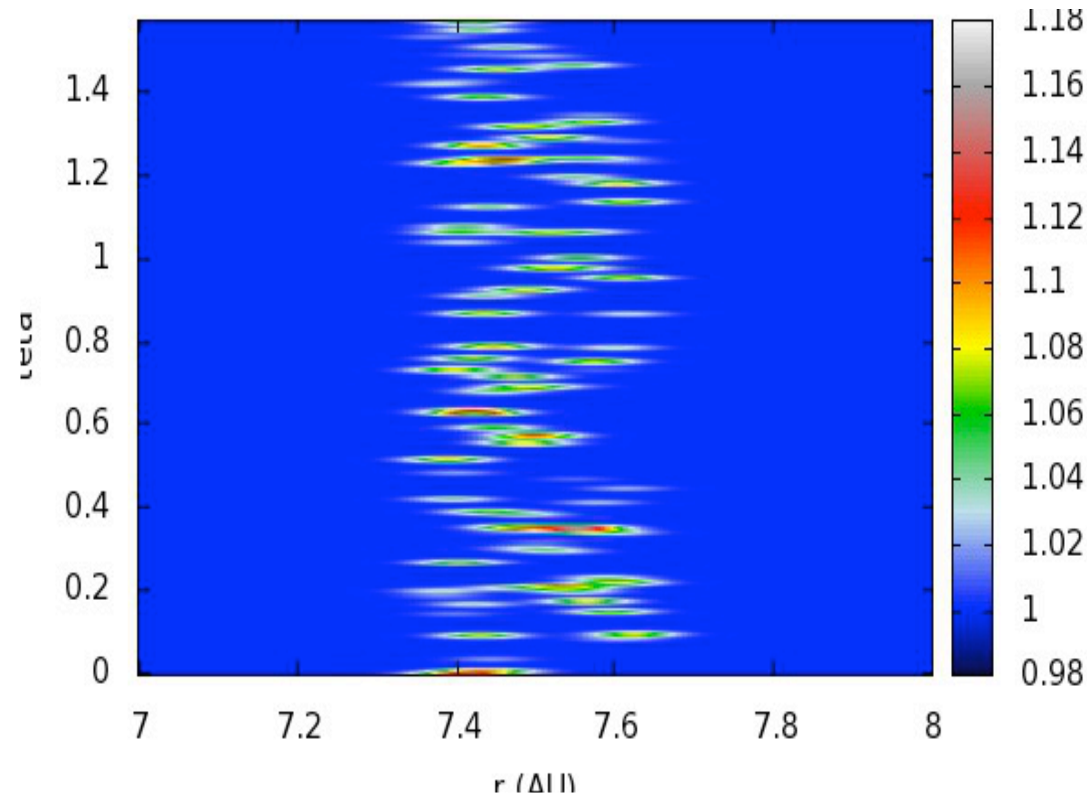
$$P_e(r)=\sigma_0 R T_0\left(\frac{r}{r_0}\right)^{-(p+q)}$$

$$u_e=0$$

$$v_e(r)=\sqrt{\frac{GM}{r}+\frac{r}{\sigma_e}\frac{\partial P_e}{\partial r}}$$

- Perturbed state :

small density bump around 7.5 Au



Heat transfert

Two cases will be considered :

- Heat transfert is due to thermal relaxation :

$$Q = \sigma C_v \frac{T - T_e}{\tau}$$

where Te is the equilibirum temperature and τ is the cooling time

- Heat transfert is due to thermal diffusivity in the gas :

$$Q = \frac{1}{Pe} \Delta \left(T - T_e \right)$$

where Δ is the laplacian

Schwarzschild stability criterion

- Brunt-Väisälä frequency :

$$N^2 = -\frac{1}{\gamma\rho}\frac{\partial P}{\partial r}\frac{\partial}{\partial r}\ln\left(\frac{P}{\rho^\gamma}\right)$$

