Thermal instability induced by the gas dust temperature difference

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1.Introduction

We carry out linear stability analysis of gas-dust two phase fluid.

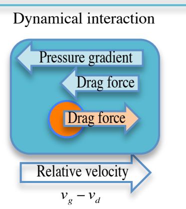
In protoplanetary disks, there can be regions in which gas temperature is higher than the dust temperature. Behind the shocked regions is a possible candidate [e.g., 1].

In such situations the dust acts as coolant for the gas because the gas temperature is higher.

We focus on the cooling effect as a trigger of the thermal instability. It may lead to the gas condensation and further dust accumulation by drag force. Although actual situations are not steady, we consider a steady state back ground to understand the nature of the instability clearer.

2.Model

Gas, Dust continuity equations +



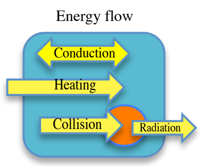
Equations of motion

$$\rho_g \frac{dv_g}{dt} = -\frac{\partial p_g}{\partial x} - \frac{\rho_g (v_g - v_d)}{\tau_{g,s}} \quad : \text{Gas}$$

Drag force

$$\rho_d \frac{dv_d}{dt} = \frac{\rho_d (v_g - v_d)}{\tau_{d,s}} \quad : \text{Dust}$$

Drag force



Equations of energy

$$c_g \rho_g T_g \frac{dT_g}{dt} - c_s^2 \frac{d\rho_g}{dt} = \Gamma(T_g, \rho_g) - \frac{c_g \rho_g (T_g - T_d)}{\tau_{g,t}} + K \frac{\partial^2 T_g}{\partial x^2} + \frac{\rho_g v_g (v_g - v_d)}{\tau_{g,s}} \quad : \text{Gas}$$

Heating   Collision   Conduction

$$c_d \rho_d T_d \frac{dT_d}{dt} = \frac{c_d \rho_d (T_g - T_d)}{\tau_{d,t}} - 4\pi r_d^2 \varepsilon \sigma_{SB} T_d^4 \frac{\rho_d}{m_d} - \frac{\rho_d v_d (v_g - v_d)}{\tau_{d,s}} \quad : \text{Dust}$$

Collision   Radiation

Ideal fluid

$$p_g = c_s^2 \rho_g$$

Time scales

$$\tau_{g,s} = \frac{r_d \rho_{\text{int}}}{\sqrt{\frac{8}{\pi}} c_s \rho_d}, \quad \tau_{d,s} = \frac{\rho_d}{\rho_g} \tau_{g,s}, \quad \tau_{g,t} = \frac{r_d \rho_{\text{int}}}{3\pi \sqrt{\frac{8}{\pi}} c_s \rho_d}, \quad \tau_{d,t} = \frac{c_d \rho_d}{c_g \rho_g} \tau_{g,t}$$

Unperturbed state

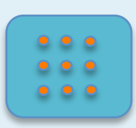
- Static
- Steady
- Uniform
- Infinite

$v_g = v_d = 0$

$T_g > T_d$

$\rho_g = \text{const.}$

$\frac{\rho_d}{\rho_g} = \text{mass ratio}$



Steady condition: $\Gamma(T_g, \rho_g) = \frac{c_g \rho_g (T_g - T_d)}{\tau_{g,t}} = \frac{c_d \rho_d (T_g - T_d)}{\tau_{d,t}} = 4\pi r_d^2 \varepsilon \sigma_{SB} T_d^4 \frac{\rho_d}{m_d}$

Once T_g and T_d are given, ρ_g is determined by the steady condition.
The gas-dust mass ratio can have arbitrary value.

3.Linear stability analysis

We assume that perturbation of each variable is the plane wave as:

$$f_{\text{perturbation}} = f_{\text{amplitude}}(k) \exp(-i\omega_k t + ikx)$$

Thus, when $\text{Im}\,\omega(k) > 0$ the mode is unstable for the given k. The value of $\text{Im}\,\omega(k)$ denotes the linear growth rate of instability.

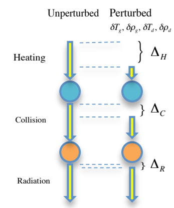
We derive the dispersion relation which is the 5th order polynomial of ω.

We find essentially two thermal instability modes.[2]

One is the condensation mode and the other is the overstable wave mode. We focus on the condensation mode which has possibility of dust accumulation.