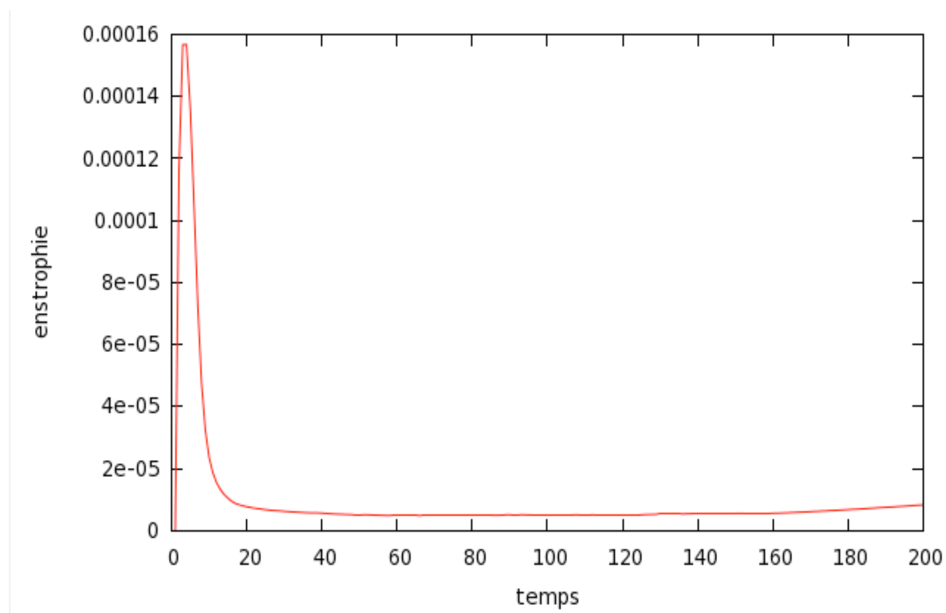
$$N^2 = \frac{p+q}{\gamma r^2}\frac{P}{\rho}\left(\left(\gamma-1\right)p-q\right)$$

- Stratification of the disk:
  - If $p>q/(\gamma-1)$ ==> $N^2>0$ ==> Stable
  - If $p<q/(\gamma-1)$ ==> $N^2<0$ ==> Unstable

Stable disk ($N^2>0$) with thermal relaxation

- P=1.5 and q=0.5

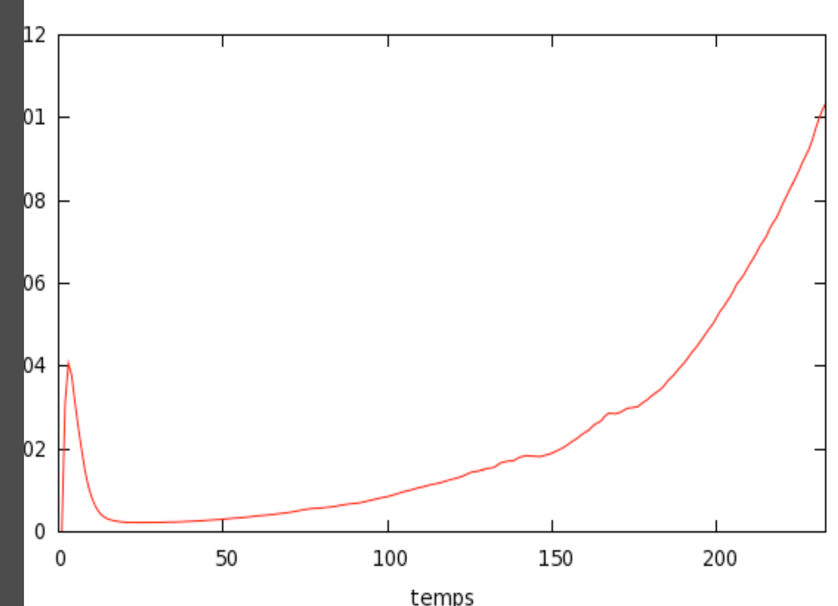
Vortices form and merge but cannot grow



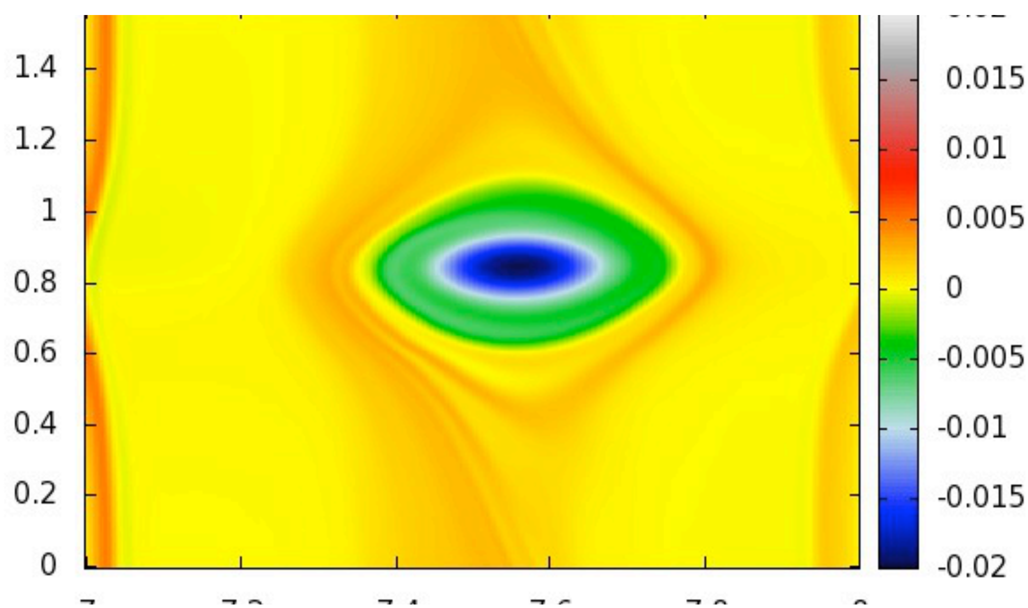
Unstable disk ($N^2<0$) with thermal relaxation