References
[1] Iida, A, et al. 2001, Icarus 153:430–450
[2]Field, G.B 1965,Astrophysical Journal, vol. 142, p.531

4.Condensation mode



$$\Delta_H = \delta T_g \frac{\partial \Gamma}{\partial T_g} + \delta \rho_g \frac{\partial \Gamma}{\partial \rho_g}$$

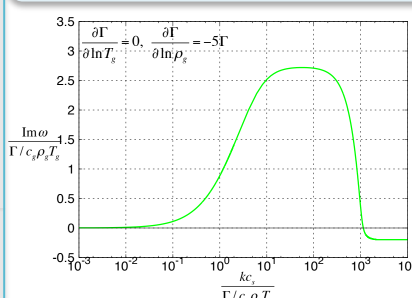
$$\Delta_C = -\delta T_g \frac{c_g \rho_g}{\tau_{g,t}} \left(\frac{3T_g - T_d}{2T_g} \right) + \delta T_d \frac{c_d \rho_d}{\tau_{d,t}} - \delta\rho_g \frac{c_g (T_g - T_d)}{\tau_{g,t}} - \delta\rho_d \frac{c_d (T_g - T_d)}{\tau_{d,t}}$$

$$\Delta_R = -\delta T_d 16\pi r_d^2 \varepsilon \sigma_{SB} T_d^3$$

When $\Delta_H < \Delta_C < \Delta_R < 0$

- Gas energy decrease
  →Perturbation $\delta T_g < 0$ growth
  →Perturbation $\delta\rho_g > 0$ growth
  →$\delta\rho_d > 0$ growth via drag force
- Dust energy decrease
  →Perturbation $\delta T_d < 0$ growth

- The mode is unstable when $|\Delta_H|$ is large enough.
- The thermal instability of gas fluid triggers dynamical flow
  → dust accumulation toward the gas condensation region by drag



$T_g = 2000\text{K}, T_d = 1000\text{K}, \rho_g = \rho_d = 6\times10^{-9}\text{g/cm}^3, \rho_{\text{int}} = 1 \text{ g/cm}^3, r_d = 10^{-2}\text{cm}$
$m_d = \frac{4\pi}{3} r_d^3 \rho_{\text{int}}, K = 10^4 \text{erg/s}\cdot\text{cm}\cdot\text{K}, \varepsilon = 1.0$

The figure shows the dispersion relation of the case in which the heating function has negative dependence of ρ_g.

In the case, when $\delta\rho_g > 0$, $|\Delta_H|$ is large enough. Therefore $\delta\rho_g > 0$ region in the gas lose its energy. That leads to further $\delta\rho_g$ increase and δT_g decrease.

The gas pressure perturbation is

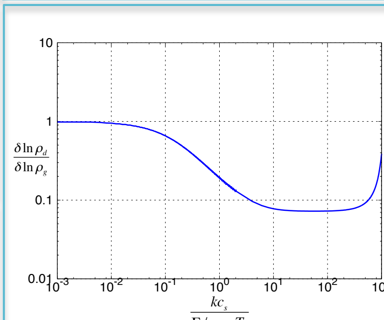
$$\frac{\delta p_g}{p_g} = \frac{\delta T_g}{T_g} + \frac{\delta \rho_g}{\rho_g}.$$

In small k (large scale perturbation) cases, $\delta\rho_g > 0$ growth is much slower than $\delta T_g < 0$ variation because sound crossing time is long. Thus δp_g is negative. Since $\delta\rho_g$ induces the instability, the $\delta\rho_g$ growth rate is limited by $\sim kc_s$.

As k becomes large crossing becomes smaller, and δp_g approaches to 0 because the pressure gradient tends to be averaged. When crossing time is much shorter than the gas energy variation time scale, perturbation is isobaric and the growth rate attains maximum.

But even larger k case, thermal conduction gradually stabilizes the instability.

5.Dust accumulation



The relation between $\delta\rho_g$ and $\delta\rho_d$ can be written as $\frac{\delta \ln \rho_d}{\delta \ln \rho_g} = \frac{1}{(1 + \tau_{d,s} \text{Im}\,\omega)}$

When the growth rate is maximum, the value is 7.5×10^{-2}.

Even though gas and dust are not tightly coupled small fraction of the dust accumulate by drag.

Suppose that dust accumulates by the condensation mode of scale λ and forms a dust small body of radius R and internal density ρ_{int} .

By assuming the tight coupling limit, we obtain an estimation of upper limit of R:

$$R^3 \rho_{\text{int}} \sim \lambda^3 \rho_g,$$

$$R \sim \sqrt[3]{\frac{\lambda^3 \rho_g}{\rho_{\text{int}}}} = 10 \sim 10^2 \text{cm}.$$

$\lambda = 10^4 \sim 10^5 \text{cm}$

$\rho_g = 10^{-9} \text{g/cm}^3$

$\rho_{\text{int}} = 1 \text{ g/cm}^3$

6.Summary

- We carried out linear analysis of gas-dust fluid of steady state back ground.
- Gas thermal instability occurs when cooling of gas by dust exceeds heating after the perturbation.
- Gas density growth has a maximum at the wave number large enough where the perturbation is isobaric. (But not too large)
- The dust accumulates by drag force accompanying with the gas condensation.