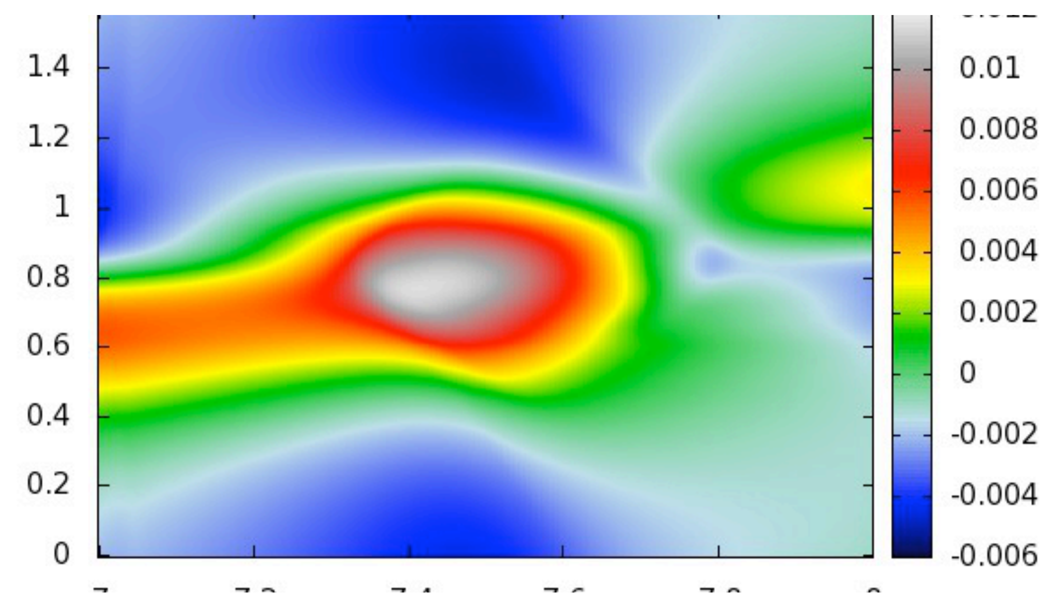
- p=1 and q=0.5

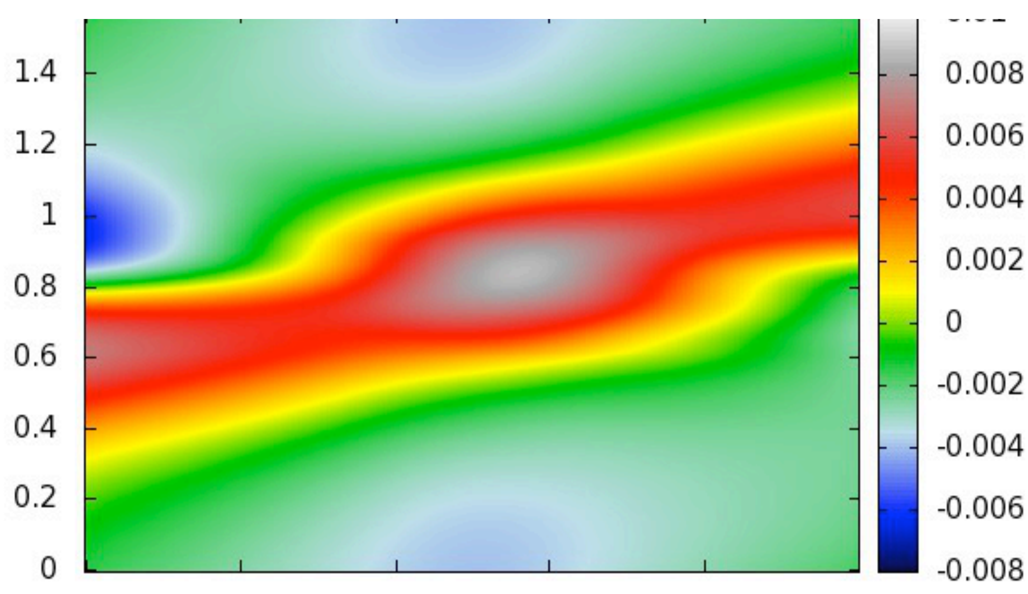
Vortices merge and grow

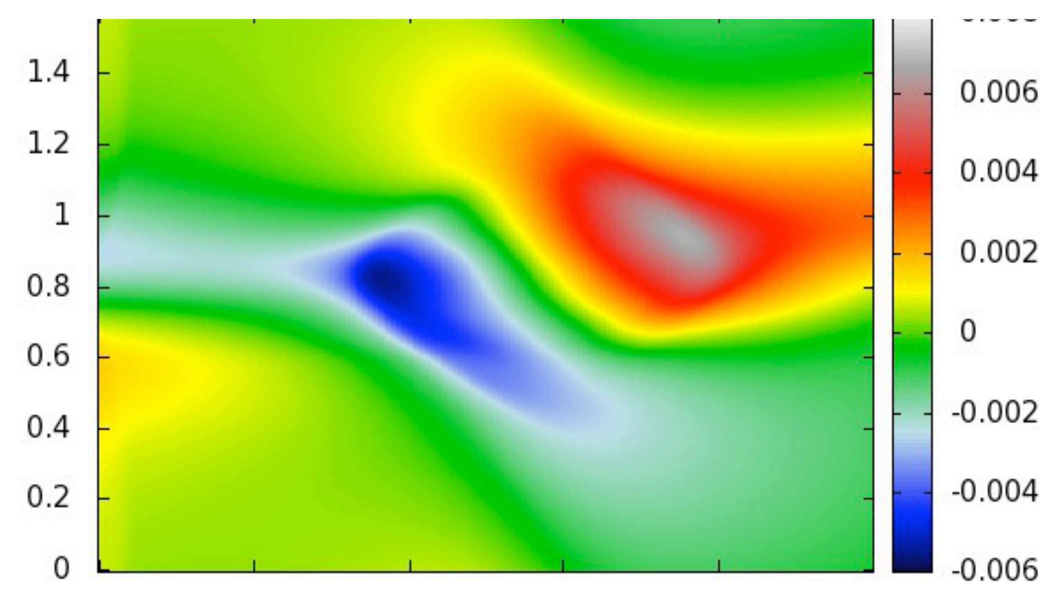


Structure of the formed vortex



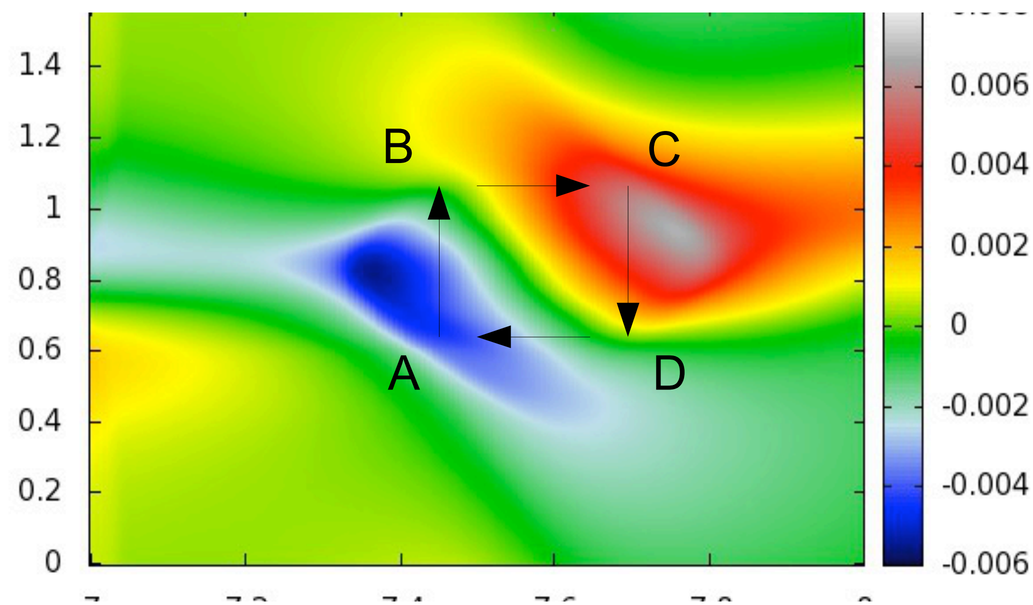






Instability mechanism

- At point A : $T<T_e$
- Along AB : T increases due to thermal relaxation so σ decreases due to thermal dilatation
- At point B : as σ is smaller than density equilibium, gravity is smaller than pressure gradient
- Along BC : a positive radial force is applied on fluid particles
- At point C : $T>T_0$
- Along CD : T decreases et σ increases
- At point D : gravity is stronger than pressure gradient
- Along DA : a negative radial force is applied on the fluid particles



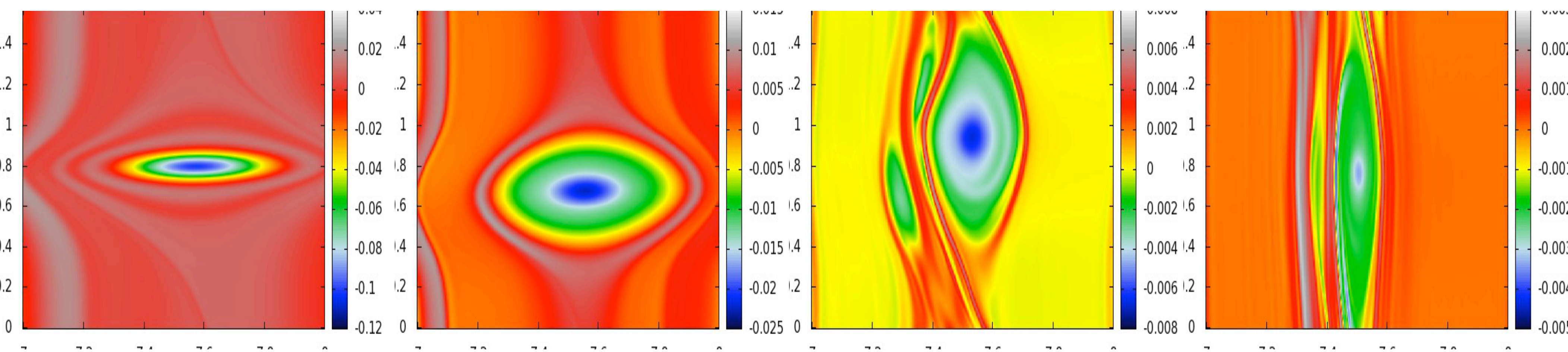
> Fluid particles are accelerated along AB and CD

> Fluid particles are thermalized along AB and CD

Cooling time versus aspect ratio

Cooling time $\tau\Omega_0$

12.10^3 12.10^4 12.10^5 12.10^6

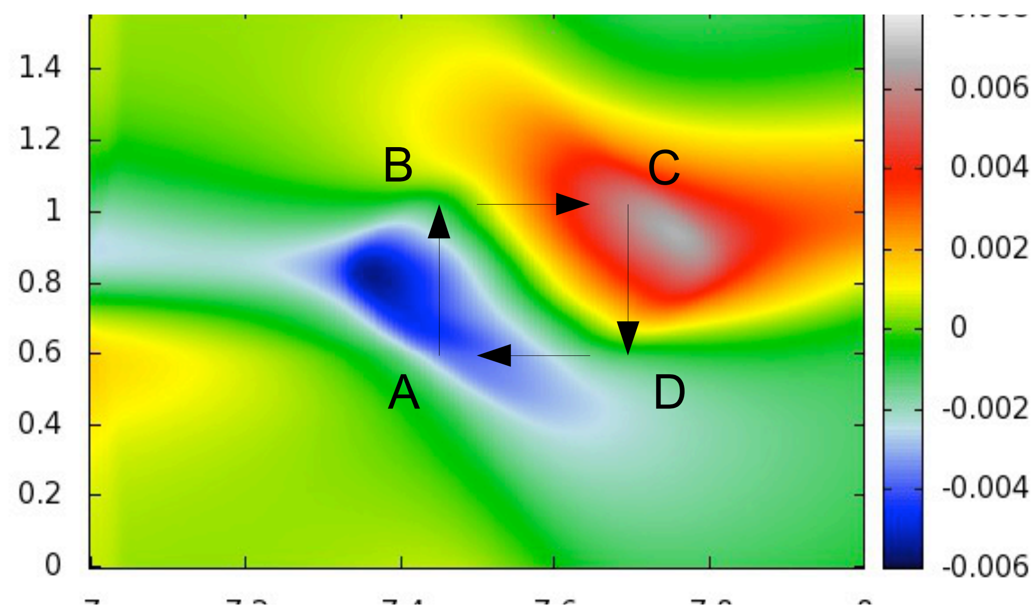


Aspect ratio

2 6 20 50

Aspect ratio

- Fluid particules are thermalized along AB and CD
- When cooling is slow, fluid needs a long time to be thermalized ; so, AB and CD are longer than BC and AD
- When cooling is fast, fluid is quickly thermalized ; so, AB and CD are shorter than BC an AD.

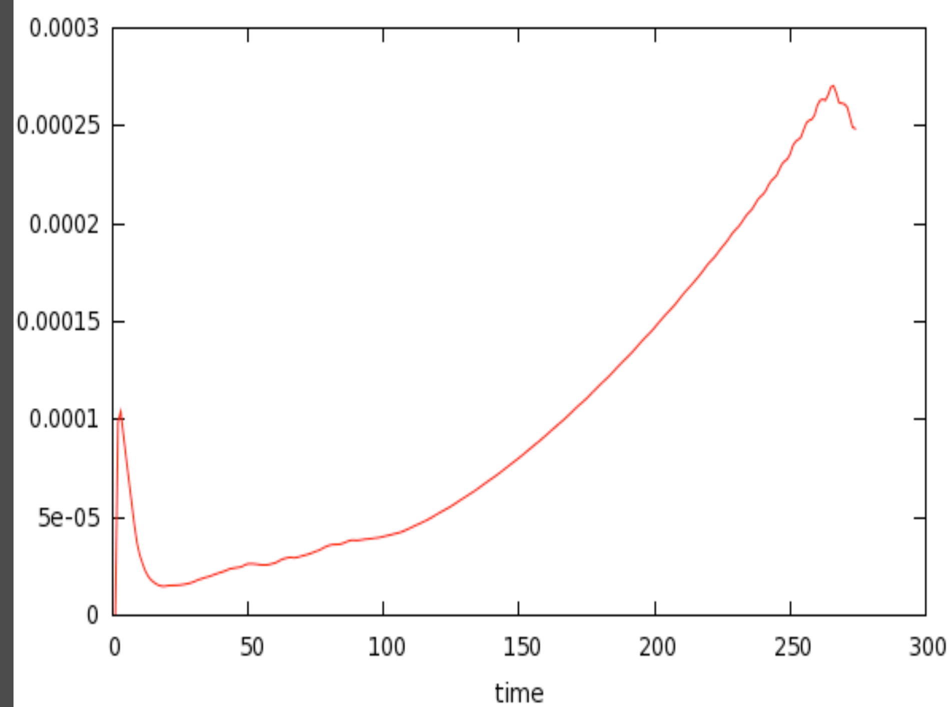


In consequence :

the longer the cooling time (τ), the greater the aspect ratio

Baroclinic instability with thermal diffusivity

- Formation of hollow vortex which is destabilized when it size increases
- Growth of enstrophy : production of vorticity

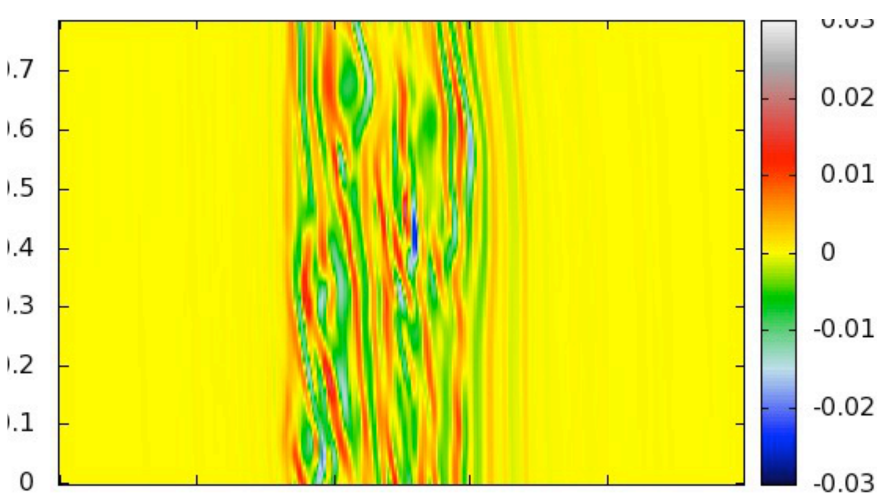


3D baroclinic instability

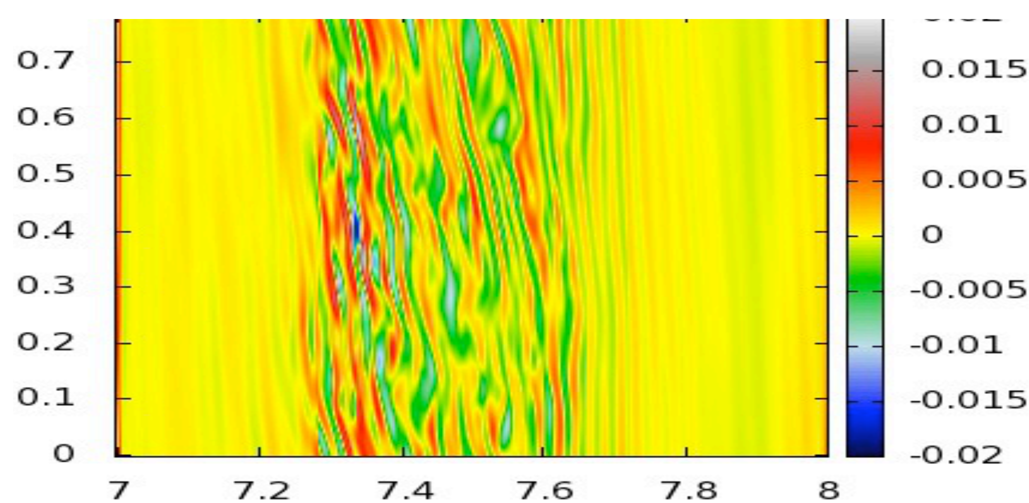
– 3D simulations begin in a similar way as 2D ones:
small vortices are formed

– Vortices do not survive :
they are distroyed and vanish

Vorticity after 30 roations :



2D simulation



Horizontal section in 3D simulation

3D Mechanism

- Each baroclinic vortex is accompagnied by an over-pressure centered on the vortex
- In the horizontal direction :
  the pressure gradient is balanced by Coriolis forces
- In the vertical direction :
  the pressure gradient is unbalanced
  and the vortex is strechted out
- The elliptical instability may also play a non negligible role in the vortex destruction

Conclusion

- 2D baroclinic instability leads to various vortices that can grow and became very large.
- These vortices are powered by the instability as long as the instability conditions are available.
- Different kind of vortices are found for different heat transfert
- However, it is not yet clear whether such vortices can also exist in 3D.